Nonlinear Unsteady Aeroelastic Analysis

**Fighter Wing Example**

- **Structural Finite Element Model**
- **Small Disturbance Aerodynamic Model**
Nonlinear Unsteady Aeroelastic Analysis

*Fighter Wing Example*

Mode Shapes and Frequencies

- **Mode 1** (5.62 Hz)
- **Mode 2** (19.28 Hz)
- **Mode 3** (24.12 Hz)
- **Mode 4** (36.83 Hz)

*Kolonay*
Nonlinear Unsteady Aeroelastic Analysis

**Fighter Wing Example**

Flutter Velocity and Flutter Frequency vs. Mach Number

**Graph 1:** Fighter Wing $U_f$ vs Mach Number
- $(\alpha=0.0, \text{Modes } 1, 2, 3, \text{and } 4)$

**Graph 2:** Fighter Wing $\omega_f$ vs Mach Number
- $(\alpha=0.0, \text{Modes } 1, 2, 3, \text{and } 4)$
Nonlinear Unsteady Aeroelastic Analysis

**Fighter Wing Example**

Static Aeroelastic Effects on IRM Flutter

\[ M_\infty = 1.1, \alpha_0 = 1.0^\circ, q_\infty = 50. \text{ psi}, U_\infty = 14736 \text{ in/sec} \]

Static Aeroelastic Deflections

\[ \Delta C_p \text{ rigid} - \Delta C_p \text{ aeroelastic} \]
**Nonlinear Unsteady Aeroelastic Analysis**

**Fighter Wing Example**

Static Aeroelastic Effects on IRM Flutter

\[ M_\infty = 1.1, \, \alpha_0 = 1.0^\circ, \, q_\infty = 50. \, \text{psi} \, , \, U_\infty = 14736 \, \text{in/sec} \]
**Nonlinear Unsteady Aeroelastic Analysis**

**Fighter Wing Example**

Static Aeroelastic Effects on IRM Flutter

### Nonlinear Unsteady Aeroelastic Analysis

<table>
<thead>
<tr>
<th>Method</th>
<th>$M_\infty$</th>
<th>$\alpha_0$ (deg.)</th>
<th>$\rho_\infty$ (slinches/in$^3$)</th>
<th>$q_f$ (psi)</th>
<th>$U_f$ (in/sec)</th>
<th>$\omega_f$ (Hz)</th>
<th>7% Difference in Flutter $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Integration</td>
<td>1.1</td>
<td>1.0</td>
<td>4.9272x10$^{-7}$</td>
<td>53.50</td>
<td>14736.48</td>
<td>20.1</td>
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<tr>
<td>IRM Rigid</td>
<td>1.1</td>
<td>1.0</td>
<td>4.9272x10$^{-7}$</td>
<td>57.24</td>
<td>15243.28</td>
<td>20.38</td>
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<tr>
<td>IRM Aeroelastic</td>
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<td>1.0</td>
<td>4.9272x10$^{-7}$</td>
<td>53.61</td>
<td>14751.37</td>
<td>20.07</td>
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</tbody>
</table>
Nonlinear Unsteady Aeroelastic Sensitivities

Sensitivity Analysis

- Flutter Constraint Definition

\[ \gamma_{lj} \leq \gamma_{j\text{REQ}} \quad j = 1, 2, \ldots, \text{number of velocities} \]
\[ l = 1, 2, \ldots, \text{number of modes} \]

or

\[ Z_j = \frac{\gamma_{lj} - \gamma_{j\text{REQ}}}{\text{GFACT}} \leq 0 \]
Differentiating w.r.t. $v_s$ gives

$$\frac{\partial Z_j}{\partial v_s} = \frac{1}{\text{GFAC}T} \frac{\partial \gamma_{ij}}{\partial v_s} \quad (11)$$

Recalling the definition for $p \equiv k(\gamma + i)$, $\frac{\partial \gamma}{\partial v_s}$ (dropping subscript) can be represented as

(Note: since $\gamma$ is real $\frac{\partial \gamma}{\partial v_s}$ is real)

$$\frac{\partial \gamma}{\partial v_s} = \frac{1}{k} \left[ \frac{\partial}{\partial v_s} \Re(p) - \gamma \frac{\partial}{\partial v_s} \Im(p) \right] \quad (12)$$

The sensitivity of $p$ can be found by differentiating Eq. (9).

