FUNDAMENTALS OF THE THEORY OF ROCKET AND SPACECRAFT FLIGHT

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PART 7.

ORBITAL FLIGHT OF A SPACECRAFT
7.1. The basic aspects
7.2. No perturbed motion of a space flight vehicle
7.1.1. THE BASIC ASPECTS

The motion of a spacecraft by orbit takes place in a composite force field. This field is characterized not only large number forces, which are affecting on flight of a spacecraft, but also their completely different physical nature.
7.1.2. THE BASIC ASPECTS

A physical picture of a motion pattern it is good illustrates by calculation which is taking into account only main force, operational on spacecraft, - Newtonian force of a gravitational attraction of the Earth.
7.1.3. THE BASIC ASPECTS

At such calculation the form of the Earth is considered as spherical, the mass distribution inside it is even on radius. It is considered also, that there is not an atmosphere.
7.1.4. THE BASIC ASPECTS

The parameters, counted with allowance for such assumptions, of motion of a spacecraft are called as non-perturbed parameters, and orbit, on which one implements in this case flight of a spacecraft, - non-perturbed orbit, and motion of a spacecraft on this orbit – non-perturbed motion.
7.1.5. THE BASIC ASPECTS

All remaining forces, which are operational on a spacecraft on orbit, are changing some parameters of its motion, form of orbit, position of orbital plane in absolute space. These forces result a disturbance of a spacecraft motion.
7.1.6. THE BASIC ASPECTS

Therefore these forces often call as *disturbances*, orbit of a spacecraft counted with allowance for of these disturbances, - *perturbed orbit*, and motion of spacecraft on such orbit - *disturbed motion*.
7.1.7. THE BASIC ASPECTS

The position of a spacecraft at non-perturbed motion at any moment of time is determined by six independent values, which are called as the elements or orbit parameters.
7.1.8. THE BASIC ASPECTS

Orbit parameters:
- Inclination of orbital plane
- Longitude of an ascending node
- Lager semi axis
- Eccentricity
- Argument of a perigee
- Time
The main disturbing source of a motion of the majority of satellites is *non-sphericity of an Earth figure* - its compression. The compression of the Earth produces the constant rotation of orbital plane around of a Earth's axis, otherwise, moving on the equator of a nodal line.
7.1.10. THE BASIC ASPECTS

This moving has a title of *precession of a nodal line*. At the same time there is a rotation of a point of a perigee in orbital plane (change of argument of a perigee), i.e. rotation of apsides. These disturbances have permanent nature, therefore are called *monotonic*. 
7.1.11. THE BASIC ASPECTS

The second relevant cause producing disturbances of orbit parameters is the Earth's atmosphere. To conduct estimations of influencing of atmosphere on flight of a spacecraft, it is necessary to select its model: stationary or dynamic.
7.1.12. THE BASIC ASPECTS

The stationary model guesses, that the Earth's atmosphere is motionless concerning the rotated Earth and at increase of an altitude density it descends by exponential law. The real atmosphere differs from described above by so-called fluctuations.
7.1.13. THE BASIC ASPECTS

Thus, the parameters of orbit during flight are changes. Therefore, that a spacecraft has executed scheduled tasks in a full volume, apart from a reliable operation of onboard instrumentation, it is required permanently to watch its position in space. If it is necessary it needs to conduct orbit correction.
Observation of a spacecraft flight and measurement of parameters of its motion are a composite technical problem, which one is decided by a ground-based command-measuring complex. The trajectory measurements can be made by optical and radio means of observation.
7.1.15. THE BASIC ASPECTS

As measured parameters can be, for example, radial distance up to a spacecraft, azimuth, angle of please located, altitude of flight of a spacecraft, and also speed of change of radial distance, azimuth etc.
7.2.1. No perturbed motion of a space flight vehicle

The calculation of no perturbed motion of a spacecraft is concerned into a problem of two bodies, which one is formulated as follows: to find motion of a mass point, the central force acts on which one.
7.2.2. No perturbed motion of a space flight vehicle

In this case the trajectory of a spacecraft at any initial conditions will be a plane curve, and its plane is determined by vector of initial velocity $v_0$ of a spacecraft and center of attractive force.
7.2.3. No perturbed motion of a space flight vehicle

