AE554 Applied Orbital Mechanics

Week 6
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Today's menu

- Orbit Perturbations
  - Non-spherical Earth
  - Lunisolar effects
  - Atmospheric drag
  - Solar radiation pressure

- Osculating elements

- Effects of J2
In an ideal world, everything moves according to two-body problem.

Two body assumptions:
- Point mass; spherically symmetric force field
  - Earth is NOT a sphere!
- Two bodies only
  - Other masses (Sun, Moon etc) $> n$-body problem
- Only force on satellite is gravitational pull
  - Atmospheric Drag
  - Solar Radiation Pressure (and albedo)
Non-spherical Earth

- Squashed from the poles, bulging on the equator
- Equator itself is squashed from the sides
- Local mass variations
- Ocean and atmosphere tides
- Solid Earth tides
  - Effects of ellipticity and of the Coriolis force due to Earth rotation on tidal deformations
- Potential Variations caused by Rotational Deformation (Pole Tides)
  - Generated by the centrifugal effect of polar motion
- Potential Variations caused by Rotational Deformation of Ocean Masses (Ocean Pole Tides)
Geopotential models

- Mass distribution of Earth as series expansion (of sorts)
  - spherical harmonics expansion

\[
U_s(r, \phi, \lambda, t) = \frac{GM_e}{r} C_{00} + \frac{GM_e}{r} \sum_{l=2}^{N_{\text{max}}} \left( \frac{a_e}{r} \right)^l \sum_{m=0}^{l} P_{lm}(\sin \phi) [C_{lm}(t) \cos m\lambda + S_{lm}(t) \sin m\lambda]
\]

- \((\phi, \lambda)\) are geocentric latitude and longitude, respectively.
- \(a_e\) is the Earth radius (part of the package!)
- \(J_n = -C_{n0}\)

- Geopotential model is simply the geopotential coefficients of the expansion!
Spherical harmonics

- Gravitational potential description via superposing spherical harmonics with gravitational coefficients
  - coefficients computed via on-orbit measurements
  - can be interpreted as adding or subtracting ‘bands’ of mass onto a perfectly spherical central body

- **Zonal harmonics**: secular, short-periodic (orbital period) and long periodic (longer than an orbital period) effects

- **Tesseral harmonics**: \(m\)-daily variations and resonance

- **Sectoral harmonics**: subset of tesserals

\[
U = -rac{\mu}{r} \left[ 1 - \sum_{l=2}^{\infty} J_l \left( \frac{R_\oplus}{r} \right)^l P_l(\cos \theta) + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{R_\oplus}{r} \right)^n P_n^m(\cos \theta) J_{nm} \cos m(\varphi - \psi_{nm}) \right]
\]
Geopotential models

- Various models through
  - Satellite tracking (GRACE, CHAMP)
  - Surface Gravimetry
  - Altimetry (Satellite altitude measurements)

- JGM-3, EGM96, GRIM5 etc.
  - “200x200” coefficients exist
  - 70x70 more than adequate for most applications
    - l=70, m=70 E-129 level!!! (precision? really?)
Lunisolar perturbations

- Other bodies exist in space
  - Weak gravitational pull from everything else in space
  - Most important for Earth orbiting satellites are the Sun and the Moon
  - Simple underlying equations: $F = GMm/r^2$
  - Three-body problem “unsolved” unless some assumptions made!
  - Extremely complicated dynamics
  - Conservative force!
Atmospheric drag

- Atmosphere is very thin but it exists up to about 1200km altitude
  - Difficult to calculate the density
  - Extremely difficult to make long term predictions
    - solar flares
    - solar cycles
    - day/night variations

\[ F = \frac{1}{2} \rho C_D A_{\perp} V^2 \]

- semimajor axis “decay”
- eccentricity decrease (aerobraking)
- Various models
Atmospheric drag

Figure 3.4: AlSat semimajor axis decay due to drag (1 Dec 2003 - 8 Jan 2004)
Atmospheric drag

Figure 3.5: AlSat semimajor axis decay due to drag with Solar flare (8 Oct - 8 Nov 2003)
Variation of drag

BilSat

- $C_d = 2.2$ (??)
- mass = 130kg
- $A = 0.36m^2$

BilSat scaled average orbit decay

15/11/2007
Solar radiation pressure

- Photons impinging on satellite
  - absorbed
  - bounced
- Geometry of the satellite
  - TurkSat
  - solar sails?
- Non-conservative!

\[ F_{SR} = p_{SR} C_R A_0 r_\odot \]

- radiation pressure
- solar reflectivity

- Major effect on eccentricity
  - Oscillations on orbital elements
# Orbit perturbations

- **Accuracy requirements**
- **Satellite orbit**
- **Satellite geometry**
- $J_4$ and $(J_2)^2$ are of same order!