Restating Eq. (9) as

$$[\Phi]^T [W] [\Phi] \{q\} = 0 \quad (13)$$

with the adjoint relation

$$\{y\}^T [\Phi]^T [W] [\Phi] = 0 \quad (14)$$

With $\{y\}^T$ the left had eigenvectors and $[W]$ the system matrices in physical coordinates.
Differentiating Eq. (13) w.r.t. $v_s$, pre-multiplying by $\{y\}^T$ and employing Eq (14) gives

$$\{y\}^T \frac{\partial}{\partial v_s} [\Phi]^T [W][\Phi]\{q\} + \{y\}^T [\Phi]^T \frac{\partial}{\partial v_s} [W][\Phi]\{q\} + \{y_h\}^T [\Phi]^T [W] \frac{\partial}{\partial v_s} [\Phi]\{q\} = 0 \quad (15)$$

Assuming $[\Phi]$ is a basis for the system[13],[14] $\frac{\partial}{\partial v_s} [\Phi]$, $\frac{\partial [\Phi]}{\partial v_s}^T$ are 0 (valid for small moves of $v_s$) results in

$$\{y\}^T \left[ 2p \left( \frac{U_\infty}{b} \right)^2 \frac{\partial p}{\partial v_s} [\bar{M}] + \left( \frac{U_\infty}{b} \right)^2 p^2 \frac{\partial}{\partial v_s} \bar{M} + \left( \frac{U_\infty}{b} \right) \frac{\partial p}{\partial v_s} [\bar{B}] + p \left( \frac{U_\infty}{b} \right) \frac{\partial}{\partial v_s} [\bar{B}] + \frac{\partial}{\partial v_s} [\bar{K}] - \frac{1}{2} \rho_\infty U_\infty^2 \frac{\partial}{\partial v_s} [\bar{Q}(\rho)] \right] \{q\} = 0 \quad (16)$$

- For Doublet Lattice $\bar{Q}(\rho)$ depends on only $M, k$.
- For non-linear unsteady $\bar{Q}(\rho)$ depends on initial conditions and the static aeroelastic equilibrium position.
Nonlinear Unsteady Aeroelastic Sensitivities

Let $\bar{Q}(p) = C_R^2 \sum_{r=1}^{n} 2p \left( \frac{C_{r_{ij}}}{2p + b_{r_{ij}}} \right)$ then

$$\frac{\partial}{\partial \upsilon_s} [\bar{Q}(p)] = C_R^2 \left[ \frac{\partial}{\partial p} [\bar{Q}(p)] \left( \frac{\partial p}{\partial \upsilon_s} \right) + \sum_{l=1}^{m} \left( \frac{\partial}{\partial \omega_l} [\bar{Q}(p)] \right) \left( \frac{\partial \omega_l}{\partial \upsilon_s} \right) \right]$$ (17)

Substituting (17) into (16) and solving for $\frac{\partial p}{\partial \upsilon_s}$ yields

$$\frac{\partial p}{\partial \upsilon_s} = \left[ -p^2 \left( \frac{U_\infty}{b} \right)^2 \{y\}^T \frac{\partial}{\partial \upsilon_s} [\bar{M}] \{q\} - p \left( \frac{U_\infty}{b} \right) \{y\}^T \frac{\partial}{\partial \upsilon_s} [\bar{B}] \{q\} ight.$$

$$- \{y\}^T \frac{\partial}{\partial \upsilon_s} [\bar{K}] \{q\} + \frac{1}{2} \rho_\infty U_\infty^2 C_R^2 \{y\}^T \left[ \sum_{l=1}^{m} \left( \frac{\partial}{\partial \omega_l} \bar{Q}(p) \right) \left( \frac{\partial \omega_l}{\partial \upsilon_s} \right) \right] \{q\} \left. \right]$$

$$\left[ 2 p \left( \frac{U_\infty}{b} \right)^2 \{y\}^T [\bar{M}] \{q\} + \left( \frac{U_\infty}{b} \right) \{y\}^T [\bar{B}] \{q\} - \frac{1}{2} \rho_\infty U_\infty^2 C_R^2 \{y\}^T \frac{\partial}{\partial p} [\bar{Q}(p)] \{q\} \right]$$

