Contents

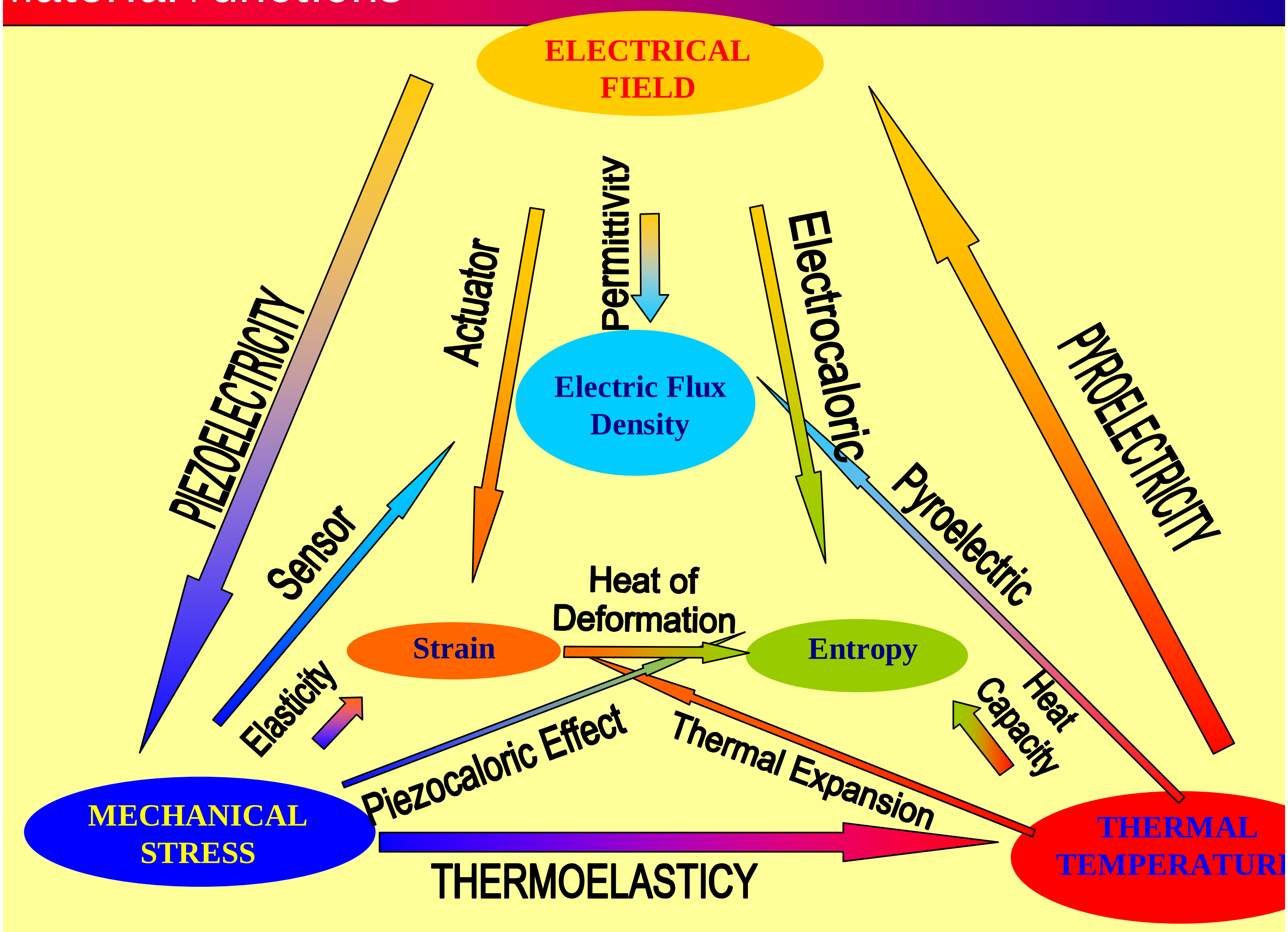
Constitutive Relations

❷ Beam, Plate and Shell Models

❸ Applications
- Panel Flutter
- Noise Attenuation

ADAPTIVE STRUCTURES

Material Functions

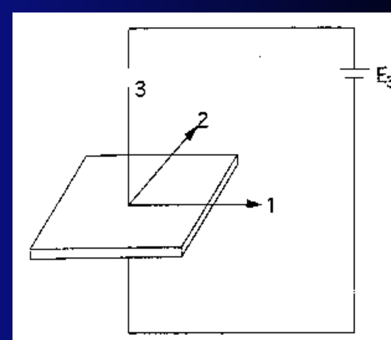


Constitutive Relations

The constitutive relations are based on the assumption that the total strain in the actuator is the sum of the mechanical strain induced by the stress, the thermal strain due to temperature and the controllable actuation strain due to the electric voltage.

$$\bar{s} = \bar{e}^{T}\mathbf{E} + \mathbf{C}\,\bar{e} + \bar{a}\ ?T$$

$$\bar{e} = \bar{d}^{T}\mathbf{E} + \mathbf{S}\,\bar{s} + \bar{a}\ ?T$$



Constitutive Relations

Re-writing the stress-strain equation:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta T$$

In a plane perpendicular to the piezo-polarization, it has isotropic properties, i.e. transversely isotropic material in the plane 1-2.

For orthotropic material, there is no temperature shear strain. However there is a shear strain induced due to the electrical fields E_1 and E_2.

Constitutive Relations

For piezoceramics, the actuation strain is:

$$\Lambda = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$

d_{33}, d_{31} and d_{15} are called piezoelectric strain coefficients of a mechanical free piezo element.

d_{31} characterizes strain in the 1 an 2 directions to an electrical field E_3 in the 3 direction

d_{33} relates strain in the 3 direction due to field in the 3 direction

d_{15} characterizes 2-3 and 3-1 shear strains due a field E_2 and E_1, respectively.

Block Force Model

- If an electric field V is applied, then the maximum actuator strain (free strain) will be:

$$e_{\max} = \Lambda = d_{31}\left(\frac{V}{t_c}\right)$$

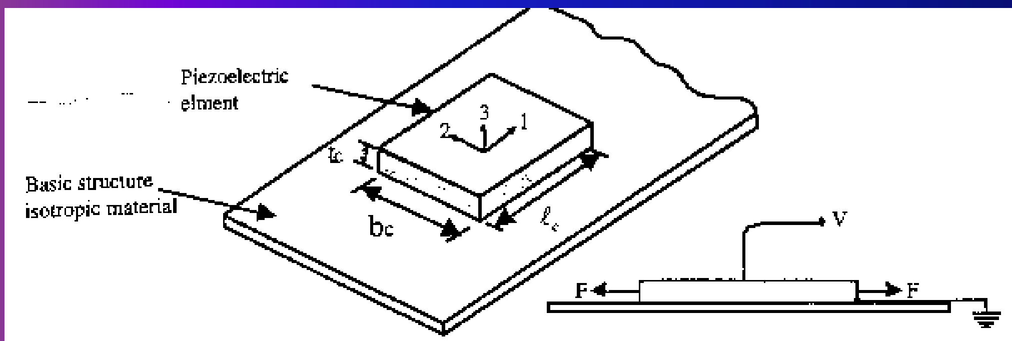
- The maximum block force (zero strain condition) is:

$$F_b = d_{31} E_c b_c V$$

Block Force Model

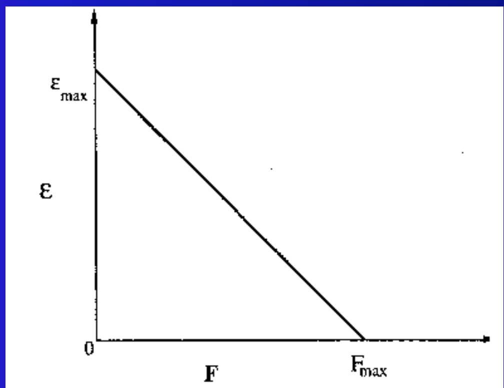
A piezo patch attached to the beam structure results in an axial force F in the beam due to potential V. The reactive force in the piezo element will be –F. Then the strain in the piezo becomes

$$e = \frac{\Delta l_c}{l_c} = d_{31}\frac{V}{t_c} - \frac{F}{b_c t_c E_c}$$



Block Force Model

Force-strain relation for constant field V:



