FUNDAMENTALS OF THE THEORY OF ROCKET AND SPACECRAFT FLIGHT

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PART 2.

COORDINATE AND ANGLES SYSTEMS. FORCES AND MOMENTS THAT ARE OPERATE ON A ROCKET IN FLIGHT
2.1. Coordinate and angles systems determining a position of a rocket in space
2.2. Thrust force, gravity and forces created by control devises
2.3. Aerodynamic forces
2.4. Moments which are act on a rocket
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2.1.1. COORDINATE AND ANGLES SYSTEMS
DETERMINING A POSITION OF A ROCKET IN SPACE

Launching coordinate system

Axis $O_c x_c$ is directed on tangent to a surface of the Earth in a point of start in a direction of targeting.

Axis $O_c y_c$ - vertically up in a point of start.

Axis $O_c z_c$ – perpendicularly to axes $O_c x_c$ and $O_c y_c$ so that to receive the right coordinate system.

Fig. 2.1. Launching coordinate system
2.1.2. COORDINATE AND ANGLES SYSTEMS
DETERMINING A POSITION OF A ROCKET IN SPACE

Polar coordinate system $Fr\varphi$ is used for calculation of a passive segment of a trajectory lying in hardly rarefied atmospheric condition. The pole $F$ of a system is selected in center of mass of the Earth, the position vector $r$ connects it to center of mass of a rocket, and the polar angle $\varphi$ is count out from earth radius-vector of a point of start $r_0$ in the direction of a rocket motion.

Fig. 2.2. Polar coordinate system
2.1.3. COORDINATE AND ANGLES SYSTEMS DETERMINING A POSITION OF A ROCKET IN SPACE

The connected coordinate system $Ox_1y_1z_1$ is fixed relative a rocket. The origin of system coin-cides mass center of a rocket.

Axis $Ox_1$ is directed on a centerline of a rocket to the direc-tion of nose cone;

Axis $Oy_1$ is located in a plane of symmetry of a rocket I - III and is directed to the direction of the stabilizer III.

Axis $Oz_1$ lies in a plane of symmetry of a rocket II — IV.

Fig. 2.3. Connected coordinate system: I, II, III, IV - number of rocket fins
2.1.4. COORDINATE AND ANGLES SYSTEMS DETERMINING A POSITION OF A ROCKET IN SPACE

Fig. 2.4. Angles - pitch $\Theta$, yaw $\psi$ and roll $\gamma$:
- a - view from the end of an axis $Oz_1$;
- b - view from the end of an axis $Oy_1$;
- c - view on an axis $Ox_1$
2.1.5. COORDINATE AND ANGLES SYSTEMS
DETERMINING A POSITION OF A ROCKET IN SPACE

The position of a rocket in space is determined in three coordinates of its center of mass in launching coordinate system and three angles determining a mutual direction of axes of connected and launching coordinate systems.

The angle between a centerline of a rocket and plane of launching horizon (plane $x_cO_cz_c$) is named as pitch angle of a rocket $\varphi$.

The angle between a centerline of a rocket and plane of targeting (plane $x_cO_cy_c$) is named as yaw angle $\psi$.

The angle between a normal axis of a rocket $Oy_1$ and plane of targeting (plane $x_cO_cy_c$) is named as angle of roll $\gamma$. 
2.1.6. COORDINATE AND ANGLES SYSTEMS
DETERMINING A POSITION OF A ROCKET IN SPACE

Fig. 2.5. Fast-track and connected coordinate systems
2.1.7. COORDINATE AND ANGLES SYSTEMS
DETERMINING A POSITION OF A ROCKET IN SPACE

The fast-track coordinate system is connected to flight trajectory of a rocket and is applied at aerodynamic calculation. The beginning of coordinate system places in center of mass of a rocket.

