1. Perifocal coordinate system is defined as one where P axis points to perigee, W axis points to the orbit normal and Q axis orients itself according to the right-hand rule.
a) Find the conversion matrix that converts a position vector in PQW frame $\hat{r}_{PQW}$ to a position vector in inertial frame $\hat{r}_{IJK}$. (Use vectors) [8 points]
b) If you were carrying out this transformation via angles, which Keplerian elements would you use? Explain briefly. [10 points]

2. You work as the systems engineer responsible for mission analysis and design at a major satellite company. Following projects are currently under review. Propose orbits and justify your choices briefly; calculate critical orbital elements or give possible ranges/values where applicable.
a) AlHaberiSat: Qatari satellite to broadcast news and provide high speed internet services to home users within the Arabian peninsula. [12 points]
b) ParanoiaSat: A classified country would like to gain global SIGINT\(^1\) capability i.e. every single mobile or satellite phone or similar transmitting device can be listened via your satellite. The receiver antenna on the satellite covers a small area on the earth (roughly a 10 km\(^2\) area). Also note that signal power received by the antenna is proportional to the antenna dish area (which is limited by size and weight considerations) but decreases with the square of the distance. [12 points]

3. Explain briefly what is meant by a *sidereal day* and a *solar day*. Why are the lengths of these days different? Why do we have to apply a correction to sidereal time? [12 points]

4. Given inertial position and velocity vectors $\hat{r}, \hat{v}$ calculate the six Keplerian elements. [18 points]

5. In November 1998, astronomers at Mauna Kea, Hawaii, discovered a small moon orbiting the asteroid (45) Eugenia. This was the first time an asteroidal moon had been discovered by a ground-based telescope. Eugenia's moon has been named (45) Eugenia I PetitPrince, after Empress Eugenia's son, the Prince Imperial.

The moon is much smaller than the 214 km diameter Eugenia, about 13 km in diameter. Petit-Prince's orbit is near-circular, with an orbital radius of 1184 km, taking 4.766 days to complete an orbit around Eugenia.

a) Assuming Petit-Prince to be massless, compute a mass estimate for Eugenia. Compute the density of Eugenia, assuming it is spherical. [12 points]
b) For a measurement error of 0.1 days in the orbital period, what is the percent relative error in the estimated mass? [8 points]
c) Show that, for a small error in the estimated semimajor axis, the error in the period is:
\[ \delta T = 3\pi \sqrt{\frac{a}{\mu}} \delta a \] [18 points]

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\(^1\) Signals intelligence (often contracted to SIGINT) is intelligence-gathering by interception of signals, whether between people (i.e., COMINT or communications intelligence) or between machines (i.e., ELINT or electronic intelligence), or mixtures of the two. As sensitive information is often encrypted, signals intelligence often involves the use of cryptanalysis. However, traffic analysis—the study of who is signalling whom and in what quantity—can often produce valuable information, even when the messages themselves cannot be decrypted.