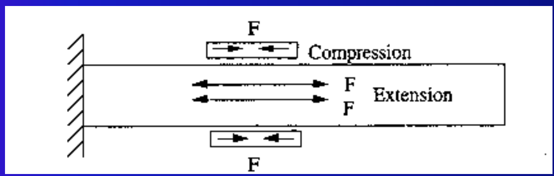
This plot can also be used to determine the properties of piezo materials experimentally.

$$d_{31} = e_{\max} \frac{t_c}{V}$$

$$E_c = \frac{F_{\max}}{e_{\max}} \frac{1}{b_c t_c}$$

Pure Extension

Two identical patches mounted on the surface of a beam, one on either side can produce pure extension



For pure extension, same potential is applied to top and bottom actuators. The induced force is

$$F = \frac{d_{31}V}{2t_c} \frac{E_b A_b E_c A_c}{E_b A_b + E_c A_c} = F_b \frac{E_b A_b}{E_b A_b + E_c A_c}$$

$$E_c A_c = 2E_c b_c t_c; \qquad E_b A_b = E_b b_b t_b$$

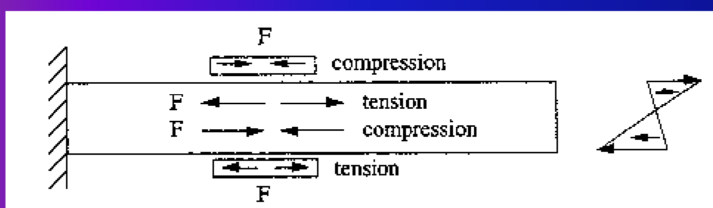
F_b is the block force for each piezo patch.

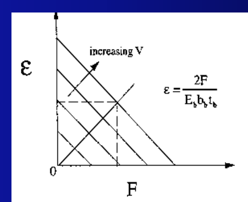
If piezo stiffness $E_c A_c >> E_b A_b$ (beam stiffness), actuation force becomes zero though actuation strain equals free strain;

If $E_c A_c << E_b A_b$ the actuation strain becomes zero though actuation force equals block force

Pure Bending

For pure bending, an equal and opposite potential is applied to top and bottom actuators





The induced bending is

$$M = 2\frac{d_{31}V}{t_c t_b}\frac{E_b I_b E_c I_c}{E_b I_b + E_c I_c} = M_b \frac{E_b I_b}{E_b I_b + E_c I_c}$$

$$E_c I_c = 2E_c b_c t_c \left(\frac{t_b}{2}\right)^2$$

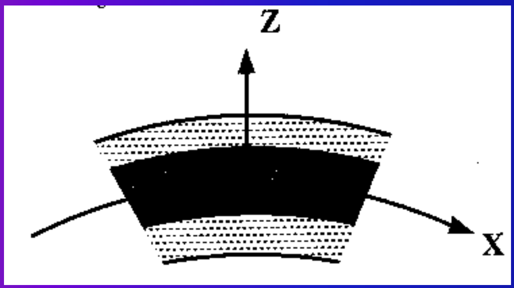
M_b is the block moment for each piezo patch.

If $E_c I_c >> E_b I_b$ actuation moment becomes zero

If $E_c I_c << E_b I_b$ actuation strain becomes zero

Euler-Bernoulli Beam Model

- Beam, adhesive and actuator form a continuous structure
- Bernoulli´s assumption: a plane section normal to the beam axis remains plane and normal to the beam axis after bending
- Linear distribution of strain in actuator and host structure
- Generally gives more accurate results than uniform strain model



$$e(z) = e_0 - z\,k, \qquad k = -w,_{xx}$$

$$e_{net} = e(z) - \Lambda(z)$$

$$s_{xx}(z) = E(z)\,e_{net}$$

Bernoulli-Euler Beam Model

Axial force and bending moment expressions are:

$$\begin{Bmatrix} F + F_{\Lambda} \\ M + M_{\Lambda} \end{Bmatrix} = \begin{bmatrix} E_0 & E_1 \\ E_1 & E_2 \end{bmatrix} \begin{Bmatrix} e_0 \\ w,_{xx} \end{Bmatrix}$$

where

F is the axial force in the beam
M is the bending moment in the beam
b(z) is the beam width

$$F = \int_{-\frac{h}{2}}^{\frac{h}{2}} b(z) s_{xx}(z) dz$$

$$M = \int_{-\frac{h}{2}}^{\frac{h}{2}} b(z) s_{xx}(z) z dz$$

$b(z)$ is beam width

Euler–Bernoulli Beam Model

Axial force and bending moment due to induced stress:

$$F_{\Lambda} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b(z)E(z)\Lambda(z)dz\,, \quad M_{\Lambda} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b(z)E(z)\Lambda(z)zdz$$

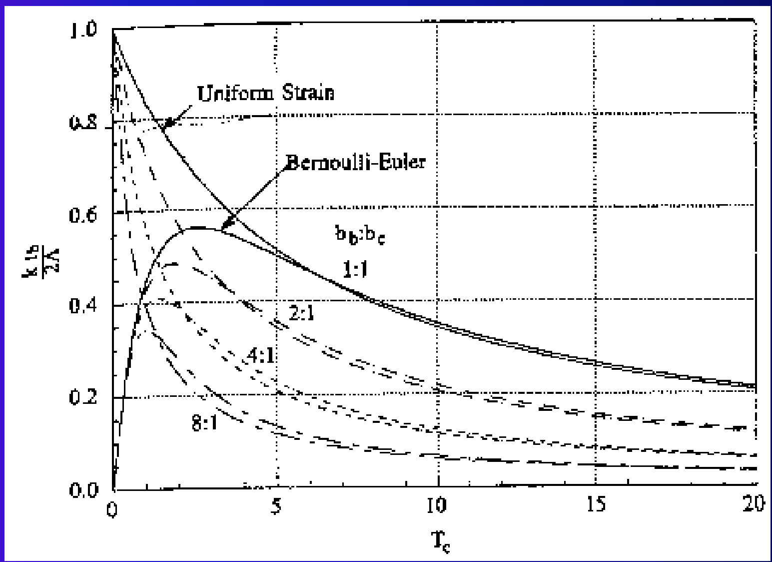
If the placement of the actuators is symmetric, the coupling term will be zero; if not, this term will be non-zero: extension-bending coupling

$$E_j = \int_{-\frac{h}{2}}^{\frac{h}{2}} b(z)E(z)z^j dz\,, \quad j = 0,1,2$$

Uniform Strain and Euler-Bernoulli Beam Models

The thickness ratio, T, determines if the strain variation across the piezo affects the analysis:

- for small T, the uniform strain model overpredicts strain (curvature)
- for large T, the predicted induced bending curvatures are identical for both models



$$T = \frac{t_b}{t_c}$$

Plate with Induced Strain Actuation

Induced strain actuation is used to control the extension, bending and twisting of a plate

Using tailored anisotropic plates with distributed piezo actuators, the control of specific static deformation can be augmented

Plate with Induced Strain Actuation

Assumptions to develop a consistent plate model:
- Actuators and substrates are integrated as plies of a laminated plate
- A consistent deformation is assumed in the actuators and substrates
- Generally, a thin classical laminated plate theory is adopted

For systems actuated in extension:
- Assume strains are constant across the thickness of actuators and plate

For systems actuated in pure bending:
- Assume strains vary linearly through the thickness