Axis $Ox$ is directed on tangent to a trajectory of motion of center mass of a rocket and is named as a fast-track axis;

Axis $Oy$ is directed on an external normal to a trajectory of motion of mass center of a rocket and is named as a lift force axis;

Axis $Oz$ is perpendicular to axes $Ox$ and $Oy$ so, that will derivate the right coordinate system and is named as an axis of lateral force.
2.1.8. COORDINATE AND ANGLES SYSTEMS DETERMINING A POSITION OF A ROCKET IN SPACE

The position of a rocket relatively of a velocity vector is determined by an angle of attack $\alpha$ and angle of slide $\beta$.

An angle of attack $\alpha$ is an angle between a centerline of a rocket $Ox_1$ and projection of velocity vector to a plane of symmetry of a rocket $Ox_1y_1$.

A slide angle $\beta$ is an angle between velocity vector and plane of symmetry of a rocket $Ox_1y_1$.

On a segment of controlled flight in atmosphere these angles are rather small and usually do not exceed $3^\circ - 4^\circ$. 
Fig. 2.6. Change of thrust force LPE in flight time of a rocket: 1 - command on start, 2 - inflammation, 3 - separation of a rocket from a launching device, 4 - 5 - segment of a nominal thrust, 5 - cut-off command (moment $t_k$), 5 - 6 - segment of an aftereffect, 6 - moment of a zero thrust $t_f$. 

2.2.1. THRUST FORCE, GRAVITY AND FORCES CREATED BY CONTROL DEVICES
2.2.2. THRUST FORCE, GRAVITY AND FORCES CREATED BY CONTROL DEVICES

Gravity. Resultant gravity of elements of a rocket \( G = mg \) is affixed in center of mass of a rocket and is directed to center of the Earth.

\[
m(t) = m_0 - mt;
\]
\[
F = f \frac{Mm}{r^2};
\]
\[
g = \frac{fM}{r^2};
\]
\[
k = fM = 3,9862 \cdot 10^{14} \text{ } m^3 / \text{sec}^2
\]
\[
g_0 = fM / R^2;
\]
\[
\frac{g}{g_0} = \frac{R^2}{r^2}
\]

Value of acceleration of gravity in a gravitational field of the Earth decreases in process of a rocket rise. It is inversely proportional to a square of distance between center of a rocket mass and center of mass of the Earth.
2.2.3. GAS-DYNAMICS CONTROL DEVICES

a) jet vane

b) deflector

c) mount-nozzle piece
2.2.4. GAS-DYNAMICS CONTROL DEVICES

- d) shaking engine
- e) shaking nozzles
- f) gas or liquid injection
2.2.5. GAS-DYNAMICS CONTROL DEVICES

Jet vanes. Such control surfaces are rather simple on a design. Their efficiency is linearly connected to a fleet angle (at least, at $\delta = 20^\circ$). Advantage of jet vanes is also that by differential control by them it is possible to create not only pitching moments and yaws, but also rolling moments. The large drag is the main of its disadvantage. This large drag is equivalent decrease of thrust force of the engine on (3 - 5%). Also the disadvantage of these devices is the fast burning, especially in flow of LPT, containing the firm particles.
Deflectors. The deflectors represent rings arranged around of a shear of a nozzles and which are turned down relatively of one or two orthogonal related axes. As the deflector comes into contact to a flow only at deviation from a neutral position, the burning of it is insignificant. The hinge moments of deflectors are small. As contrasted to jet vanes the deflectors have by smaller efficiency. Besides the relation of the control moment to a fleet angle of a deflector has non-linear nature. Some type of deflectors is the mouth-nozzle piece. They are more effective, but at deviation such a mount-nozzle piece there are large hinge moments.
2.2.7. GAS-DYNAMICS CONTROL DEVICES

Shaking engines. This way of creation of the control moments with success can be applied to flight vehicles with LPT, the chamber that one is set on the gimbal mount. At deviation of the chamber in any plane together with it the gas jet and thrust vector deviates.

