A COMPUTATIONAL STUDY OF FLOW ENTRAINMENT OVER
A STATIONARY/FLAPPING AIRFOIL COMBINATION IN TANDEM

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ABSTRACT

Unsteady flowfields over a stationary/flapping airfoil combination in tandem are computed using a Reynolds averaged Navier-Stokes solver. The stationary leading airfoil has an experimental profile which promotes flow separation at its trailing edge. The effect of the flapping airfoil on the leading airfoil and the flow reattachment at the trailing edge are investigated by varying the amplitude and frequency of the flapping motion. A single deforming C-H type grid and overset grids are employed in the computations. Unsteady flowfields are presented in terms of Mach number contours and time histories of unsteady aerodynamic loads. The high frequency flapping motions, where the experimental data show complete flow reattachment at the trailing edge, could not be performed due to the time step limitation of the solver. However, the flow configurations studied produced a significant amount of flow entrainment at the trailing edge.

INTRODUCTION

The aerodynamic interference effects between two airfoils are of a practical interest in many aeronautical applications. Prominent examples are the interference effects between the leading or trailing edge devices and the main airfoil on high-lift systems, the interference effects on canard-wing and wing-tail configurations, the blade-vortex interactions on helicopter blades, and the blade row interactions which occur in axial jet engines. In these applications the primary emphasis has been on the understanding of the steady interference effects or on the gust response of the complete airfoil-flap combinations. Yet little is known about the oscillatory interference effects due to small amplitude oscillations of the leading or trailing-edge flaps. It was shown theoretically1,2,3 and demonstrated experimentally4 that a flapping airfoil generates thrust. Freynuth5 identified the thrust producing vortical flow pattern behind a sinusoidally flapping airfoil as compared to the drag producing pattern behind a stationary cylinder. He observed that in the wake of a flapping airfoil counterclockwise rotating vortices are located above the mean flap position and vice versa, which produces a jet-like flow in the wake. Recently, Platzer et al.6,7,8 parametrically studied the flows over single flapping and flapping/stationary airfoil combinations by inviscid and viscous flow computations and confirmed the thrust generation and the associated vortical wake patterns due to airfoil flapping. In our previous computational work9, we have already shown that a flapping airfoil placed in the wake of a stationary airfoil has a potential to energize boundary layer flows of the upstream airfoil.

Recently, Dohring10 and Lai and Platzer11 observed that a flapping airfoil entrains the flow upstream and forms a jet-like flow in its wake. This jet-like behavior of a flapping airfoil has been exploited by Lai et al.12 to con-
fluence of the flapping airfoil are studied parametrically with respect to the amplitude and the frequency of the flapping motion.

**NUMERICAL METHOD**

The steady and unsteady flowfields over the airfoil combination in tandem are computed by a compressible Navier-Stokes solver as the trailing airfoil undergoes an oscillatory flapping motion with varying amplitude and frequency. The unsteady computed flowfields are analyzed in terms of instantaneous Mach number contours, vortical wake patterns, and the time variation of integrated aerodynamic loads.

**Computational Domain**

The computational domain around the airfoil combination in tandem is discretized either with a single C-H-grid, or two overset C-grids as shown in Figure 3. The farfield boundaries extend about ten main airfoil chord lengths away. The flapping motion of the airfoil is implemented by moving the trailing airfoil and the computational grid around it in the transverse flow direction as specified by the flapping motion. The flapping motion is defined by

$$ h_f = -A \cos(\omega t) $$

where $A$ denotes the mean amplitude and $\omega$ is the frequency of the oscillatory flapping motion, which is given in terms of the reduced frequency, $k = \omega c / U_{\infty}$. Here $c$ is the chord length of the trailing airfoil, and $U_{\infty}$ is the free-stream velocity. The flapping amplitude $A$ is non-dimensionalized with the chord length of the leading airfoil, $C$.