Plate with Induced Strain Actuation

Strain in the system: $e = e^0 + z\,k$

Mid-plane strain:

$$e^0 = \{ e_x^0 \quad e_y^0 \quad e_{xy}^0 \}^T = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}^T$$

Curvature:

$$k = \{ k_x \quad k_y \quad k_{xy} \}^T = \left\{ -\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x \partial y} \right\}^T$$

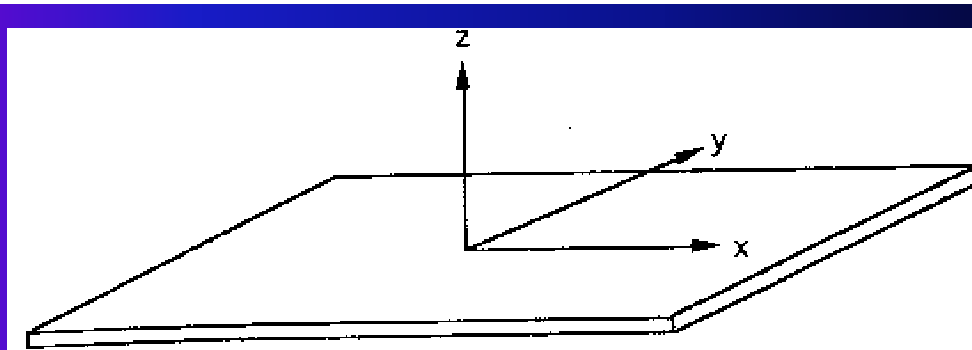


Plate with Induced Strain Actuation

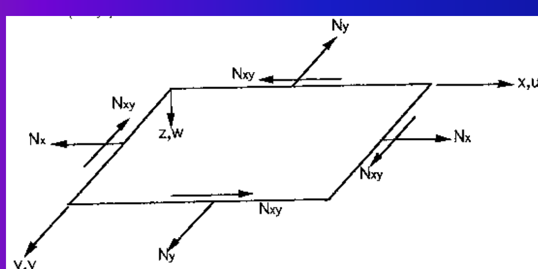
Constitutive relation for any ply: $s = \overline{Q}(e - \Lambda)$

$\overline{Q}$ is the transformed reduced stiffness of the plate

The second term represents an equivalent stress due to the actuation

Stress vector: $s = \{s_x \quad s_y \quad t_{xy}\}^T$

Actuation strain vector $\Lambda = \{\Lambda_x \quad \Lambda_y \quad \Lambda_{xy}\}^T$



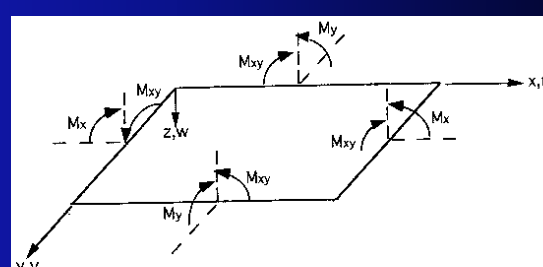
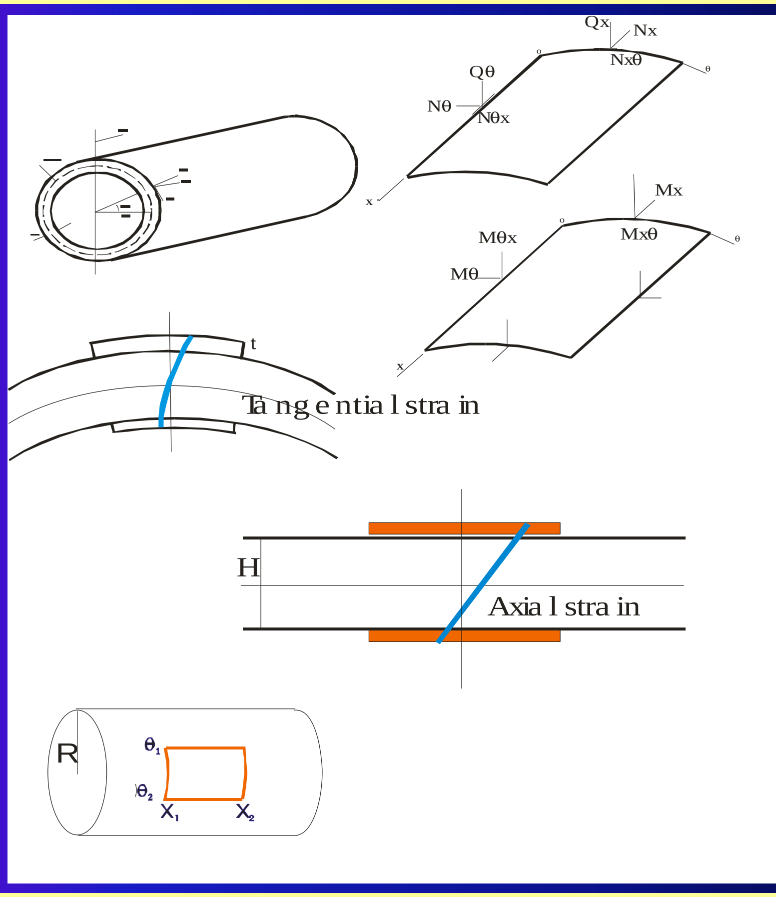


Plate with Induced Strain Actuation

Net forces and moments

$$\begin{Bmatrix} N_x \\ N_y \\ N_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{11} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{6} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{6} \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{6} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{6} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} e_x^0 \\ e_y^0 \\ g_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

Shells


Qx
Nx
Nxθ
Qθ
Nθ
Nθx
x
Mx
Mθx
Mxθ
Mθ
x
t
Ta ng e ntia l stra in
H
Axia l stra in
R
θ1
θ2
X1
X2

Shells

Strain-Displacement Relations

$$\varepsilon_x = \frac{\partial u}{\partial x}; \qquad \kappa_x = -\frac{\partial^2 w}{\partial x^2};$$

$$\varepsilon_\theta = \frac{\partial v}{\partial \theta} + \frac{w}{R}; \qquad \kappa_\theta = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \underline{\frac{1}{R^2}\frac{\partial v}{\partial \theta}};$$

$$\varepsilon_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}; \qquad \tau = -\frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta} + \underline{\frac{2}{R}\frac{\partial v}{\partial x}}$$

Piezo Patch Contributions

Finite Patches

$$M_{x_p} = \left[M_{x_{Pinner}} + M_{x_{Pouter}} \right] \left[H_1(x) - H_2(x) \right] \left[H_1(\theta) - H_2(\theta) \right]$$

$$M_{\theta_p} = \left[M_{\theta_{Pinner}} + M_{\theta_{Pouter}} \right] \left[H_1(x) - H_2(x) \right] \left[H_1(\theta) - H_2(\theta) \right]$$

$$N_{x_p} = \left[N_{x_{Pinner}} + N_{x_{Pouter}} \right] \left[H_1(x) - H_2(x) \right] \left[H_1(\theta) - H_2(\theta) \right] S_{1,2}(x) \, \hat{S}_{1,2}(\theta)$$

$$N_{\theta_p} = \left[N_{\theta_{Pinner}} + N_{\theta_{Pouter}} \right] \left[H_1(x) - H_2(x) \right] \left[H_1(\theta) - H_2(\theta) \right] S_{1,2}(x) \, \hat{S}_{1,2}(\theta)$$

Concluding Remarks

Analytical models for beam, plate and shell type elements have been presented.

The weak form of the equations of motion are desirable since they circumvent the need to differentiate terms with patch force and moment terms.