$\frac{\partial p}{\partial \upsilon_s}$ - in general is a complex quantity which can be used to complete the evaluation of Eq. (12)
Sensitivity of Structural Matrices \( \frac{\partial}{\partial v_s}[\tilde{K}], \frac{\partial}{\partial v_s}[\tilde{M}], \frac{\partial}{\partial v_s}[\tilde{B}] \)

The left and right had eigenvectors can be expressed in physical coordinates as

\[
\{ y_g \} = [\Phi] \{ y \} \\
\{ q_g \} = [\Phi] \{ q \}
\]  \hspace{1cm} (19)

Using (19) the moves the derivatives of the mass, stiffness, and damping matrices into the \( g \)-set

- Reference Eq. 30-32 of Aeroelastic Optimization Notes for \([M], [K], [B]\) sensitivities
**Sensitivity of Structural Natural Frequencies** \( \frac{\partial \omega_l}{\partial v_s} \)

Let \( \omega_l^2 = \lambda_l \), where \( \lambda_l \) \( lth \) undamped eigenvalue of the system excluding external loads.

\[
\frac{\partial \omega_l}{\partial v_s} = \left( \frac{\partial \omega_l}{\partial \lambda_l} \right) \left( \frac{\partial \lambda_l}{\partial v_s} \right) = \frac{1}{2 \omega_l} \left( \frac{\partial \lambda_l}{\partial v_s} \right) \tag{20}
\]

Where \( \frac{\partial \lambda_l}{\partial v_s} \) is determined by differentiating \([ [K] - \lambda_l[M] ] \{ \phi_l \} = 0 \) w.r.t. \( v_s \) yielding

\[
\left[ \frac{\partial}{\partial v_s} [K] - \lambda_l \frac{\partial}{\partial v_s} [M] - \lambda_l \frac{\partial}{\partial v_s} [M] \right] \{ \phi_l \} + [[K] - \lambda_l[M]] \frac{\partial}{\partial v_s} \{ \phi_l \} = 0 \tag{21}
\]

Pre-multiplying by \( \{ \phi \}_T \) and taking advantage of the self-adjoint nature \( \{ \phi_l \}_T [[K] - \lambda_l[M]] = 0 \)

\[
\frac{\partial \lambda_l}{\partial v_s} = \frac{\{ \phi_l \}_T \left[ \frac{\partial}{\partial v_s} [K] - \lambda_l \frac{\partial}{\partial v_s} [M] \right] \{ \phi_l \}}{\{ \phi_l \}_T [M] \{ \phi_l \}} \tag{22}
\]
Nonlinear Unsteady Aeroelastic Sensitivities

**Sensitivity of Aerodynamic Terms**

\[
\frac{\partial}{\partial p}[\tilde{Q}(p)], \frac{\partial [\tilde{Q}(p)]}{\partial \omega_l}
\]

\[
\frac{\partial}{\partial p}[\tilde{Q}(p)] = 2 \sum_{r=1}^{n} \frac{C_{r,ij}}{2p + b_{r,ij}} - \sum_{r=1}^{n} \frac{4pC_{r,ij}}{(2p + b_{r,ij})^2}
\]  \hspace{1cm} (23)

• \(\frac{\partial [\tilde{Q}(p)]}{\partial \omega_l}\) can be found by one of the following methods

  - Differentiating the modal static aeroelastic equation of motion w.r.t. \(\omega_l\) and solving for \(\frac{\partial [\tilde{Q}(p)]}{\partial \omega_l}\)

  - Finite Difference
Computational Cost of Sensitivity for IRM Analysis

- Retaining $m$ structural modes

- $m + 1$ CFD Time integrations to get unsteady aerodynamics into Laplace domain

- Additional time integrations for $\partial [\bar{Q}(p)] / \partial \omega_l$ calculations
  - Finite Difference $m(m + 1)$
    - $m$ - perturbations of each $\omega_l$ to determine static aeroelastic equilibrium
    - $m \times m$ time integrations to perform the indicial response about these perturbed static aeroelastic states.
  - Analytically if possible $m$

If static equilibrium is a rigid solution instead of static aeroelastic

$$\partial [\bar{Q}(p)] / \partial \omega_l = 0$$
Sensitivity Analysis Example

Moderate Aspect Ratio Wing (Goland Wing)

