A test needs to be performed in your lab to analyze the structural properties of an aircraft structure. The engineers in the structures group have reported that they need to measure the response and the associated frequencies of the structure to a particular excitation. They think the associated frequencies should be around 5-20Hz. You are the engineer assigned to do the measurements and provide the frequency analysis.

Assume the actual displacement response of that structure to be
\[ x(t) = 3^{-t} \left[ 2 \cos(2 \pi 8t) + \cos(2 \pi 12t) \right] \] eqn.(1)

Generate the “actual response” by plotting \( x(t) \) vs. \( t \) for a 1s time period with increments of \( \Delta t = 10^{-3} \). (Note that, in reality this function is not known!)

Follow the following steps to perform your assignment:

a) You are ready for measurements. What sampling rate would you use to collect data? Why?

b) Use your selected sampling rate and eqn.(1) to generate a set of discrete “measured data”, by generating data with time increments of \( 1/(\text{sampling rate}) \).

c) Perform a Fourier Transform (FT) on the measured data and plot the Frequency Spectrum. What dominant frequencies did you find? Do they agree with the frequencies of the actual response?

d) Now use a sampling rate of 16 Hz and 128Hz. Perform a FT and plot the Freq. Spectrum for both sampling rates. Did you find the same dominant frequencies? Why/why not?

e) Unfortunately, your measurement set-up is known to generate noise due to interference at 1000Hz and can be represented with an additional sine term in the response function:
\[ x(t) = 3^{-t} \left[ 2 \cos(2 \pi 8t) + \cos(2 \pi 12t) + 0.2 \sin(2 \pi 1000t) \right] \] eqn.(2)

a. Generate sampling data at 128Hz using eqn.(2). Perform a FT and plot the Freq. Spectrum. Can you still see the dominant frequencies? Why/why not?

b. Does the Freq. Spectrum improve if you use >2000Hz as your sampling rate?

c. Is it possible to use a sampling rate of 128Hz, if a low pass filter was used? Write a simple low pass filter and use a sampling rate of 128Hz and plot the Freq. Spect.

NOTE: You may use any computer language, or spread sheet to do the calculations. Do not use a pre-programmed library functions to calculate the FTs, however you may use them to check your results. Provide all your code, plots and calculations. You may talk and discuss the HW questions with your friends, but anything you submit...
(codes, plots, writings, etc.) should be your own work.