The analytical models provide a physical appreciation of the interaction between the structure and the actuating piezo patches

Finite Element Models

Piezoelectric Finite Elements

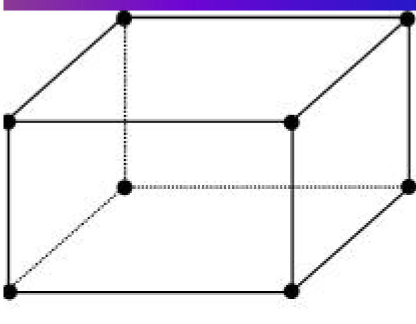
- ☞ **Solid, Plate and Beam Models**

❷ **Simple Plate Finite Element Model**

❸ **Actuation and Sensing Examples**

- ☞ **Bimorph beam**
- ☞ **Adaptive Composite Plate**

Solid Elements

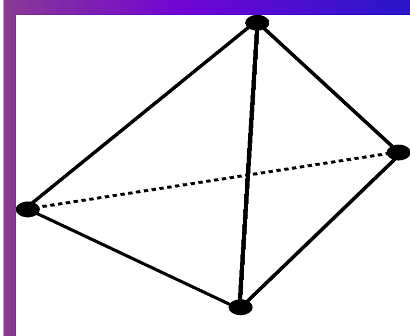

Hex8

Allik and Hughes (1970)

u,v,w, φ : linear

16 dof

Static condensation of the electric dof


Teta4

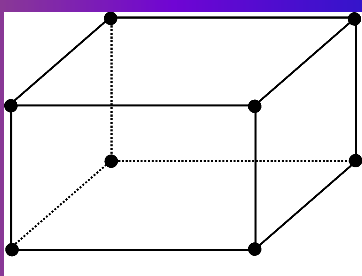
Gandhi and Hagood (1997)

u,v,w, φ : linear

16 dof + internal dof

Nonlinear constitutive relations

Solid Elements



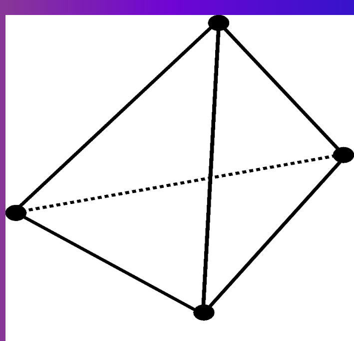
Hex8

Tzou and Tseng (1990)

u,v,w,φ : linear + quadratic incompatible mode

32 dof

Static condensation of the electric dof



Teta4

Ha and Keilers (1992)

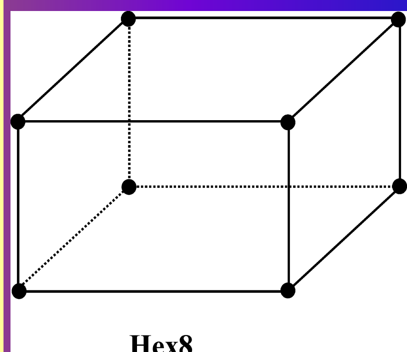
u,v,w,φ : linear + quadratic incompatible mod

32 dof

Equivalent single layer model

Static condensation of incompatible modes

Solid Elements

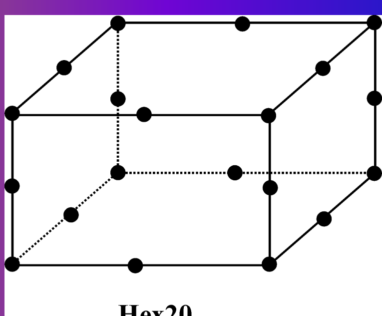


Chin and Varadan (1994)

u,v,w,φ : linear

32 dof

Lagrange method



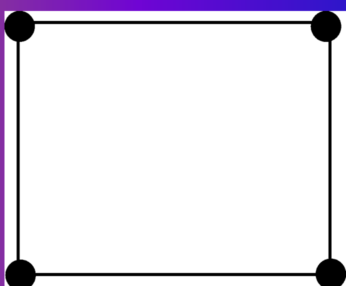
Allik and Webman (1974)

u,v,w,φ : quadratic

80 dof

Sonar transducers

Shell Elements



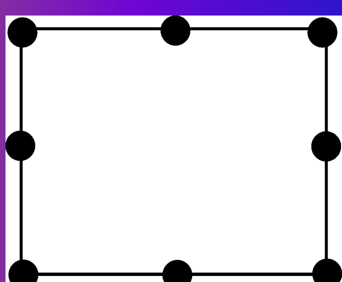
Lammering (1991)

$u, v, w, \beta_x, \beta_y$: linear

28 dof

Shallow shell theory

Upper-lower nodal electric potential dof



Thirupati et al (1997)

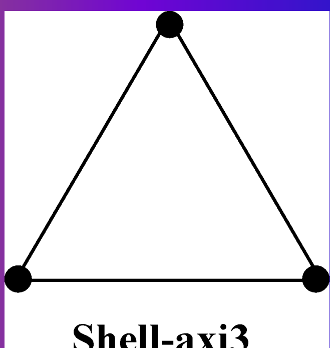
u, v, w, ϕ: quadratic

32 dof

3D degenerated shell theory

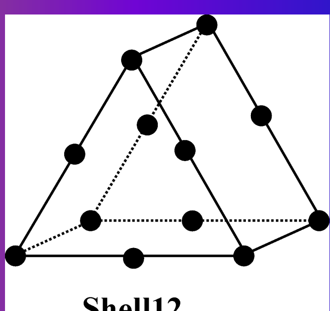
Piezo effect as initial strain problem

Shell Elements



Varadan et al (1993)

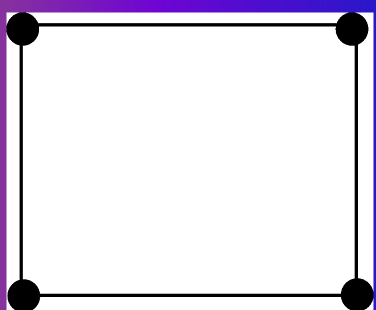
u,w,ϕ : linear
9 dof
Lagrange formulation
Mooney transducers



Tzou and Ye (1993)

u,v,w,ϕ : in-plane quadratic, thickness linear
48 dof
Layerwise constant shear angle theory
Laminated piezo shell continuum

Plate Elements



Suleman and Venkayya (1995)

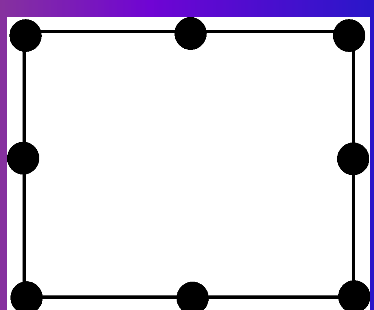
u,v,w, θ_x, θ_y, θ_z : bilinear

ϕ: linear

24 dof

Mindlin plate element C^0

1 dof per piezo patch/layer



Ray et al (1994)

w: cubic

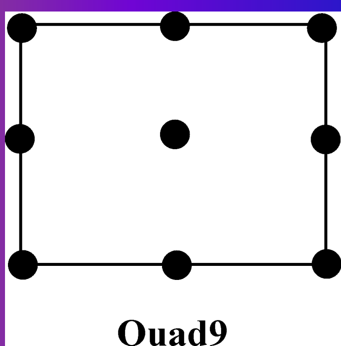
ϕ: linear

104 dof

Linear potential in thickness

1 dof per piezo patch/layer

Plate Elements



Yin and Shen (1997)

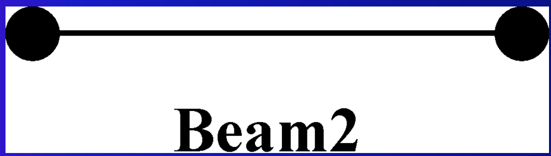
u,v,w, βx, βy, ϕ : quadratic

54 dof

Mindlin plate theory C^0

Linear voltage but transverse field dof

Beam Elements



Shen (1994)

U: linear

W: cubic hermite

B: linear

8 dof

Timoshenko beam theory with Hu-Washizu Principle (Mixed)