- 6% parabolic airfoil
  
  Modes
  
  - 1.789 hz 1st Bnd.
  - 3.922 hz 1st Trsn.
Nonlinear Unsteady Aeroelastic Sensitivities

Sensitivity Analysis Example

Rectangular Wing Sensitivity Analysis

\( M_\infty = 0.85, \alpha_0 = 0.5^\circ \)

Static Aeroelastic Equilibrium

Rectangular Wing Constraint Values

<table>
<thead>
<tr>
<th>Mode #</th>
<th>800. (fps)</th>
<th>850. (fps)</th>
<th>1000. (fps)</th>
<th>1150. (fps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.01035</td>
<td>-0.00576</td>
<td>0.01742</td>
<td>0.06940</td>
</tr>
<tr>
<td>2</td>
<td>-0.28870</td>
<td>-0.31910</td>
<td>-0.42490</td>
<td>-0.56820</td>
</tr>
</tbody>
</table>
# Sensitivity Analysis Example

## Rectangular Wing Sensitivity Analysis

Rectangular Wing Gradient Terms ($M_\infty = 0.85, \alpha_0 = 0.5^\circ$)

<table>
<thead>
<tr>
<th>$\frac{\partial Z_j}{\partial v_s}$</th>
<th>Analytical</th>
<th>Finite Difference</th>
<th>Analytical</th>
<th>Finite Difference</th>
<th>Finite Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Includes $\frac{\partial [\bar{Q}]}{\partial [\Phi]} \frac{\partial [\Phi]}{\partial v_s}$</td>
<td>No $\frac{\partial [\bar{Q}]}{\partial v_s}$</td>
<td>No $\frac{\partial [\bar{Q}]}{\partial v_s}$</td>
<td>Includes $\frac{\partial [\bar{Q}]}{\partial [\Phi]} \frac{\partial [\Phi]}{\partial v_s}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_1}$ = 800.</td>
<td>-0.5442</td>
<td>-0.5453</td>
<td>-0.5653</td>
<td>-0.5648</td>
<td>-0.5423</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_1}$ = 850.</td>
<td>-0.7262</td>
<td>-0.7267</td>
<td>-0.7455</td>
<td>-0.7445</td>
<td>-0.7220</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_1}$ = 1000.</td>
<td>-1.7099</td>
<td>-1.7091</td>
<td>-1.7345</td>
<td>-1.7332</td>
<td>-1.7104</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_1}$ = 1150.</td>
<td>-4.4704</td>
<td>-4.4656</td>
<td>-4.5144</td>
<td>-4.5110</td>
<td>-4.4948</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_2}$ = 800.</td>
<td>0.0156</td>
<td>0.0149</td>
<td>0.0191</td>
<td>0.0188</td>
<td>0.0166</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_2}$ = 850.</td>
<td>0.0136</td>
<td>0.0130</td>
<td>0.0183</td>
<td>0.0183</td>
<td>0.0146</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_2}$ = 1000.</td>
<td>0.0000</td>
<td>-0.0011</td>
<td>0.0069</td>
<td>0.0069</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\frac{\partial Z_{u_c}}{\partial v_2}$ = 1150.</td>
<td>-0.0532</td>
<td>-0.0549</td>
<td>-0.0456</td>
<td>-0.0465</td>
<td>-0.0431</td>
</tr>
</tbody>
</table>
Nonlinear Unsteady Aeroelastic Sensitivities

Sensitivity Analysis Example

Fighter Wing Sensitivity Analysis

\( M_\infty = 0.93, \alpha_0 = 0.5^\circ \)

Static Aeroelastic Equilibrium

<table>
<thead>
<tr>
<th>Fighter Wing Constraint Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode #</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Kolonay
# Nonlinear Unsteady Aeroelastic Sensitivities

## Sensitivity Analysis Example

**Fighter Wing Sensitivity Analysis**

Fighter wing gradient terms ($M_\infty = 0.93, \alpha_0 = 0.5^\circ$)