In the case of a single grid, the grid distribution downstream of the trailing edge of the leading airfoil is deformed to accommodate the flapping motion of the trailing airfoil. The grid deformation is gradually introduced between the trailing edge of the stationary leading airfoil and the leading edge of the flapping airfoil by means of a hyperbolic tangent function and extends in the transverse direction by a chord length from the flapping airfoil. The outer boundaries are kept stationary. The additional terms due to the grid motion and the grid deformation are incorporated into the governing equations. In the case of overset grids, the trailing airfoil and the grid around it is traversed over the background grid.

**Navier-Stokes Solver**

The strong conservation-law form of the compressible 2-D, thin-layer, Reynolds averaged Navier-Stokes equations

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is solved using an approximately factored, implicit algorithm. The convective terms are evaluated using the third order accurate Osher’s upwind biased flux difference splitting scheme\textsuperscript{15}. The governing equations in a curvilinear coordinate system, \((\xi, \zeta)\), are given as follows

\[ \partial_t \hat{Q} + \partial_\xi \hat{F} + \partial_\zeta \hat{G} = Re^{-1} \partial_\zeta \hat{S} \]  

(2)

where \( \hat{Q} \) is the vector of conservative variables, \( 1/J(\rho, \rho u, \rho w, e) \), \( \hat{F} \) and \( \hat{G} \) are the inviscid flux vectors, and \( \hat{S} \) is the thin layer approximation of the viscous fluxes in the \( \zeta \) direction normal to the airfoil surface. The pressure is related to density and total energy through the equation of state for an ideal gas, \( p = (\gamma - 1) \left[ e - \rho (u^2 + w^2) / 2 \right] \).

In the overset grid solution method, two C-grids around each airfoil are overlaid on each other as shown in Figure 3. The Navier-Stokes equations are solved implicitly in each subgrid with the proper boundary conditions. In addition to the boundaries of the computational domain, each grid now has intergrid boundaries with the other grid and the background grid contains a hole created by the grid around the flapping airfoil. Additional boundary conditions therefore need to be satisfied at the intergrid boundaries, and the hole points are to be excluded from the integration of the Navier-Stokes equations.

For turbulent flow computations the Baldwin-Lomax and the Baldwin-Barth turbulence models are used.

**Boundary Conditions**

Boundary conditions are applied on the airfoil surfaces, at the farfield boundaries, and at the intergrid boundaries for overset grid solutions. On the airfoil surfaces the no-slip boundary condition is applied. For the flapping airfoil, the surface fluid velocity is set equal to the prescribed airfoil lapping velocity derived from Eqn. 1. The density and the pressure values on the airfoil surfaces are extrapolated from the interior solution. At the farfield inflow and outflow boundaries the flow variables are evaluated using the zero-order Riemann invariant extrapolation.

In the case of overset grids, the intergrid boundary conditions are applied by first localizing the intergrid boundary points on the donor subgrid, and then interpolating the flow variables. The boundary point localization process is based on a directional search algorithm\textsuperscript{16}. The search direction follows the local geometry gradients of the donor grid, and a boundary point is localized in a triangular stencil on the donor grid. The boundary point localization process also supplies the geometric interpolation weights at the boundary point in terms of the function values at the vertices of the triangular stencil. Thus, once an intergrid boundary point is localized, interpolation weights are readily available to interpolate flow variables from the donor grid.

**RESULTS AND DISCUSSION**

We computed the steady and unsteady flowfields over the airfoil combination in tandem (Fig. 1) as the trailing airfoil, which is placed \( dx \) distance away from the trailing edge of the leading airfoil, undergoes a periodic flapping motion defined by the mean amplitude, \( A \), and the reduced frequency, \( k \) (Eqn. 1). An experimental airfoil profile which promotes the flow separation at the trailing edge is employed for the leading airfoil. The airfoil
Fig. 4 Flowfield computed with the C-H grid at $h_f = 0$, $dx = 0.035C$.

has a 16 percent thickness at 75 percent chord location. The trailing airfoil has a NACA0012 profile with a chord length of $c = 0.075C$, where $C$ is the chord length of the leading airfoil. The airfoil combination is set at $\alpha = 0^\circ$ incidence. All the flowfields were computed at a free stream Mach number of 0.2, and a Reynolds number of $30 \cdot 10^3$, which is approximately the same as the water tunnel experiments.