If this plane coincides a plane \( OXY \) of the Cartesian coordinate system with the beginning in center of the Earth, the equality called as an integral of the squares takes place:

\[
XY - YX = r^2 \dot{\phi} = rv \sin (\vec{r}, \vec{v}) = C,
\]

Where \( r, \phi \) - polar coordinates of a spacecraft.
7.2.4. The Kepler's Laws:

1. All the planets are moving by elliptical orbits, in common focus which one is the Sun.
2. The squares (sectorial velocity) depicted by a radius-vector in unit of time, is value a constant.
3. For all planets the squares of cycle times of a planet around of the Sun correspond as cubes of semi major axes of ellipses
Problem 7.1.1.

Two spacecrafts are removed on complanar circular orbits according to altitudes $h_1$ and $h_2$ ($h_1 < h_2$). In the initial moment the spacecrafts are on one straight line passing through attracting center (figure 7.1).
Problem 7.1.2.

1. With what angular velocity one spacecraft will displace concerning other?
2. On what angular distance spacecrafts diverge for one full revolution of the first spacecraft?
Problem 7.1.3.

Fig. 7.1. The scheme of roundabout of spacecrafts
Problem 7.1.4.

The angular velocities of spacecrafts are determined from ratio

\[ \lambda_1 = \sqrt{\frac{\mu}{r_1^3}} \quad \lambda_2 = \sqrt{\frac{\mu}{r_2^3}} \]  \hspace{1cm} (7.1.1)

Where \( r_1 = R + h_1, \ r_2 = R + h_2 \).

Then the difference of angular velocities will make

\[ \Delta \lambda = \sqrt{\frac{\mu}{r_2^3}} - \sqrt{\frac{\mu}{r_1^3}} \]  \hspace{1cm} (7.1.2)
Problem 7.1.5.

If the cycle time of the first spacecraft is equal $T_1$, the angular distance between spacecrafts through one revolution of the first spacecraft will make

$$\Delta n = T_1 \cdot \Delta \lambda = 2\pi \left[ \sqrt{\left( \frac{R + h_1}{R + h_2} \right)^3} - 1 \right] \quad (7.1.3)$$
Problem 7.1.6.

Shall suspect, that difference is small in compare with value $R$. Then the equations (7.1.2) and (7.1.3) can be simplified. After a linearization is received

$$\Delta \lambda = -\frac{3}{2} \sqrt{\frac{\mu}{(R + h)^3}} \cdot \frac{\Delta h}{R + h} \quad (7.1.4)$$

$$\Delta n = -3\pi \cdot \frac{\Delta h}{R + h} \quad (7.1.5)$$
Problem 7.1.7.

Example 1. Is given: \( h_1 = 6378 \text{ km} \); \( h_2 = 200 \text{ km} \). To determine \( \Delta \lambda \) and \( \Delta n \).

The solution:

a) It is ground ratio (7.1.2) we have \( \Delta \lambda = (1,185 - 0,439) \cdot 10^{-3} \text{ 1/s} \approx 0,746 \cdot 10^{-3} \text{ 1/s}, \)
i.e. the second spacecraft will overtake the first with relative angular velocity \( 0,746 \cdot 10^{-3} \text{ radian/s} \) or \( \approx 2,6 \text{ °/min}; \)

b) It is ground ratio (7.1.3) is discovered \( \Delta n \approx 2 \cdot \pi \cdot 1,75 \), i.e. to the moment, when the first spacecraft will make one full revolution, the second will make 1,75 revolutions.
Problem 7.1.8.

Example 2. Is given: \( h_1 = 300 \) kms, \( h_2 = 250 \) kms. To determine \( \Delta \lambda \) and \( \Delta n \).

As \( \Delta h << r_1 \), we shall take advantage of the formulas (7.1.4) and (7.1.5) and we shall receive \( \Delta \lambda = 0,013 \cdot 10^{-3} \) 1/s angle minutes per one minute and \( \Delta n \approx 2 \cdot \pi \cdot 0,01 \), i.e. the second spacecraft will overtake the first only on the 100-th part of a revolution.
Problem 7.2.1.