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th>Finite Difference</th>
<th>Analytical</th>
<th>Finite Difference</th>
<th>Finite Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial Z_j}{\partial v_s}$</td>
<td>Includes $\frac{\partial [\bar{Q}]}{\partial v_s}$</td>
<td>No $\frac{\partial [\bar{Q}]}{\partial [\Phi]}$</td>
<td>Includes $\frac{\partial [\bar{Q}]}{\partial [\Phi]}$</td>
<td>No $\frac{\partial [\bar{Q}]}{\partial [\Phi]}$</td>
<td>Includes $\frac{\partial [\bar{Q}]}{\partial [\Phi]}$</td>
</tr>
<tr>
<td>$\frac{\partial Z_{\bar{v}<em>1}}{\partial v</em>{16}}$</td>
<td>$-0.1742$</td>
<td>$-0.182$</td>
<td>$-0.1665$</td>
<td>$-0.1875$</td>
<td>$-0.1725$</td>
</tr>
<tr>
<td>$\frac{\partial Z_{\bar{v}<em>2}}{\partial v</em>{16}}$</td>
<td>$-0.3170$</td>
<td>$-0.3424$</td>
<td>$-0.3191$</td>
<td>$-0.3390$</td>
<td>$-0.320$</td>
</tr>
<tr>
<td>$\frac{\partial Z_{\bar{v}<em>3}}{\partial v</em>{16}}$</td>
<td>$-0.5675$</td>
<td>$-0.5895$</td>
<td>$-0.5654$</td>
<td>$-0.5960$</td>
<td>$-0.550$</td>
</tr>
<tr>
<td>$\frac{\partial Z_{\bar{v}<em>4}}{\partial v</em>{16}}$</td>
<td>$-0.8418$</td>
<td>$-0.8600$</td>
<td>$-0.8380$</td>
<td>$-0.8600$</td>
<td>$-0.8150$</td>
</tr>
</tbody>
</table>

*For cases tested Non-linear term in sensitivity found negligible*
Nonlinear Unsteady Aeroelastic Optimization

Rectangular Wing Design Example

\( M_\infty = 0.85, \alpha_0=0.5^\circ, U_\infty = 948. \text{ ft/sec}, \) 20\% Increase in \( U_f \)

**Linear**

Initial Design

\( U_f = 987. \text{ ft/sec} \)
\( \omega_f = 2.10 \text{ Hz} \)
\( \rho_\infty = 2.465\text{E-4 slugs/ft}^3 \)
\( v_1 = 0.2590 \text{ ft}, v_2 = 0.9092 \text{ ft} \)

Final Design (7 iterations 0.71 hr YMP)

\( U_f = 1184. \text{ ft/sec} \)
\( \omega_f = 2.27 \text{ Hz} \)
\( v_1 = 0.2711 \text{ ft}, v_2 = 0.9275 \text{ ft} \)

(objective 6.7\% greater than initial)

**Nonlinear**

Initial Design

\( U_f = 899. \text{ ft/sec} \)
\( \omega_f = 2.07 \text{ Hz} \)
\( \rho_\infty = 1.910\text{E-4 slugs/ft}^3 \)
\( v_1 = 0.2590 \text{ ft}, v_2 = 0.9092 \text{ ft} \)

Final Design (3 iterations 0.36 hr YMP)

\( U_f = 1078. \text{ ft/sec} \)
\( \omega_f = 2.17 \text{ Hz} \)
\( v_1 = 0.2642 \text{ ft}, v_2 = 0.9161 \text{ ft} \)

(objective 2.7\% greater than initial)
Nonlinear Unsteady Aeroelastic Optimization

Rectangular Wing Design Example

$M_\infty = 0.85$, $\alpha_0 = 0.5^\circ$, 20% Increase in $U_f$

![Graph showing $C_p$, $C_U$, $C_L$, and $dC_p$ for nonlinear and linear aeroelastic optimization at 8% span.](image)
Nonlinear Unsteady Aeroelastic Optimization

Rectangular Wing Design Example

\( M_\infty = 0.85, \alpha_0 = 0.5^\circ, 20\% \text{ Increase in } U_f \)
Nonlinear Unsteady Aeroelastic Optimization

