FUNDAMENTALS OF THE THEORY OF ROCKET AND SPACECRAFT FLIGHT

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PART 6.

6. THE LAUNCHING OF THE SPACE VEHICLE INTO ORBIT
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6.1. The basic aspects
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6.1.1. THE BASIC ASPECTS

Flight of a space flight vehicle (spacecraft) begins with its launching into orbit with the help of the launcher. Orbit of a spacecraft is determined by parameters of motion of the launcher in an end of propulsion flight trajectory.
6.1.2. THE BASIC ASPECTS

One of main problems of the launching into orbit of a spacecraft consists in selection of such parameters of flight control on the active segment, which provide the greatest efficiency of the solution of preset tasks.
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6.1.4. THE BASIC ASPECTS

The basic problems of ballistics of launchers that are used for start of a spacecraft on adjusted orbits are following:

11. Research of relation of flight characteristics of the launcher from design data with the purpose of selection of the most rational combination of these parameters (ballistic designing).
6.1.5. THE BASIC ASPECTS

2. Determination of a trajectory and other characteristics of motion of the launcher with known design parameters and a control system, which provide preset final conditions of the launching of a spacecraft.
6.1.6. THE BASIC ASPECTS

3. Selection of a nominal trajectory of motion that provides an effective utilization of capabilities of the launcher (selection of a time schedule control and compiling of the flying mission).
6.1.7. THE BASIC ASPECTS

4. Separate independent tasks are problems of selection of a point of start of the launcher, flight course, impact areas of separable parts of the launcher, a lift-off time etc.
6.2.1. Boundary conditions.
Equations of motion

One of basic conditions of space flight is the message to a space flight vehicle of a necessary velocity. If not to reach a required level of a velocity, it is impossible to put a spacecraft into orbit of an artificial satellite of the Earth.
6.2.2. Boundary conditions. Equations of motion

One of basic conditions of space flight is the message to a space flight vehicle of a necessary velocity. If not to reach a required level of a velocity, it is impossible to put a spacecraft into orbit of an artificial satellite of the Earth.
6.2.3. Boundary conditions. Equations of motion

If motion of a spacecraft is considered in a central field of forces the circular velocity on distance $r$ from center of the Earth is evaluated under the formula

$$v_1 = \sqrt{\frac{\mu}{r}}$$

$\mu = 3,986 \cdot 10^5 \text{ km}^3/\text{sec}^2$ - gravitational parameter of the Earth
6.2.4. Boundary conditions. Equations of motion

At reaching this velocity on distance $r$ from center of the Earth and provided that the velocity vector is directed perpendicularly to radius $r$, the spacecraft goes into circular orbit of a satellite of the Earth.
6.2.5. Boundary conditions. Equations of motion

Velocity of flight, which is determined by relation

\[ v_2 = \sqrt{\frac{2\mu}{r}} \]  

(6.2)

is named as lift-off speed or a parabolic velocity (at \( r = R \) - this velocity is named as the second space velocity).
6.2.6. Boundary conditions. Equations of motion

At development of the mathematical mission model of the launcher known classical theorems of change of momentum of a skew field in weight $m$ under an operation of external force $F$ are used:

$$m\frac{d\vec{v}}{dt} = \overline{F}dt$$  \hspace{1cm} (6.3)

And about change of a moment of momentum:

$$\frac{d}{dt}(\vec{r} \times m\vec{v}) = \overline{M}$$  \hspace{1cm} (6.4)
6.2.7. Boundary conditions. Equations of motion

In that case when it is possible to consider progress irrespective of a rotation for the description of a trajectory of a spacecraft such equation may be used:

\[
m \frac{d\vec{v}}{dt} = \vec{F} + \frac{dm}{dt} (\vec{v}_e - \vec{v})
\]  

(6.5)

Where \( \vec{v}_e \) - exhaust velocity of working substance from the engine.
6.2.8. Boundary conditions. Equations of motion

At motion of a spacecraft outside of a field of attraction $\vec{F} = 0$ and at constant exhaust velocity $\vec{v}_e = \text{const}$ The equation (6.5) is resulted in the known formula of the Ziolkovsky:

$$v = v_e \ln \frac{m_0}{m}$$  (6.6)

Value of a velocity $v$, obtained under the formula (6.6), is named as characteristic (an ideal) velocity.
6.2.9. Boundary conditions. Equations of motion