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<th>750km</th>
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J₂ Secular Effects

• Secular (y=C.t) and Periodic Effects (y=C.sin(t))

• Secular effects more important (long term)

• J₂ (Equatorial mass band) has two major secular effects:
  – Change in Ω (precession)
  – Change in ω (node shift)
  – Change in M (mean anomaly)

\[
P_2(γ) = \frac{3}{2} \left( γ^2 - \frac{1}{3} \right)
\]

\[
U = -\frac{\mu}{r} \left[ 1 - \sum_{l=2}^{\infty} J_l \left( \frac{R_\oplus}{r} \right)^l P_l(\cos θ) + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{R_\oplus}{r} \right)^n \frac{P^m_n(\cos θ)}{m!} J_{nm} \cos m(\varphi - \psi_{nm}) \right]
\]
J$_2$ Secular Effects - Precession

- Inclinations NOT equal to 0 or 90 deg: equatorial “extra” mass pulls the orbit

- Satellite has very high angular momentum $>>$ fast!

- This results in “precession” of the orbit, rather than a secular drift to a lower inclination

- Right Ascension of Ascending Node (RAAN) changes linearly with time!
$J_2$ Secular Effects - Precession

Angular momentum vector

Extra pull due to $J_2$ over northern hemisphere

Node shift direction

Extra pull due to $J_2$ over southern hemisphere

Equator pass $t=t_2$

Equator pass $t=t_1$
$J_2$ Secular Effects – Node Shift

Rotation of apsides (apogee and perigee)
J₂ Secular Effects - Equations

Secular rate of change of precession

\[ \dot{\Omega}_{\text{secular}} = -\frac{3nR^2 J_2}{2 p^2} \cos(i) \]

Secular rate of apsidal rotation

\[ \dot{\omega}_{\text{secular}} = \frac{3nR^2 J_2}{4 p^2} \left( 4 - 5 \sin^2(i) \right) \]

Secular effects to the rate of change in Mean Anomaly

\[ \dot{M}_0 = -\frac{3nR^2 J_2 \sqrt{1-e^2}}{4 p^2} \left( 3 \sin^2(i) - 2 \right) \]
Sun-synchronous orbits (SSO)

- The Earth covers $360/265.25 = 0.986$ degrees in a solar day.
- If the rate of change of $\Omega$ can be made equal to $0.986$ deg/day, the angle between the orbit plane and the Sun will stay constant!

\[
d(\Omega)/dt = -9.95 \left(\frac{R}{a}\right)^{3.5} \frac{\cos(i)}{(1-e^2)^2} \quad \text{(formulation for deg/day)}
\]

- Correct combination of $(a,e,i)$!

- Negative sign in the eqn: Orbit plane rotates around $-Z$ axis (counter to that of the Earth!)
Sun-synchronous orbits (SSO)

- Orbit plane
- Earth (seen from north pole)
- Earth rotation direction (positive direction)
- Sun

Keplerian case
Sun-synchronous orbits (SSO)

- Sun
- Earth rotation direction (positive direction)
- Orbit plane
- Earth (seen from north pole)
- J2 case with $i < 90$ deg
Sun-synchronous orbits (SSO)

J2 case with proper selection of initial conditions

i > 90deg for correct rotation direction

Orbit plane

Earth (seen from north pole)

Earth rotation direction (positive direction)

Sun
Circular SSO
SSOs – What are they good for?

- SSOs are named after their Locat Time of Ascending Node (LTAN) values (10.30AM)

- Determines when they will cross the equator – always same local time!

- Consistent lighting conditions – ideal for Earth Observation
  - Near-global coverage is a plus!
Repeat groundtrack orbits

- Concept of groundtrack
  - Where nadir vector intersects the Earth surface
- Earth rotates underneath the satellite
  - Uydu yerde bir kez izlediği yolu bir sonraki geçişte izlemez

Int’l Space Station
two consecutive orbits
Repeat groundtrack orbits

- Retracing the groundtrack over a certain period
  - Revisit the same point on the Earth repeatedly
- Osculating elements!
- Anomalistic period: \( P_K = \frac{2\pi}{n} = 2\pi \sqrt{\frac{a^3}{\mu}} \)
  - Perturbed (osculating) Keplerian period
- Ref to Vallado for nodal (node-to-node) period
- Groundtrack shift \((\lambda_s)\) (in radians!) (westward shift!)

\[
\lambda_s = (\omega_\oplus - \dot{\Omega})P_\Omega
\]
Osculating elements

- See files