We first computed the flow over the airfoil combination where the trailing airfoil was placed at a distance $dx = 0.035C$ from the leading airfoil. A single C-H type grid, which is of $357 \times 91$ size with 146 and 81 grid points around the leading and trailing airfoils, respectively, was used (Figure 3). It should be noted that the clustering of the grid in the flow direction around the leading edge of the trailing airfoil imposes some restrictions on

Fig. 6 Variation of the drag coefficient on the leading airfoil computed with the C-H grid at $A = 0.02C$, $dx = 0.035C$. 

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the time step size of the numerical integrations. The flow is assumed to be fully turbulent and the Baldwin-Lomax model is used. The solution when the trailing airfoil is stationary is given in Figure 4. It is seen that the flow completely separates at the trailing edge. In addition, the computed flowfield experiences unsteadiness due to the periodic vortex shedding in the wake.

The unsteady computation was then carried out as the trailing airfoil undergoes a sinusoidal flapping motion with an amplitude of $A = 0.02C$, and reduced frequencies of $k = 0.75, 1.5$ and $5$. The unsteady solution is initiated when the trailing airfoil is at its minimum position. The computed flowfield for $k = 1.5$ is shown in Figure 5 in terms of instantaneous Mach number contours. The flowfield is seen to be highly vortical with the presence of leading and trailing edge vortices shed from the trailing airfoil. During the down stroke, it is observed that the flow tends to reattach at the upper trailing edge of the leading airfoil under the influence of the leading edge vortex. The same trend is similarly observed during the upward stroke.

Figure 6 shows the time history of the pressure drag coefficient on the leading airfoil for all the flapping frequencies. At low flapping frequencies the mean value of the unsteady drag is greater than the stationary value, the time variation of the drag shows an asymmetric behavior during the up and down strokes. It may therefore be concluded that at the low frequency flapping motions, the vortices in the wake act like a flow blockage, and the overall effect is towards increasing the drag on the leading airfoil. Increasing the amplitude to $A = 0.03C$ or the reduced frequency to $k = 5$ produced a divergent solution as shown in Figure 6 for $k = 5$. We attributed this numerical difficulty to the excessive grid deformation and very small time step requirements at these relatively large amplitude and frequency flapping motions.

Next, we employed an overset grid approach to alleviate the computational difficulties associated with the grid deformation and the grid clustering at the leading edge of the trailing airfoil, and for a better leading edge resolution of the trailing airfoil. Two overset grid systems were employed; one with 221x81 and 155x24 size grids for the leading and trailing airfoils, respectively, and the other with 241x91 and 157x32 size grids. In the finer grid system, the background grid resolution was approximately doubled around the trailing airfoil (Figure 3). Laminar and turbulent unsteady flowfields were then computed on both grids at $dz = 0.05C$, $A = 0.02C$ and $k = 3$. It should be noted that the overset grid topology prevented us from reducing the $dz$ any further. Figure 7 shows the Mach number contours at $h_f = 0$ during the down stroke of the fifth period. As observed, although the general trend is similar, the predictions differ from each other in detailed flow features. Vortices in laminar flow computations

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are sustained longer in the flowfield. The Baldwin-Barth (BB) turbulence model predicts the leading edge vortex stronger than the Baldwin-Lomax (BL) model does.

Figure 8 shows the turbulent (BB) flowfield at the sixth period of computations. It is noted that the flowfield does not experience a symmetric behavior during the down stroke and the upstroke. A stronger leading edge vortex forms during the down stroke. However, a slight flow entrainment behind the leading airfoil is clearly observed. Figure 9 shows the time variation of the pressure drag coefficient on the leading airfoil. The drag coefficient when the trailing airfoil is stationary converges to 0.046 for the laminar and turbulent computations. The general trend in the flapping case is therefore an increase in the average pressure drag with respect to the stationary value. It is also noted that while none of the solutions converge to a periodic behavior in time, the turbulent flow solution with the Baldwin-Barth model predicts larger and erratic variations. Computations with higher reduced frequencies proved to be impractical due to a prohibitively low timestep size requirement which would require on the order of 100 hours of CPU usage per period on workstations.