For compiling differential equations of motion of the launcher in some cases it is convenient to use equations of Lagrange of the second kind:

\[
\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{q}_i} \right) - \frac{\partial E_c}{\partial q_i} = F_i \quad \text{for } i = 1, 2, \ldots, n, \tag{6.7}
\]

Where \( q_i \) - generalized coordinates; \( F_i \) - the generalized forces; \( E_c \) - a kinetic energy.
6.2.10. Boundary conditions. Equations of motion

Problem 6.1. To determine summarized characteristic velocity, which provides a capability of the ascent of a spacecraft into circular orbit of a satellite of the Earth with an altitude $h$. 
6.2.11. Boundary conditions. Equations of motion

1. The energy necessary for raising of a spacecraft from distance \( R \) from center of the Earth (from its surface) up to distance \( r = R + h \) (potential energy)

\[
E_p = \int_{R}^{r} \frac{\mu}{r^2} \, dr = \mu \left( \frac{1}{R} - \frac{1}{r} \right)
\]
6.2.12. Boundary conditions. Equations of motion

2. Activity, which needs to be made to give a spacecraft a circular velocity (kinetic energy). For a unit mass it is equal

\[ E_c = \frac{v^2}{2} \]

\[ E_c = \frac{\mu}{2r} \]

\[ E = E_p + E_c = \mu \left( \frac{1}{R} - \frac{1}{2r} \right) \]
6.2.13. Boundary conditions. Equations of motion

The first item of characteristic velocity will be determined from a ratio

\[
\frac{v_1^2}{2} = \mu \left( \frac{1}{R} - \frac{1}{2r} \right)
\]

\[
v_1 = \left[ \frac{2\mu}{R} \left( 1 - \frac{R}{2r} \right) \right]^{\frac{1}{2}}
\]
### 6.2.14. Boundary conditions. Equations of motion

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6.2.15. Boundary conditions. Equations of motion

The second component of summarized characteristic velocity is stipulated by losses on overcoming of resistance of atmosphere. Value of it appreciably depends on aerodynamic characteristics of the launcher and a velocity of motion on the active segment of a trajectory. It makes usually 100 - 500 m/s.
6.2.16. Boundary conditions. Equations of motion

The third item of summarized characteristic velocity is connected to gravitational losses. They are stipulated by that it is necessary to spend energy not only for ascent and the give of a velocity of a spacecraft, but also on movement of constructions items of the launcher and fuel. For circular orbits with an altitude of 200 kms gravitational losses make 1000 - 1200 m/sec
6.2.17. Boundary conditions. Equations of motion

Problem 6.2. To determine boundary conditions on the right end of an active segment trajectory of a spacecraft into stationary orbit in a central and normal gravitational field of the Earth.
6.2.18. Boundary conditions. Equations of motion

\[ T = 2\pi \frac{r^{3/2}}{\sqrt{\mu}} \]

\[ \Delta T = \frac{3T}{2r} \Delta r \]

\[ v_1 = \sqrt{\frac{\mu}{r}} \]

\[ \Delta r = \frac{4\varepsilon_E}{3r\mu} \]

\[ \Delta T = -\frac{4\pi}{\sqrt{\mu r}} \cdot \frac{\varepsilon_E}{\mu} \]

\[ \Delta v = -\frac{v}{2} \cdot \frac{\Delta r}{r} \]

Problem 6.3. Using equations of Lagrange of the second kind to make differential equations of progress of a center of mass of the launcher in a normal gravitational field of the Earth in cylindrical coordinate system at absence of influence of atmosphere.
6.2.20. Boundary conditions. Equations of motion

Let \( r, u, b \) - generalized coordinates in cylindrical coordinate system (\( r \) - radius, \( u \) - argument of a latitude, \( b \) - distance on a normal to orbital plane).

\[
E_c = \frac{m}{2} \left( \dot{r}^2 + r^2 \dot{u}^2 + b^2 \right)
\]

\[
\dot{m}v_u = - \frac{mv_u v_r}{r} + F_u
\]

\[
\dot{m}v_r = \frac{mv_u^2}{r} + F_r
\]

\[
\dot{m}v_b = F_b
\]
Equations of motion

As the gravitational field of the Earth is conservative,

\[
F_r = \frac{\partial U_0}{\partial r} \quad F_u = \frac{\partial U_0}{\partial u} \quad F_b = \frac{\partial U_0}{\partial b}
\]