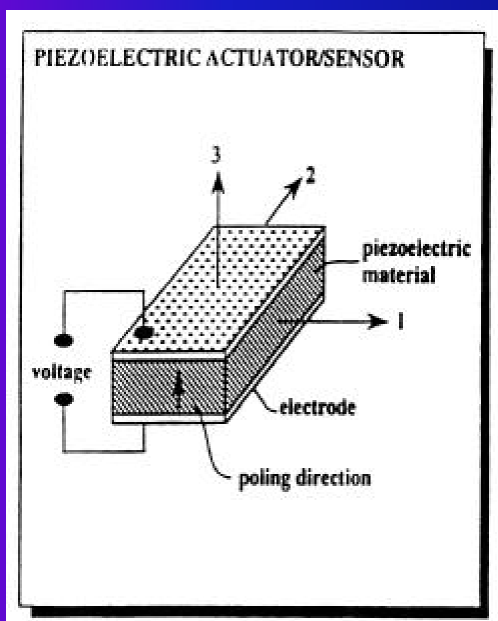
Offset nodes

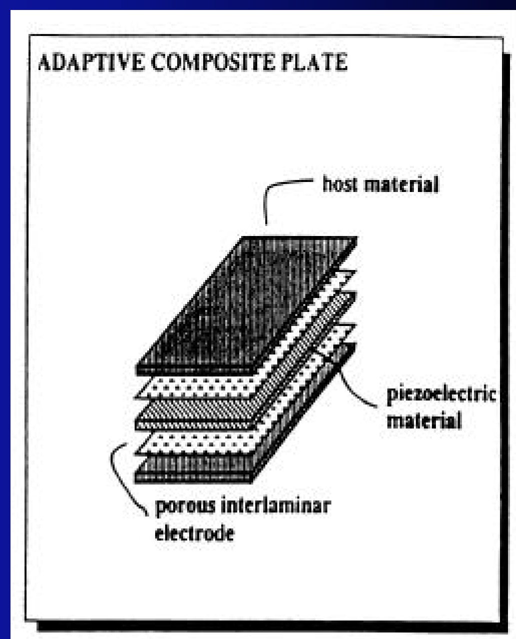
Summary of Available Elements

Elements	Shape and approximations	
Solid	4-nodes linear tetrahedron 8-nodes linear hexahedron 20-nodes quadratichexahedron	available available available
Shell	3-nodes linear axisymmetric flat triangle 8-nodes quadratic axisymm. quadrangle 4-nodes linear flat quadrangle 8-nodes 3D-degenerated quadratic quad 12-nodes 3D-degenerated quadratic prism	available available available not available available
Plate	3-nodes linear triangle 4-nodes linear quadrangle 8-nodes quadrangle 9-nodes quadrangle	not available available available available
Beam	2-nodes linear element 3-nodes quadratic element	available not available

Adaptive Composite Plate Model

- If an electric field V is applied, then maximum actuator strain (free strain) will be:





Kinetic, Potential and Electrical Energies

- **The Hamiltonian for the system is**

$$d\int_{t_1}^{t_2} \left[T - \Pi + W_e\right] dt = 0$$

$$T = \int_V \frac{1}{2}\, r\, \dot{\bar{u}}^T\, \dot{\bar{u}}\; dV; \qquad \Pi = \int_V \frac{1}{2}\, \bar{S}^{c^T}\, \bar{T}^c\, dV$$

$$W_e = \int_{V_p} \frac{1}{2}\, \bar{S}^{e^T}\, \bar{T}^e\, dV_p$$

Stress-Strain Relations

$$\bar{T}^{e} = \mathbf{e}^{T}\ \bar{S}^{c} + \boldsymbol{e}\ \bar{S}^{e}$$
$$\bar{T}^{c} = \mathbf{c}\ \bar{S}^{c} - \mathbf{e}\ \bar{S}^{e}$$

$$\begin{aligned}\bar{S} &= \left\{\bar{S}^{m} \quad \bar{S}^{b} \quad \bar{S}^{ts} \quad \bar{S}^{e}\right\} \\ &= \left\{S_{x}^{m} \quad S_{y}^{m} \quad S_{xy}^{m} \quad S_{x}^{b} \quad S_{y}^{b} \quad S_{xy}^{b} \quad S_{xz}^{ts} \quad S_{yz}^{ts} \quad -E_{1} \quad \ldots \quad -E_{n_p}\right\}\end{aligned}$$

$$\begin{aligned}\bar{T} &= \left\{\bar{T}^{m} \quad \bar{T}^{b} \quad \bar{T}^{ts} \quad \bar{T}^{e}\right\} \\ &= \left\{T_{x}^{m} \quad T_{y}^{m} \quad T_{xy}^{m} \quad T_{x}^{b} \quad T_{y}^{b} \quad T_{xy}^{b} \quad T_{xz}^{ts} \quad T_{yz}^{ts} \quad D_{1} \quad \ldots \quad D_{n_p}\right\}\end{aligned}$$

Stress-Strain Relations

$$\bar{T} = \begin{Bmatrix} \bar{T}^{c} \\ \bar{T}^{e} \end{Bmatrix} = \begin{bmatrix} \mathbf{c} & \mathbf{c} & \mathbf{0} & \mathbf{e} \\ \mathbf{c} & \mathbf{c} & \mathbf{0} & \mathbf{e} \\ \mathbf{0} & \mathbf{0} & \mathbf{g} & \mathbf{0} \\ \mathbf{e}^{T} & \mathbf{e}^{T} & \mathbf{0} & e \end{bmatrix} \begin{Bmatrix} \bar{S}^{m} \\ \bar{S}^{b} \\ \bar{S}^{ts} \\ \bar{S}^{e} \end{Bmatrix} - \begin{Bmatrix} \bar{a}^{m} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta T$$

Strain-Displacement Relations

$$\bar{S}^b = \begin{Bmatrix} S_x^b \\ S_y^b \\ S_{xy}^b \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} w_{,x}^2 \\ w_{,y}^2 \\ 2w_{,x} w_{,y} \end{Bmatrix} + z \begin{Bmatrix} -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{Bmatrix}$$

$$\mathbf{N} = \frac{1}{4} \left(\bar{C} + \bar{x}\, x + \bar{h}\, h + \bar{H} x h \right)$$

$$\bar{q}_i^s = \{ u \quad v \quad w \quad q_x \quad q_y \}_i ;$$
$$\bar{q}^e = \{ f_1 \quad \dots \quad f_{n_p} \}$$

Strain-Displacement Relations

$$\bar{S} = \begin{Bmatrix} \bar{S}^s \\ \bar{S}^e \end{Bmatrix} = \begin{bmatrix} \mathbf{b}^s & \mathbf{0} \\ \mathbf{0} & \mathbf{b}^e \end{bmatrix} \begin{Bmatrix} \bar{q}^s \\ \bar{q}^e \end{Bmatrix}$$

$$\mathbf{b}_i^s = \begin{bmatrix} \frac{\partial N_i^s}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i^s}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_i^s}{\partial y} & \frac{\partial N_i^s}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z\frac{\partial N_i^s}{\partial x} \\ 0 & 0 & 0 & -z\frac{\partial N_i^s}{\partial y} & 0 \\ 0 & 0 & 0 & -z\frac{\partial N_i^s}{\partial x} & z\frac{\partial N_i^s}{\partial x} \\ 0 & 0 & \frac{\partial N_i^s}{\partial x} & 0 & N_i^s \\ 0 & 0 & \frac{\partial N_i^s}{\partial x} & -N_i^s & 0 \end{bmatrix}; \; i = 1, \cdots, n_{el}$$