Let the spacecraft goes on a circular orbit at the altitude $h$ around of the Earth. To determine the size of maximum duration of the spacecraft stay in a shade of the Earth in the supposition, that a shade of the Earth cylindrical.

To find also minimum value of maximum duration of the spacecraft stay in a shade of the Earth in altitude function of orbit.
Problem 7.2.2.

Fig. 7.2. The scheme of transit of a spacecraft through a shade of the Earth.
Problem 7.2.3.

\[ 2\alpha = 2 \cdot \arcsin \frac{R}{R + h} \quad (7.2.1) \]

\[ \lambda = \sqrt{\frac{\mu}{(R + h)^3}} \quad (7.2.2) \]

\[ t_{\max} = \frac{2\alpha}{\lambda} = 2 \sqrt{\frac{(R + h)^3}{\mu}} \arcsin \frac{R}{R + h} \quad (7.2.3) \]

\[ \frac{\partial t_{\max}}{\partial h} = 0 \quad (7.2.4) \]
Problem 7.2.4.

\[
\frac{\partial t_{\text{max}}}{\partial h} = \frac{3(R + h)^2 \arcsin\left(\frac{R}{R + h}\right)}{\sqrt{\mu(R + h)^3}} - \frac{2R \left(\frac{(R + h)^3}{\mu}\right)^{1/2}}{(R + h)(h^2 + 2hR)^{1/2}} = 0
\]

\[
\frac{1}{(h^2 + 2hR)^2} \arcsin\left(\frac{R}{R + h}\right) = \frac{2}{3} R \quad (7.2.5)
\]

The solution of this non-linear equation gives minimum value of maximum duration of stay of a spacecraft in a shade of the Earth in altitude function of a circular orbit.
Problem 7.3.1.

To determine a possible date and time of approach of an eclipse of a stationary satellite of the Earth provided that the shade of the Earth represents the cylinder with radius, equal radius of the Earth.
Problem 7.3.2.

The feature of a stationary satellite is, that it always is in a plane of the earth's equator and the longitude it is constant. The maximum Sun's declination, at which one a stationary satellite will be on border of a shade and light, will be determined by expression

$$\sin \delta = \frac{R}{R + h}$$

where $\delta$ - Sun's declination; $h$ - altitude of flight of a stationary satellite.
Problem 7.3.3.

Having substituted $R = 6371$ km and $h = 36000$ km, we shall find, what a maximum value of a Sun's declination $\delta = 8,6^\circ$. Thus, if a Sun's declination $\delta > -8,6^\circ$ and $\delta < +8,6^\circ$, the stationary satellite will go into daily shade of the Earth. These conditions are executed annually from February 26 till April 12 and from September 1 till October 15.
Problem 7.3.4.

Let us to determine time of day in these intervals of dates, when there comes an eclipse of a stationary satellite. As the stationary satellite constantly is on one longitude, also middle of an eclipse it is necessary for true solar midnight for the given longitude.
Problem 7.3.5.

For a spacecraft located in a point of stand 30° east longitude, the eclipse will come about 22 hours by Kiev time. For more precise prediction of time of an eclipse it is necessary to take into account an equation of time, which one gives on 16 minutes earlier time of an eclipse at the end of October and beginning of November and later on 14 minutes in February.
Problem 7.4.1.

The space vehicle is removed into circular polar orbit with an altitude $h$ for realization of television filming of a surface of the Earth. What least look angle $\varepsilon$ the television camera should have to receive the map of all surface for minimum time?
Problem 7.4.2.

The Earth is gyrated concerning the axis uniformly with angular velocity $\omega_E$. Then at a command altitude $h$ for each revolution of a space vehicle continuing $T(h)$ of units of time, the Earth will turn on an angle

$$\varphi = \omega_E \cdot T. \quad (7.4.1)$$

Greatest linear displacement of a surface conforming to this angle will be on equator. It will make

$$I = \varphi R = \omega_E \cdot T \cdot R. \quad (7.4.2)$$
Problem 7.4.3.

That the time of filming was least, the minimum value of a look angle of a television camera will be determined from a condition: on equator interturn spacing interval \( l = ab \) (fig. 7.3) should be completely enveloped by a television camera. On all remaining segments of a trajectory overlap observation zones will take place. Then, as follows from a fig. 7.3
Problem 7.4.4.

