1. a) Consider two satellites in close proximity and on a near-circular orbit. Derive the equations of relative motion in inertial frame, assuming the satellites move in a spherically symmetric gravitational potential. [15 points]
   b) What is(are) the condition(s) that two satellites stay in close proximity for a long duration, e.g. a year? (No limitations to the orbit shape or the forces acting on the satellite) [5 points]

2. When launching GEO satellites, the launchers are capable of reaching a transfer orbit (called GTO), where they release their payload. The satellites will then have to reach GEO using their on-board propulsion systems. As a systems engineer, you will have to make a first estimate of the $\Delta V$ required for propulsion system sizing.
   a) GTO is simply a highly elliptical orbit with perigee at about 200km altitude and apogee at GEO altitude. Calculate the $\Delta V$ required to reach GEO. [10 points]
   b) The launch site is at Kourou, French Guiana is at a 5 degree latitude. Calculate the $\Delta V$ required to correct this inclination. Calculate the difference between firing at the apogee or the perigee. [6 points]

3. For a satellite in an almost circular low Earth orbit (such as BilSat), the effect of drag is very small, causing a semimajor axis loss of a few kilometres per year. Nevertheless it has to be taken into account for long term orbital analyses as well as design issues such as propellant mass required for orbit maintenance.
   a) Calculate the energy loss of the satellite as a function of time. How is the energy loss related to time? [8 points]
   b) Calculate the semimajor axis decay rate. [5 points]
   c) Calculate the change in the mean anomaly of the satellite. How is this change related to time? [7 points]
   d) Calculate the $\Delta V$ required to counter this semimajor axis loss. [7 points]
   e) Many satellite constellations are made up of a number of satellites sharing an orbital plane with a constant angular separation. In other words, each satellite keeps its angular “slot”. However, unless the satellite platforms are exactly the same, a “differential drag” is unavoidable, where different satellites experience different drag forces. This was the case for BilSat where the other members of the Disaster Monitoring Constellation were identical platforms with a higher drag. Briefly explain why this is a problem. Propose a stationkeeping strategy where the satellite with the different drag coefficient is kept at its “slot” within a mean anomaly error of $\delta M$. (Hint: visualising the problem and drawing figures might be helpful, though no need to derive a full mathematical expression.) [8 points]

4. Answer the following questions using no figures or no formulas. A couple of sentences should be adequate for each.
   a) Explain the difference between an batch estimator and a sequential estimator. [5 points]
   b) What is state in an orbit estimator? What are its elements? What does the covariance matrix signify? [5 points]
   c) Explain the difference between the Kalman Filter and the Extended Kalman Filter. Why is the latter advantageous? [5 points]

5. You are the lead systems engineer of an experimental 80kg small satellite mission for the Air Force. The satellite is going through the final stages of testing and integration, a few months short of shipment to the launch site. The representatives from the Air Force, fresh from a visit to Japanese space facilities, announce at a project review meeting that they heard that the state-of-the-art is centimetre level positional accuracy, calculated on-board, in real-time. In light of this, they would like that their satellite should be equally capable.
   a) Explain how such an accuracy would be possible, bearing in mind that the satellite carries nothing more complicated than a single channel GPS receiver, where the standard PVT (Position, Velocity, Time) output is accurate to 10-20 metres. What would be the properties of such an estimator? [7 points]
   b) Prepare a response to Air Force representatives, discussing the feasibility of this solution. [7 points]