System Matrices

$$\mathbf{M}_{cc}^{j} = \int_{V_j} r\,\mathbf{N}^{T}\,\mathbf{N}\,dV_j\,,$$

$$\mathbf{K}_{cc}^{j} = \int_{V_j} \mathbf{b}^{c^T}\,\mathbf{c}\,\mathbf{b}^{c}\;dV_j\,,$$

$$\mathbf{K}_{ce}^{j} = \int_{V_j} \mathbf{b}^{c^T}\,\mathbf{e}\,\mathbf{b}^{e}\;dV_j\,,$$

$$\mathbf{K}_{ee}^{j} = \int_{V_j} \mathbf{b}^{e^T}\,\mathbf{e}\,\mathbf{b}^{e}\;dV_j\,, \qquad \textit{\textbf{for}} \qquad j = 1,\cdots,n_{el}$$

Geometric Stiffness

$$\mathbf{K}_g = N_x \int_A \mathbf{N}_\mathbf{x}^\mathbf{T} \mathbf{N}_\mathbf{x} dA + N_y \int_A \mathbf{N}_y^\mathbf{T} \mathbf{N}_y dA + N_{xy} \int_A \mathbf{N}_x^\mathbf{T} \mathbf{N}_y dA + N_{xy} \int_A \mathbf{N}_y^\mathbf{T} \mathbf{N}_x dA$$

Equations of Motion

$$\overbrace{\begin{bmatrix} \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{\text{inertia}} \begin{Bmatrix} \ddot{\overline{U}}^{c} \\ \ddot{\overline{U}}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{K}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{\text{linear stiffness}} \begin{Bmatrix} \overline{U}^{c} \\ \overline{U}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{0} & \mathbf{K}_{ce} \\ \mathbf{K}_{ec} & \mathbf{K}_{ee} \end{bmatrix}}^{\text{piezo stiffness}} \begin{Bmatrix} \overline{U}^{c} \\ \overline{U}^{e} \end{Bmatrix} +$$

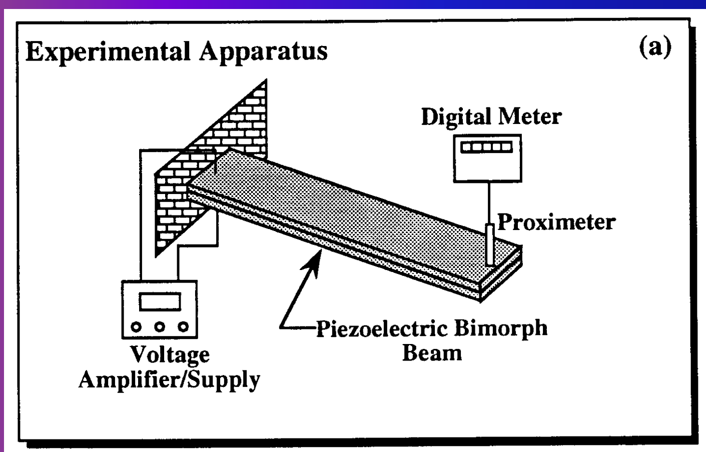
$$\overbrace{\begin{bmatrix} \mathbf{K}_{\Delta \mathbf{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{\text{thermal stiffness}} \begin{Bmatrix} \overline{U}^{c} \\ \overline{U}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{K}_{g} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{\text{nonlinear stiffness}} \begin{Bmatrix} \overline{U}^{c} \\ \overline{U}^{e} \end{Bmatrix} = \begin{Bmatrix} P_{\Delta T} \\ \mathbf{0} \end{Bmatrix}$$

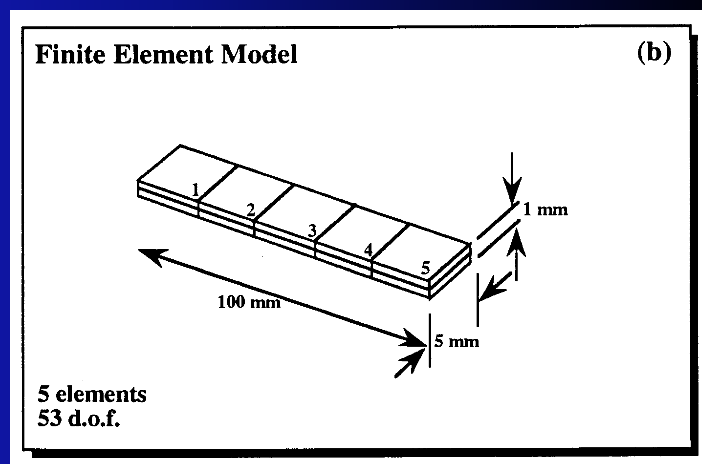
Actuation and Sensing Mechanisms

$$\overline{U}_c = \mathbf{K}_{\mathbf{cc}}^{-1}\,\mathbf{K}_{\mathbf{ce}}\,\overline{U}_e$$

$$\overline{U}_e = \mathbf{K}_{ee}^{-1}\,\mathbf{K}_{ec}\,\overline{U}_c$$

Bimorph Beam





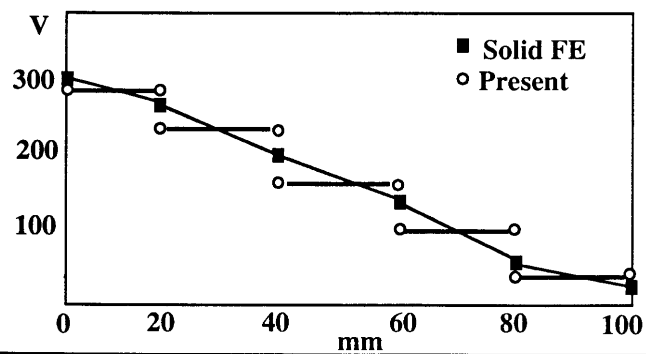
Bimorph Beam

Actuation Mechanism (c)

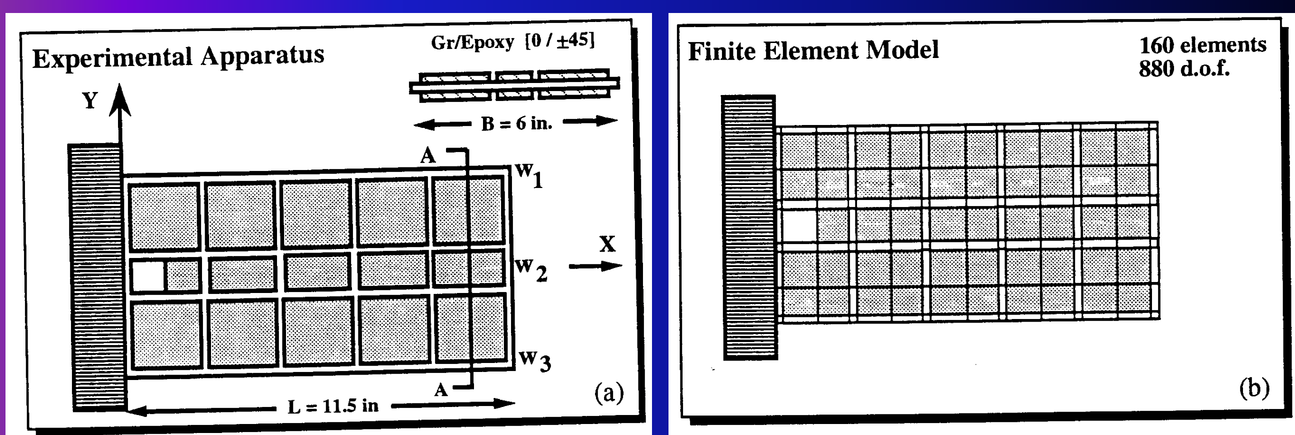
x10 $^{-7}$ m

Position	1	2	3	4	5
Theory	0.14	0.55	1.24	2.21	3.45
Beam FE[6]	0.12	0.51	1.16	2.10	3.30
Present	0.14	0.55	1.24	2.21	3.45
EXP[6]	-	-	-	-	3.15