Fig. 7.3. The scheme of the view of the Earth from a space vehicle
Problem 7.4.5.

\[
\varepsilon_{\text{min}} = 2 \arctg \left( \frac{\sin \frac{\omega_E T}{2}}{1 + \frac{h}{R} - \cos \frac{\omega_E T}{2}} \right)
\]  
(7.4.3)

To receive the image of all surface of the Earth should make one revolution concerning orbital plane of a space vehicle. Thus of the space vehicle should end capture at that latitude, on which one began, i.e. it should make an integer of turns
Problem 7.4.6.

\[ n^* = E \left[ \frac{2\pi}{\varphi} \right] = E \left[ \frac{2\pi}{\omega_E T} \right] \]  

(7.4.4)

Thus the least time, which one expends a space vehicle on filming of all surface, is determined by a ratio

\[ t_{\text{min}} = n^* T = E \left[ \frac{2\pi}{\omega_E T} \right] T \]  

(7.4.4)

where \( E \left[ \frac{2\pi}{\omega_E T} \right] \) - the greatest integer value of fraction.
Problem 7.4.6.

Example. Let $h = 1000$ kms, $T = 1$ hour 45 mines 02 sec. Under the formula (7.4.3) is received $\varepsilon_{\text{min}} = 111^\circ$. Thus $t_{\text{min}} = 88\ 228$ sec = 24 hr 30 min 28 sec.
Problem 7.5.1.

Two space vehicles operate on coplanar circular orbits accordingly at altitudes $h_1$ and $h_2$, and are moving in one direction. In the initial moment the vehicles are in a zone of visibility one another on angular distance $n_0$ (fig. 7.4, a). What is the time straight line the visibility between vehicles will be broken?
Problem 7.5.2.

Fig. 7.4. Limits of visibility of two space vehicles
Problem 7.5.3.

As follows from a fig. 7.4,b, the border of lost of a straight line of visibility corresponds to angular distance between space vehicles, equal

\[ n = n_1 + n_2 = \arccos \frac{R}{R + h_1} + \arccos \frac{R}{R + h_2} \]  \hspace{1cm} (7.5.1)
Problem 7.5.4.

If in the initial moment the vehicles were on one straight line passing through attracting center (see a fig. 7.1), the situation figured schematic in a fig. 7.4, b would be when vehicles have dispatched on angular distance \( n \). As the initial angular distance already made \( n_0 \), it is required only to overcome angular distance (in case of motion in one direction)

\[
n - n_0 = n_1 + n_2 - n_0. \quad (7.5.2)
\]
Problem 7.5.5.

Let's designate required time through $t^*$. Then, allowing the formula (7.1.2), it is possible to write

$$t^* \Delta \lambda = n_1 + n_2 - n_0. \quad (7.5.3)$$

Whence finally we have

$$t^* = \frac{\arccos \frac{R}{R + h_1} + \arccos \frac{R}{R + h_2} - n_0}{\sqrt{\mu (R + h_1)^3} - \sqrt{\mu (R + h_2)^3}} \quad (7.5.4)$$
Problem 7.6.1.

The stationary space vehicle is characterized by geographic longitude $L_C$. To determine an angle of elevation and azimuth of a direction on the vehicle from ground point arranged at a latitude $B_P$ and geographic longitude $L_P$. 
Problem 7.6.2.

Fig. 7.5. Conditions of observation of stationary space vehicle
Problem 7.6.3.

Let's accept, that the Earth represents a sphere with radius $R$ (fig. 6.5). Then the central angle between directions of observing point and space vehicle will be determined from expression

$$\beta = \arccos \left( \cos B_p \cos (L_C - L_P) \right).$$  \hspace{1cm} (7.6.1)

From a condition of observation follows, what the angle $\beta$ always is in an interval from 0 up to $90^\circ$, and consequently the definition it under the reduced formula is monosemantic.
Problem 7.6.4.