Fighter Wing Design Example

Fighter Wing Post Element IDs
Nonlinear Unsteady Aeroelastic Optimization

Fighter Wing Design Example

Fighter Wing Shear Panel Element IDs
Nonlinear Unsteady Aeroelastic Optimization

Fighter Wing Design Example

Fighter Wing Skin Element IDs
## Fighter Wing Design Variables

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>CROD 101 - 131</td>
<td>0.05 in(^2)</td>
<td>0.737</td>
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<tr>
<td>2</td>
<td>CSHEAR 309, 310, 312, 314, 316, 318, 321, 324, 328, 332</td>
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<td>3</td>
<td>CSHEAR 311, 313, 315, 317, 319, 322, 325, 329, 333</td>
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<tr>
<td>6</td>
<td>CROD 134 - 153</td>
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<td>7</td>
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<tr>
<td>9</td>
<td>CROD 204 - 209</td>
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<tr>
<td>10</td>
<td>CSHEAR 340 - 361</td>
<td>0.08 in</td>
<td>7.532</td>
<td>1.00</td>
<td>0.25</td>
<td>1.50</td>
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<tr>
<td>11</td>
<td>CTRMEM 1, 2</td>
<td>0.25 in</td>
<td>3.755</td>
<td>1.00</td>
<td>0.10</td>
<td>1.50</td>
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<tr>
<td>12</td>
<td>CTRMEM 23, 24</td>
<td>0.188 in</td>
<td>2.046</td>
<td>1.00</td>
<td>0.10</td>
<td>1.50</td>
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<tr>
<td>13</td>
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<td>14</td>
<td>CTRMEM 55, 56</td>
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<tr>
<td>15</td>
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<tr>
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<td>CQDMEM1 25 - 42</td>
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<td>1.50</td>
</tr>
<tr>
<td>17</td>
<td>CQDMEM1 45 - 54</td>
<td>0.08 in</td>
<td>26.701</td>
<td>1.00</td>
<td>0.10</td>
<td>1.50</td>
</tr>
<tr>
<td>18</td>
<td>CQDMEM1 57 - 62</td>
<td>0.04 in</td>
<td>4.578</td>
<td>1.00</td>
<td>0.10</td>
<td>1.50</td>
</tr>
<tr>
<td>19</td>
<td>CSHEAR 300, 336</td>
<td>0.135 in</td>
<td>2.833</td>
<td>1.00</td>
<td>0.25</td>
<td>1.50</td>
</tr>
<tr>
<td>20</td>
<td>CSHEAR 301 - 304, 337</td>
<td>0.12 in</td>
<td>7.405</td>
<td>1.00</td>
<td>0.25</td>
<td>1.50</td>
</tr>
<tr>
<td>21</td>
<td>CSHEAR 305, 306, 338</td>
<td>0.09 in</td>
<td>3.329</td>
<td>1.00</td>
<td>0.25</td>
<td>1.50</td>
</tr>
<tr>
<td>22</td>
<td>CSHEAR 307, 308, 339</td>
<td>0.05 in</td>
<td>1.0309</td>
<td>1.00</td>
<td>0.25</td>
<td>1.50</td>
</tr>
<tr>
<td>23</td>
<td>CROD 132, 133, 154, 155</td>
<td>1.75 in(^2)</td>
<td>12.240</td>
<td>1.00</td>
<td>0.10</td>
<td>1.25</td>
</tr>
<tr>
<td>24</td>
<td>CROD 156 - 163, 182, 183</td>
<td>1.35 in(^2)</td>
<td>32.152</td>
<td>1.00</td>
<td>0.10</td>
<td>1.25</td>
</tr>
<tr>
<td>25</td>
<td>CROD 184 - 187, 198, 199</td>
<td>1.05 in(^2)</td>
<td>19.119</td>
<td>1.00</td>
<td>0.10</td>
<td>1.25</td>
</tr>
<tr>
<td>26</td>
<td>CROD 200 - 203, 210, 211</td>
<td>0.88 in(^2)</td>
<td>10.924</td>
<td>1.00</td>
<td>0.10</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Nonlinear Unsteady Aeroelastic Optimization

Fighter Wing Design Example 1

\[ M_\infty = 0.93, \ \alpha_0=0.5^\circ, \ U_\infty = 12459. \ \text{in/\text{sec}, \ 20\% \ Increase \ in} \ U_f \]

**Linear**

Initial Design

\[ U_f = 12492.07 \ \text{in/\text{sec}} \]
\[ \omega_f = 19.72 \ \text{Hz} \]
\[ \rho_\infty = 6.764\text{E-7 \ slinches/in}^3 \]
\[ \text{designed weight} = 497.69 \ \text{lbs} \]

Final Design (15 iterations)

\[ U_f = 14990. \ \text{in/\text{sec}} \]
\[ \omega_f = 19.94 \ \text{Hz} \]
\[ \rho_\infty = 6.764\text{E-7 \ slinches/in}^3 \]
\[ \text{designed weight} = 506.98 \ \text{lbs} \]