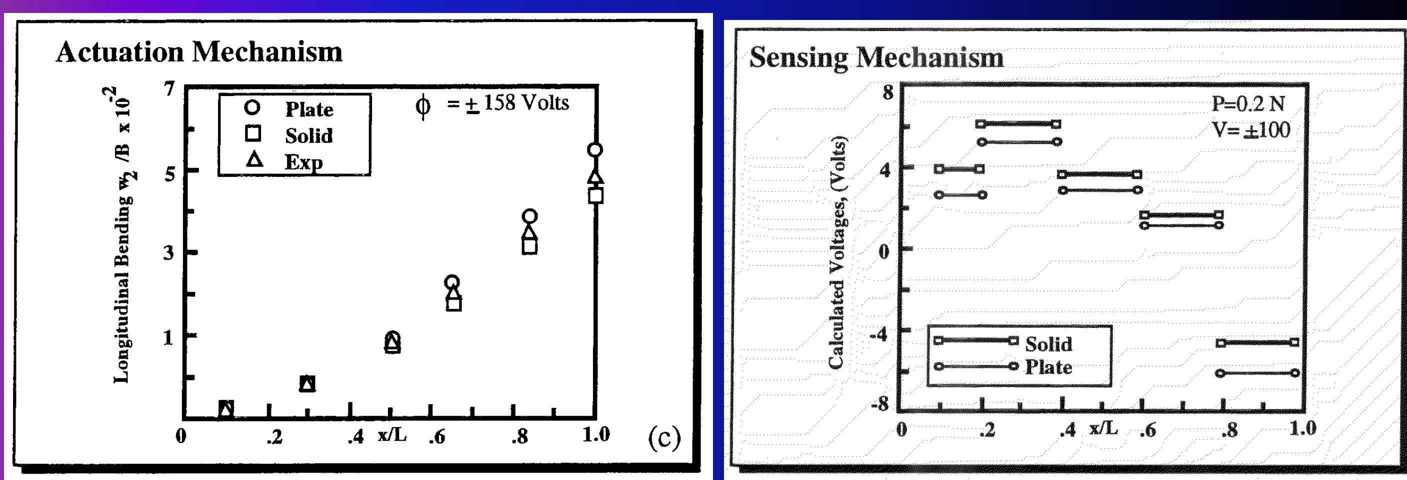
Sensing Mechanism (d)