Shaking nozzles. The deviation of a gas flow at a fixed camera of the engine can be reached by deviation of a nozzle or its part. In this version the reliable hermetic sealing of connection of mobile and fixed parts of a nozzle working in conditions of high pressures and temperatures is required.
Gas or liquid injection in a nozzle. By a gas or liquid injection inside of a nozzle through its lateral wall also it is possible to achieve deviation of a gas flow and originating of the control moment. Advantage of such way is the absence of mobile pieces of the engine or its nozzle.
2.2.9. THRUST FORCE, GRAVITY AND FORCES CREATED BY CONTROL DEVICES

At turn of a control vane on some angle $\delta$ there is gas dynamics force $R_c$.

The first of these component $X_c$, directed on an axis of a rocket and breaking its flight. Second component $Y_c$, called as lift force of a control surface, is directed perpendicularly axes of a rocket, therefore it creates the control moment relative of mass center of a rocket.

$$Y_c = Y_c \delta$$
2.2.10. THRUST FORCE, GRAVITY AND FORCES CREATED BY CONTROL DEVICES

The value $Y_c$ depends on the square of a control surface, speed of gas flow rate and angle of a control surface. Changing an angle of a control surface, receive different values $Y_c$. Approximately it is possible to consider, that the force $Y_c$ is proportional to an angle of a control surface

$$Y_c = Y_c^\delta \delta$$

where $Y_c^\delta$ - gradient of the lift force.
2.3.1. AERODYNAMIC FORCES

The aerodynamic forces represent result of effect of a flow of air on a surface of a rocket at flight in atmosphere. For the solution of the majority of the tasks of the theory of flight it is enough to take into account influence of aerodynamic forces only up to an altitude about 80 km. At altitudes more than 80 km density of air is so small, that the aerodynamic forces can be neglected.
2.3.2. AERODYNAMIC FORCES

Fig. 2.9. Aerodynamic forces which are act on a surface of a rocket: $P_n$ - normal, $P_t$ - tangent; $P_b$ - bottom; $R$ - resultant aerodynamic forces at symmetrical flow around
2.3.3. AERODYNAMIC FORCES

Subsonic speed rates of flight. At motion of a rocket in air it appears under an act of normal and tangent aerodynamic forces distributed on its surface. Resultant all aerodynamic forces is named as full aerodynamic force. To determine a mode of boundary-layer flow it is possible with the help of a dimensionless Reynold's number:

\[
Re = \frac{\nu L \rho}{\mu} = \frac{\nu L}{\nu} = \frac{\nu}{\nu} \frac{L}{\mu} \rho
\]

\(\nu\) - flow velocity; \(L\) - length of a rocket; \(\rho\) - mass density of air; \(\mu\) - dynamic viscosity of air; \(\nu = \mu/\rho\) - kinematical viscosity of air.

At \(Re < R_{CR}\) flow is laminar; at \(Re > R_{CR}\) flow is turbulent. As have shown experience, for subsonic flows \(R_{CR} = 4 \cdot 10^5\), and for supersonic \(R_{CR} = 6,5 \cdot 10^5\).
2.3.4. AERODYNAMIC FORCES

Fig. 2.10. Formation of compression shock:

- a - an affixed shock wave $\Theta < \Theta_{CR}$;
- b - a detached shock wave $\Theta < \Theta_{CR}$
Fig. 2.11. Schemes of shockwaves at flow around a rocket by supersonic flow: 1 - a head shockwave; 2 - a bottom shockwave; 3 - a vortex trace; 4 - waves of rarefaction
Fig. 2.12. Components of the full aerodynamic force in fast-track coordinate system:
X — a drag force, Y — lift force; Z — lateral force.
2.3.7. AERODYNAMIC FORCES

Fig. 2.13. Components of full aerodynamic force in fast-track and connected coordinate systems (motion of a rocket in one plane)
2.3.8. THE FORMULAS DETERMINING AERODYNAMIC FORCES

\[ R = c_R \cdot q \cdot S_M, \] where \( q = \rho v^2/2 \) – velocity head; \( \rho \) - mass density of air; \( v \) - flow velocity of air; \( S_M \) - square of the greatest cross section of a rocket, perpendicular to a filling flow of air (Midel’s cross-section); \( c_R \) - a dimensionless factor of full aerodynamic force.