**Nonlinear**

Initial Design

\[ U_f = 12137.04 \ \text{in/\text{sec}} \]
\[ \omega_f = 19.64 \ \text{Hz} \]
\[ \rho_\infty = 6.571\text{E-7 \ slinches/in}^3 \]
\[ \text{designed weight} = 497.69 \ \text{lbs} \]

Final Design (15 iterations 4.8 hr YMP)

\[ U_f = 14565.46 \ \text{in/\text{sec}} \]
\[ \omega_f = 17.64 \ \text{Hz} \]
\[ \rho_\infty = 6.571\text{E-7 \ slinches/in}^3 \]
\[ \text{designed weight} = 426.91 \]
\[ (14.22\% \ decrease \ from \ initial) \]
Nonlinear Unsteady Aeroelastic Optimization

**Fighter Wing Design Example 1**

\[ M_\infty = 0.93, \ \alpha_0 = 0.5^\circ, \ 20\% \ \text{Increase in} \ U_f \]

\[ \rho_f \ \text{Linear} = 6.764 \times 10^{-7} \ \text{slinches/in}^3 \]

\[ \rho_f \ \text{Nonlinear} = 6.571 \times 10^{-7} \ \text{slinches/in}^3 \]
Nonlinear Unsteady Aeroelastic Optimization

**Fighter Wing Design Example 1**

\[ M_\infty = 0.93, \alpha_0 = 0.5^\circ, \text{ 20\% Increase in } U_f \]
Nonlinear Unsteady Aeroelastic Optimization

**Fighter Wing Design Example 1**

\[ M_\infty = 0.93, \alpha_0 = 0.5^\circ, 20\% \text{ Increase in } U_f \]

Linear/Nonlinear % Diff., \( \Delta w \)

\[ \Delta v_1 = -0.05 \text{ lbs} \]
\[ \Delta v_6 = +1.69 \text{ lbs} \]
\[ \Delta v_7 = +0.65 \text{ lbs} \]
\[ \Delta v_8 = +0.00 \text{ lbs} \]
\[ \Delta v_9 = +0.00 \text{ lbs} \]
\[ \Delta v_{23} = +0.00 \text{ lbs} \]
\[ \Delta v_{24} = +0.20 \text{ lbs} \]
\[ \Delta v_{25} = +0.00 \text{ lbs} \]
\[ \Delta v_{26} = +0.00 \text{ lbs} \]

Kolonay
Nonlinear Unsteady Aeroelastic Optimization

**Fighter Wing Design Example 1**

\[ M_\infty = 0.93, \alpha_0=0.5^\circ, \text{20\% Increase in } U_f \]

Linear/Nonlinear % Diff., \( \Delta w \)

\[ \Delta v_2 = +0.00 \text{ lbs} \]
\[ \Delta v_3 = +0.00 \text{ lbs} \]
\[ \Delta v_4 = -3.58 \text{ lbs} \]
\[ \Delta v_5 = +0.00 \text{ lbs} \]
\[ \Delta v_{10} = +0.00 \text{ lbs} \]
\[ \Delta v_{19} = +0.00 \text{ lbs} \]
\[ \Delta v_{20} = +0.00 \text{ lbs} \]
\[ \Delta v_{21} = -1.77 \text{ lbs} \]
\[ \Delta v_{22} = +0.00 \text{ lbs} \]
Nonlinear Unsteady Aeroelastic Optimization

**Fighter Wing Design Example 1**

\[ M_\infty = 0.93, \, \alpha_0 = 0.5^\circ, \, 20\% \text{ Increase in } U_f \]

Linear/Nonlinear % Diff., \( \Delta w \)

\( \Delta v_{11} = -0.05 \text{ lbs} \)
\( \Delta v_{12} = -0.01 \text{ lbs} \)
\( \Delta v_{13} = -0.02 \text{ lbs} \)
\( \Delta v_{14} = +0.00 \text{ lbs} \)
\( \Delta v_{15} = -77.24 \text{ lbs} \)
\( \Delta v_{16} = +0.00 \text{ lbs} \)
\( \Delta v_{17} = +0.00 \text{ lbs} \)
\( \Delta v_{18} = +0.00 \text{ lbs} \)