Consider a beam that deforms in the plane of the paper due to pure bending. This beam is represented by an element that has 4 dof; a lateral displacement \( w \), and rotation \( \theta \) at each end. Nodal forces \( F_i \) and \( F_j \) and nodal moments \( M_i \) and \( M_j \) correspond to nodal displacements \( w_i \), \( w_j \), and nodal rotations \( \theta_i \) and \( \theta_j \), respectively.

Derive all the components of the element stiffness matrix for the beam element by the direct method. That is, by giving unit displacement to each degree of freedom, and by calculating the nodal forces in return. Use the positive directions as shown in the figure below!

\[
\begin{bmatrix}
  w_i \\
  \theta_i \\
  w_j \\
  \theta_j
\end{bmatrix}
= 
\begin{bmatrix}
  F_i \\
  M_i \\
  F_j \\
  M_j
\end{bmatrix}
\]