Components of full aerodynamic force on axes of fast-track coordinate system: \( Y = c_y q S_M \); \( X = c_x q S_M \); \( Z = c_z q S_M \), where \( c_y \), \( c_x \) and \( c_z \) - dimensionless factors of lift force, drag force and lateral force accordingly.

Components full aerodynamic force on axes of connected coordinate system: \( N = c_n q S_M \); \( T = c_\tau q S_M \); \( B = c_b q S_M \), where \( c_n \), \( c_\tau \) and \( c_b \) - dimensionless factors of normal, tangential and lateral forces accordingly.
2.3.9. THE FORMULAS DETERMINING AERODYNAMIC FORCES

Between factors $c_n$, $c_\tau$, $c_y$, $c_x$ there are following relations:

$$c_\tau = c_x \cos \alpha - c_y \sin \alpha;$$
$$c_n = c_x \sin \alpha + c_y \cos \alpha.$$

At a rocket flight in atmosphere the angle $\alpha$ happens small, therefore $\cos \alpha \approx 1$, and $\sin \alpha \approx \alpha$. Then we can use an approximated kind

$$c_\tau \approx c_x - c_y \alpha; \quad c_n \approx c_x \alpha + c_y.$$

At small angles of attack ($\alpha < 10^\circ$) the factor of tangential force $c_\tau$ little depends on an angle of attack, and the factors of lift $c_y$ and normal $c_n$ forces are proportional to an angle of attack:

$$c_y \approx c_y^\alpha \alpha \quad c_n \approx c_n^\alpha \alpha.$$
2.4.1. MOMENTS WHICH ARE ACT ON A ROCKET

The scheme of reduction of full aerodynamic force to center of mass of a rocket
2.4.2. MOMENTS WHICH ARE ACT ON A ROCKET

\[ M_{z1} = S_M q (x_{pc} - x_{mc}) C^\alpha_y \alpha = m^\alpha_{z1} \alpha \]

\[ M_{x1} = m^\alpha_{x1} \alpha \]

\[ M_{y1} = m^\alpha_{y1} \alpha \]
2.4.3. MOMENTS WHICH ARE ACT ON A ROCKET

\[ M_{Z1c} = Y_c (x_c - x_{mc}) = Y_c \delta \delta (x_c - x_{mc}) \]
2.4.4. MOMENTS WHICH ARE ACT ON A ROCKET

\[ M_{x1c}c = 2 \cdot Y_c \delta \cdot l_c \]
2.5.1. Testing questions:

1. What coordinate systems are applied in theory of flight of rockets?
2. Why the beginning of connected and fast track coordinate systems is placing in center of mass of a rocket?
3. What is named as pitch angle, yaw angle and rolling angle?
4. Enumerate titles of axes in fast track and connected coordinate systems.
5. What is named as an angle of attack and yaw angle?
6. What forces act on a rocket at flight in atmosphere with a running engine?
2.5.2. Testing questions:

7. How the thrust force of a controlled ballistic missile changes?
8. What is an impulse of an aftereffect of a thrust? Why it aims to reduce it to a minimum?
9. How gravity of a rocket changes at its flight?
10. What reasons of the friction resistance and resistance of pressure is called?
11. Explain physical sense of bottom resistance of a rocket.
12. What singularities of supersonic flow of a rocket?
13. What the unsymmetrical flow around of a rocket from symmetrical differ?
2.5.3. Testing questions:

14. How there is a lift at unsymmetrical flow around of a rocket?
15. How are named component full aerodynamic forces in fast track and in connected coordinate systems?
16. Write the formulas of aerodynamic forces $Y$, $X$ and $Z$.
17. What moments act on a rocket at its flight in atmosphere with a running engine?
18. What is named as center of a rocket mass?
19. What is named as center of pressure?