Problem 1:

Assumptions:
1. Uniform, internal heat generation at a rate
2. Constant properties
3. Surroundings form a large enclosure around the rod
4. Negligible spatial variations in temperature

Analysis:
(a) Establishing a control volume around a rod and writing the energy balance equation for it yields

\[ \dot{E}_{in} + \dot{E}_{cut} + \dot{E}_q = \dot{E}_{st} \]  \hspace{1cm} (1)

At steady-state, \( \dot{E}_{st} = 0 \), and keeping in mind spatial temperature variations are negligible,

\[ \dot{E}_{in} = 0 \]  \hspace{1cm} (2)
\[ \dot{E}_{cut} = -hA_s(T - T_\infty) - \varepsilon\sigma A_s(T^4 - T_{sur}^4) \]  \hspace{1cm} (3)
\[ \dot{E}_q = \dot{q}V \]  \hspace{1cm} (4)

where \( V \) is the volume of the rod and radiative heat transfer has been included as it may be very important due to large temperature difference. Recognizing that \( V/A_s = D/4 \) and substituting equations (2-4), equation (1) may be rewritten

\[ T - T_\infty + \frac{\varepsilon}{h} (T^4 - T_{sur}^4) = \dot{q}D \frac{4}{4h} \]  \hspace{1cm} (5)

This equation may be solved for temperature to find the temperature of the rod.

(b) Neglecting radiation exchange, and including the heat storage term, with negligence of the temperature gradients across the rod, the appropriate energy balance equation is

\[ -hA_s(T - T_\infty) + \dot{q}V = \rho CV \frac{dT}{dt} \]  \hspace{1cm} (6)

Integrating and applying the initial and boundary conditions yields

\[ T(t) = T_\infty + \frac{\dot{q}V}{hA_s} + (T_i - T_\infty - \frac{\dot{q}V}{hA_s}) \exp(-4ht/\rho CD) \]  \hspace{1cm} (7)

Problem 2:

Assumptions:
1. Bakelite slab can be treated as a semi-infinite solid,
2. Constant properties,
3. Heater mass (heat storage capability) is negligible

Properties:

Table A-3, Bakelite at 300 K:
\( \rho = 1300 \text{ kg/m}^3; \ C = 1465 \text{ J/kg K}; \ k = 1.4 \text{ W/mK}, \) and \( \alpha = 7.35 \times 10^{-7} \text{ m}^2/\text{s}. \)

Analysis: The geometry may be considered as semi-infinite solid. It is initially at a uniform temperature and suddenly subjected to a constant heat flux, say \( \dot{q}''_w \). Doing some research, we find that this case
corresponds to Case 2, Fig. 5.17 in the textbook. Then the temperature distribution is given by Eq. 5.59:

\[ T(x, t) = T_i + \frac{2d_i'' \sqrt{\alpha t / \pi}}{k} \exp\left(-x^2 / 4\alpha t\right) - \frac{d_i'' x}{k} \text{erfc}\left(x / 2\sqrt{\alpha t}\right) \quad (8) \]

where \text{erfc}=1-\text{erf}, \text{erf} is the Gaussian error function which is tabulated in Appendix B.2. Then the surface temperature after 10 minutes, \( T(0, 600 \text{ s}) \), is evaluated as

\[ T(0, 600 \text{ s}) = 300 \text{K} + \frac{2 \times 2500 \text{ W/m}^2}{1.4 \text{ W/mK}} \sqrt{7.35 \times 10^{-4} \text{ m}^2/\text{s} \times 600 \text{ s} / \pi \exp(0) - 0 \times \text{erf}(0)} \]
\[ = 342 \text{K} \]

The temperature at a depth of 25 mm after 10 min is found by setting \( x = 0.025 \text{ m} \) and \( t = 600 \text{ s} \) in Eq. (8) above. Doing so and performing calculations yields

\[ T(25 \text{ mm}, 600 \text{ s}) = 312 \text{K} \]