Where \( F_u \) - derivatives in the direction; \( U_0 \) - a gravitational potential of the Earth.
6.2.22. Boundary conditions. Equations of motion

\[ U_0 = \frac{\mu}{r_1} + \frac{3}{2} \cdot \frac{\varepsilon_2}{r_1^3} \left( \sin^2 B_0 - \frac{1}{3} \right) \]

Here \( B_0 \) - geocentric latitude,

\[ \sin B_0 = \frac{1}{r_1} \left( r \cdot \sin u \cdot \sin i + b \cdot \cos i \right) \]

\[ r_1^2 = r^2 + b^2 \]
6.2.22. Boundary conditions. Equations of motion

\[ \frac{\partial U_0}{\partial r} = -\frac{\mu}{r_1^3} + \frac{9 \varepsilon_2 r}{2 r_1^5} \sin^2 B_0 + \frac{3 \varepsilon_2}{r_1^5} kb \cdot \sin B_0 + \frac{3}{2} \frac{\varepsilon_2}{r_1^5} r \]

\[ \frac{\partial U_0}{\partial u} = \frac{3 \varepsilon_2}{r_1^4} r \cdot \sin B_0 \cdot \sin i \cdot \cos u \]

\[ \frac{\partial U_0}{\partial b} = -\frac{\mu b}{r_1^3} - \frac{9 \varepsilon_2 b}{2 r_1^5} \sin^2 B_0 - \frac{3 \varepsilon_2}{r_1^5} kr \cdot \sin B_0 + \frac{3}{2} \frac{\varepsilon_2}{r_1^5} b \]
6.2.23. Boundary conditions. Equations of motion

Projections of thrust vector $\overrightarrow{P}$ will be determined by formulas

$$
\begin{align*}
P_u &= P \cdot \cos \Theta \cdot \cos \psi \\
P_r &= P \cdot \sin \Theta \cdot \cos \psi \\
P_b &= P \cdot \sin \psi
\end{align*}
$$

Where $\Theta$ - an angle between thrust vector and local horizon; $\psi$ - yaw angle.
6.2.24. Boundary conditions.
Equations of motion

In outcome we obtain a set of equations of motion:

\[
\begin{align*}
\dot{\mathbf{v}}_r &= A \cdot \sin \Theta \cdot \cos \psi + \frac{v_u^2}{r} + Br + Cb \\
\dot{\mathbf{v}}_u &= A \cdot \cos \Theta \cdot \cos \psi - \frac{v_u v_r}{r} + D \\
\dot{\mathbf{v}}_b &= A \cdot \sin \psi + Bb - Cr \\
\dot{r} &= v_r \\
\dot{u} &= \frac{v_u}{r} \\
\dot{b} &= v_b
\end{align*}
\]
6.2.25. Boundary conditions. Equations of motion

Problem 6.4. To make equations of motion of a center of mass of the launcher for phase of launch in launching coordinate system in a normal gravitational field of forces of the Earth.
6.2.26. Boundary conditions. Equations of motion

\[ m \frac{d\vec{V}_a}{dt} = \vec{F} + \vec{P} \]

\[ P = P_{spp} \dot{m} g_0 + (p_a - p) F_b \]

\[ \ddot{\vec{V}}_a = \ddot{\vec{V}} + \ddot{\vec{V}}_e + \ddot{\vec{V}}_k \]

\[ g_{pr} = \frac{\mu}{r^2} + \frac{3 \cdot \varepsilon_2}{2 \cdot r^4} (5 \cdot \sin^2 B_0 - 1) \]

\[ g_{p\omega} = -3 \cdot \frac{\varepsilon_2}{r^4} \cdot \sin^2 B_0 \]
6.2.27. Boundary conditions.
Equations of motion

Designing the obtained values of accelerations on axes of launching coordinate system, we discover

\[ g_x = -\frac{gr}{r} (x - x_0) - g_\omega \cos B \cdot \cos A \]

\[ g_y = -\frac{gr}{r} (y - y_0) - g_\omega \sin B \]

\[ g_z = -\frac{gr}{r} (z - z_0) + g_\omega \cos B \cdot \sin A \]
6.2.28. Boundary conditions. Equations of motion

Where $x_0$, $y_0$, $z_0$ - coordinates of center of an earth ellipsoid in launching coordinate system, defined under formulas

$$x_0 = r_0 \cdot \sin \Delta_0 \cdot \cos A$$

$$y_0 = -r_0 \cdot \cos \Delta_0 - h_0$$

$$z_0 = -r_0 \cdot \sin \Delta_0 \cdot \sin A$$