Azimuth of a line-of-sight can be determined from relation

\[ A = \arcsin \frac{\sin(L_C - L_P)}{\sin \beta} \]  

(7.6.2)

The definition of quarters of a lateral angle will be helped with a following simple rule, which is flowing out directly from a fig. 7.5: For point located in northern hemisphere \(90^\circ < A < 270^\circ\), and in Southern - \(270^\circ < A < 90^\circ\).
Problem 7.6.5.

To determine an angle of elevation of observing $\gamma$, it is necessary previously to find distance between an observation point and space vehicle:

$$d = \sqrt{R^2 + (R + h)^2} - 2(R + h)R \cos \beta$$  \hspace{1cm} (7.6.2)

where $h = 35\,800$ km - an altitude of flight of stationary space vehicle.
Problem 7.6.6.

Having taken advantage the theorem of sin, we can write expression for an angle of elevation of observation:

$$\gamma = \arccos \left( \frac{R + h}{d} \sin \beta \right)$$  \hspace{1cm} (7.6.4)
Problem 7.6.7.

Capabilities of observation of a stationary space vehicle take place, if $0^\circ < \gamma < 90^\circ$. Thus, for definition of azimuth and angle of elevation of observation of a stationary space vehicle it is enough to know latitude of ground observing point and difference of longitude of stand of the stationary space vehicle and ground point.
Example. To the observer located in Moscow \((L_p = 38^\circ\) Eastern Longitude, \(B_p = 56^\circ\) Northern Latitude), it is necessary with the help of optical means to find a stationary space vehicle arranged in a point \(L_0 = 38^\circ\) Eastern Longitude. What azimuth and angle of elevation is necessary for establishing on the device to see a stationary space vehicle?
Problem 7.6.9.

From ground observing point, which one is on latitude 82°, is possible to observe on horizon the stationary space vehicle, if it is on longitude of observation point. If the observer is located in northern hemisphere, azimuth makes 180°, and if in southern - 0°.
Problem 7.7.1.

Elliptical orbit of a space vehicle presets by the semi major axis $a$ and focal parameter $p$ (fig. 7.6). To determine flight time of a space vehicle between two points of elliptical orbit, which one preset by true anomalies $\mathcal{O}_1$ and $\mathcal{O}_2$. 
Problem 7.7.2.

Fig. 7.6. The scheme of motion of a space vehicle on elliptical orbit
Problem 7.7.3.

\[ t = \tau + \frac{E - e \sin E}{\lambda} \]  \hspace{1cm} (7.7.1)

which one determines flight time of a space vehicle from the moment of transit of a perigee of orbit up to some point of orbit, given true anomaly \( \vartheta \).

\[ \tan \frac{E}{2} = \sqrt{1 - e} \tan \frac{\vartheta}{2} \]  \hspace{1cm} (7.7.2)

\[ \lambda = \sqrt{\frac{\mu}{a^3}} \sqrt{1 - e^2} \left( \frac{1}{1 - e \cos E} \right) \]  \hspace{1cm} (7.7.3)
Problem 7.7.4.

At calculation of value $E$ under this scheme it is necessary to mean, what angles $E/2$ and $\vartheta/2$ always are in one quarter. Depending on a position initial $\vartheta_1$ and final $\vartheta_2$ points on orbit are possible four cases of application of the formula (7.7.1):
Problem 7.8.1.

On elliptical orbit, given perigee altitudes \( h_p \), apogee \( h_A \) and argument of a perigee \( \omega \) (fig. 7.7), goes a space vehicle. To determine time in flight of a space vehicle in a shade of the Earth, if it is known, what at the moment of an entrance of the vehicle in a shade the Sun's declination is equal \( \delta \), the straight line connecting centers of the Sun and the Earth, lies in orbital plane and during transit of a shade relative movement of the Sun and Earth misses. A shade of the Earth can be accepted as cylindrical with radius \( R \).
Problem 7.8.2.

Fig. 7.7. The scheme of transit of a space vehicle through a shade of the Earth
Problem 7.9.1.

The space vehicle goes on elliptical orbit, which one is characterized by perigee altitudes $h_p$ and apogee $h_A$. It needs to determine the greatest and least velocity of a point movement under space vehicle (point of intersection of radius-vector of the vehicle with a surface) on the Earth surface. Rotation of the Earth can be neglected.
Problem 7.9.1.

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