Next, we computed the same flow at $A = 0.03C$ using both BL and BB models. The flowfield and the pressure drag variation on the leading airfoil is given in Figures
Fig. 10 Instantaneous Mach number distributions computed with the Baldwin-Lomax (BL) turbulence model at \( k = 3, A = 0.03C, \ dz = 0.05C \).

Fig. 11 Variation of the drag coefficient on the leading airfoil computed with the overset grids at \( k = 3, A = 0.03C, \ dz = 0.05C \).

10 and 11. This increase of the flapping amplitude from 26.6 percent to 40 percent of the flapping airfoil chord produces a noticeable reduction of the extent of flow separation at the trailing edge of the leading airfoil, as can be seen by overlaying the figures 8 and 10. This would indicate an increased flow entrainment due to higher amplitude flapping. Indeed, during about 30 percent of the cycle the pressure drag is significantly lower, but the average drag is still higher than both in the stationary or in the lower amplitude flapping case. Both BL and BB solutions tend toward a periodic drag variation, but they are again significantly out of phase with each other. Also, an asymmetric behavior of the drag variation during the up and down strokes can be observed.

We next reduced the mean amplitude to \( A = 0.01C \), and were able to compute the flow at \( k = 10 \). The flowfield and the drag variation are given in Figures 12 and 13. As seen, the leading and trailing edge vortices are now significantly weaker, and are mostly confined to the close vicinity of the trailing airfoil. While the BL solution predicts higher drag values than the stationary value all along the flapping cycle, the BB solution appears to break down at this relatively high flapping frequency.

Figure 14 shows the drag (thrust) variation on the flapping airfoil. The drag coefficient is based on the chord length.

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Fig. 12 Instantaneous Mach number distributions computed with the Baldwin-Lomax (BL) turbulence model at $k = 10$, $A = 0.01C$, $dx = 0.05C$.

Fig. 13 Variation of the drag coefficient on the leading airfoil computed with the overset grids at $k = 10$, $A = 0.01C$, $dx = 0.05C$.

Fig. 14 Variation of the drag coefficient on the flapping airfoil computed with BL turbulence model on the overset grids at $dx = 0.05C$.

of the leading airfoil for consistency. It is seen that in all cases, the flapping airfoil generates not only trailing edge vortices but also vortices shed from the leading edge. It is noteworthy to observe that the trailing airfoil still produces a net thrust in these circumstances.

CONCLUDING REMARKS

In this computational investigation, an attempt was made to simulate the flow reattachment observed in the experiments performed by Doling et al., where the non-dimensional flapping velocity, $v_f$, exceeds a value of approximately 12. The flapping velocity is defined by multiplying the reduced flapping frequency with the flapping amplitude which is normalized with the flapping airfoil chord length. In the present study we were only able to reach values of at most $v_f = 1.33$ (for $k = 10$, $A = 0.01C$). Therefore it is not surprising that significant drag reductions and complete flow reattachment could not be computed. The computed flow features involve the shedding of large vortices from the leading and the trailing edge of the flapping airfoil. The resulting flow blockage therefore appears to prevent flow reattachment observed in the experiments.
We have shown that overset grid solutions can be successfully applied to flows over stationary/flapping airfoil combinations in tandem. However, C-type grids, which result in a highly clustered grid distribution across the wake cut, may not be the proper choice for the flapping airfoil at high frequencies. It is also apparent from the computational results that the turbulence modeling plays a significant role in the simulation. Although the eddy viscosity computed by the BB model is continuous across the intergrid boundaries, it seems not to be suitable for high frequency unsteady flows. This may be attributed to the fact that the BB model does not account for the grid motion. The turbulence modeling together with the question of compressibility effects in flows with high frequency flapping motions need to be addressed in detail in future studies.

Finally, this investigation revealed the possibility of generating thrust with flapping in the presence of vortices shed from the leading edge. In the future, we will study this interesting effect in more detail both computationally and experimentally.

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