Composite Plate

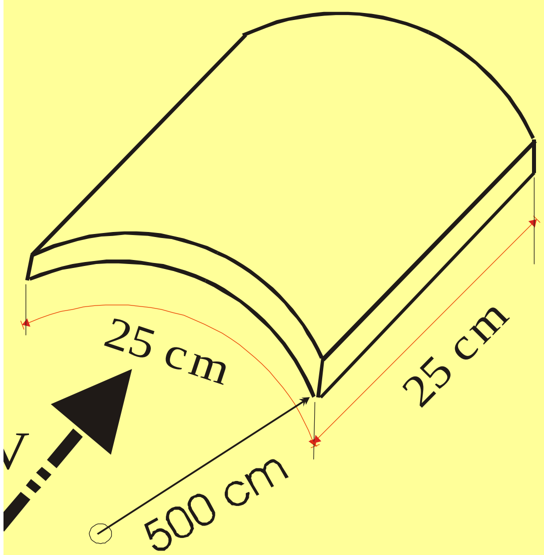


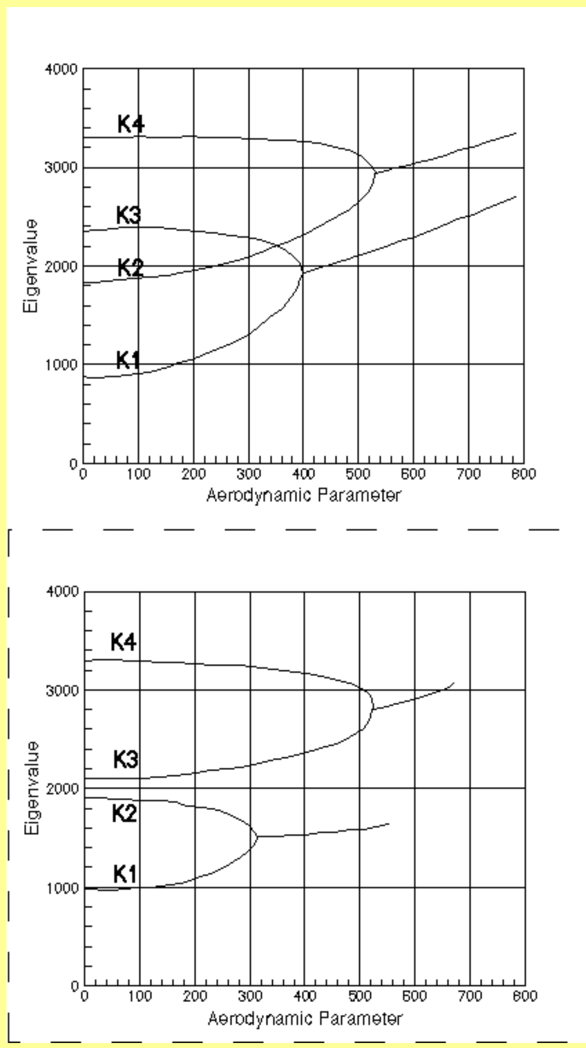
Composite Plate



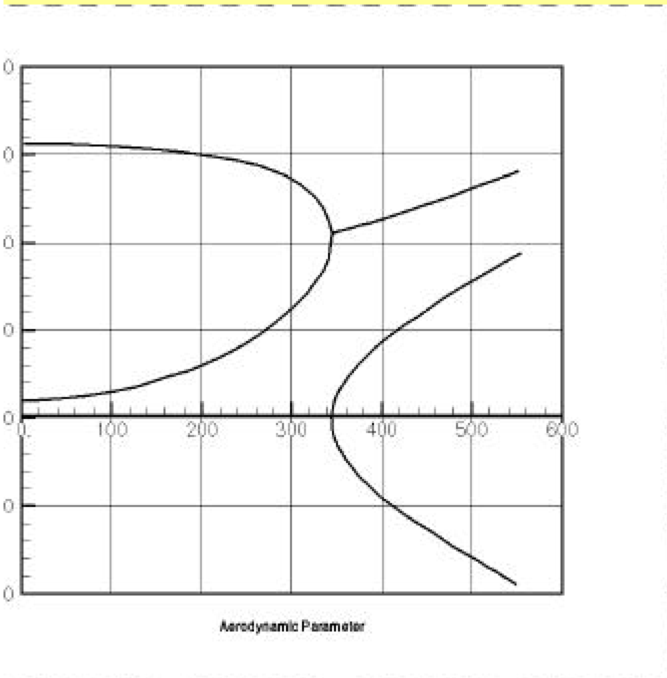
ADAPTIVE STRUCTURES

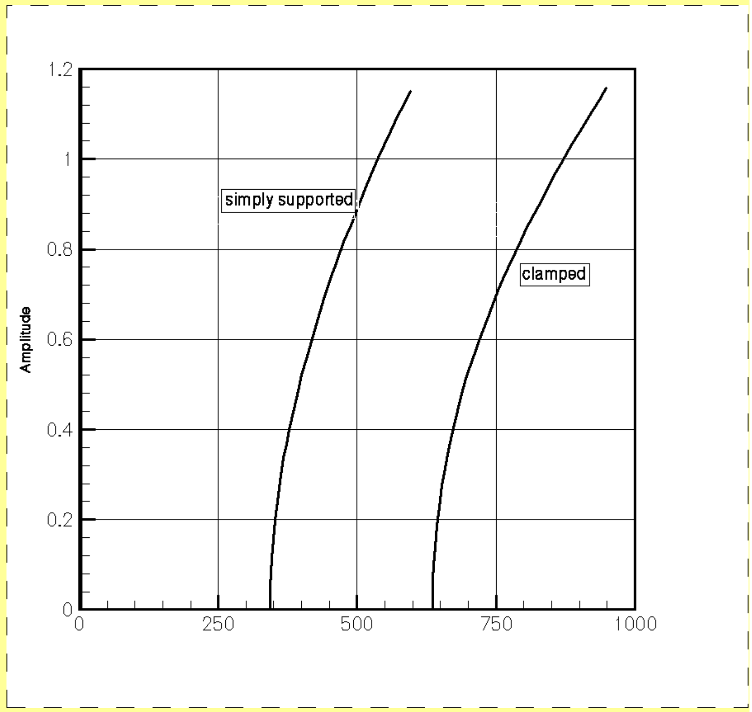
PANEL FLUTTER



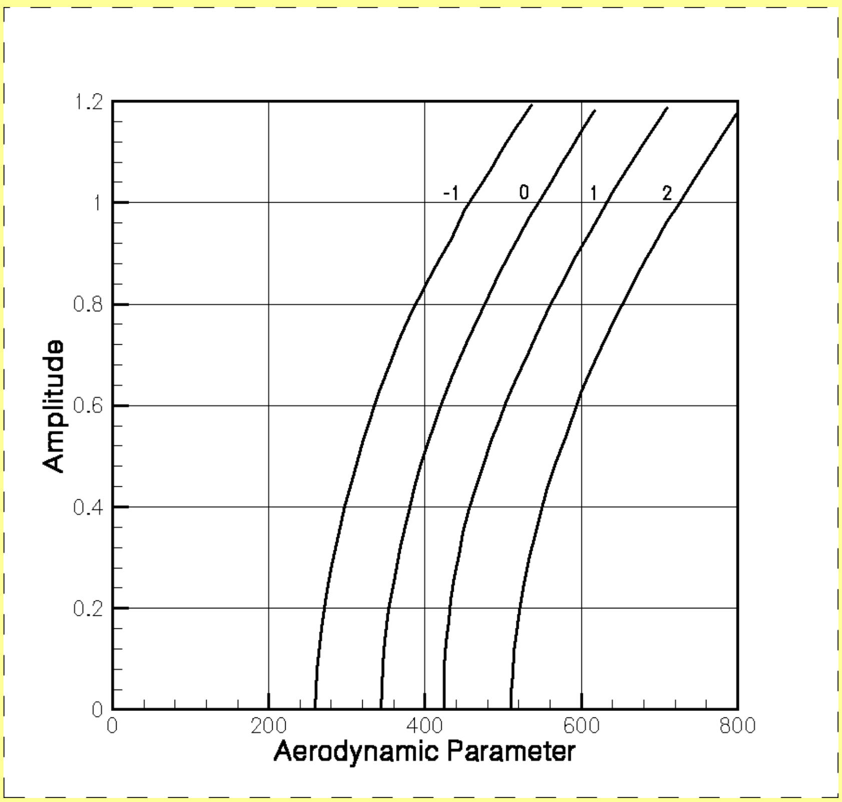


PANEL FLUTTER – BOUNDARY EFFECT





PANEL FLUTTER – IN PLANE LOADING



PANEL FLUTTER

$$\overbrace{\begin{bmatrix} \mathbf{M}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{inertia} \begin{Bmatrix} \ddot{\bar{U}}^{c} \\ \ddot{\bar{U}}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{aero\ \ damping} \begin{Bmatrix} \dot{\bar{U}}^{c} \\ \dot{\bar{U}}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{K}_{cc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{linear\ \ stiffness} \begin{Bmatrix} \bar{U}^{c} \\ \bar{U}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{0} & \mathbf{K}_{ce} \\ \mathbf{K}_{ec} & \mathbf{K}_{ee} \end{bmatrix}}^{piezo\ \ stiffness} \begin{Bmatrix} \bar{U}^{c} \\ \bar{U}^{e} \end{Bmatrix}$$

$$\overbrace{\begin{bmatrix} \mathbf{K}_{\Delta \mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{thermal\ stiffness} \begin{Bmatrix} \bar{U}^{c} \\ \bar{U}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{K}_{a} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{aero\ \ stiffness} \begin{Bmatrix} \bar{U}^{c} \\ \bar{U}^{e} \end{Bmatrix} + \overbrace{\begin{bmatrix} \mathbf{K}_{g} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}^{nonlinear\ \ stiffness} \begin{Bmatrix} \bar{U}^{c} \\ \bar{U}^{e} \end{Bmatrix} = \begin{Bmatrix} P_{\Delta T} \\ \mathbf{0} \end{Bmatrix}$$

AERODYNAMIC LOADS

$$p_a = -\frac{2q}{\sqrt{M_\infty^2 - 1}}\left(w_{,x} + \frac{M_\infty^2 - 2}{M_\infty^2 - 1}\frac{1}{V_\infty} w_{,t} - \frac{1}{2r\sqrt{M_\infty^2 - 1}} w_{,t} \right)$$

$$p_a = -\left(l \frac{D}{a^3} w_{,x} + \frac{g_a}{w_0}\frac{D}{a^4} w_{,t} - \frac{l}{2r\,b}\frac{D}{a^3} w_{,t} \right)$$

$$l = \frac{2q_a a^3}{D\sqrt{M_\infty^2 - 1}}; \qquad g_a = \sqrt{l\,\frac{m\left(M_\infty^2 - 2\right)^2}{b\left(M_\infty^2 - 1\right)}}$$

AERODYNAMIC LOADS

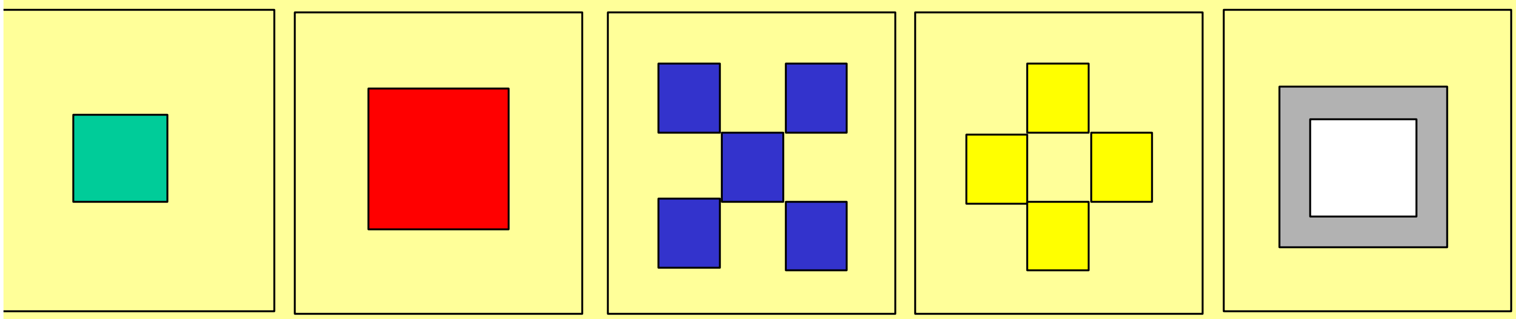
$$W_a = -\int_A \left(l \, \frac{D}{a^3}\frac{\partial w}{\partial x} + \frac{g_a}{w_0}\frac{D}{a^4}\frac{\partial w}{\partial t} - \frac{l}{2r\,b}\frac{D}{a^3}\frac{\partial w}{\partial t} \right) w \, dA$$

$$\mathbf{G}^j = \left(g_a - \frac{l}{2r\,b}\frac{D}{a^3} \right) \int_A \mathbf{N}^T \, \mathbf{N} \, dA,$$

$$\mathbf{K}_a^j = l \int_A \mathbf{N}^T \, \mathbf{N}_{,x} \, dA \qquad \textbf{\textit{for}} \qquad j = 1, \cdots, n_{el}$$

RESULTS

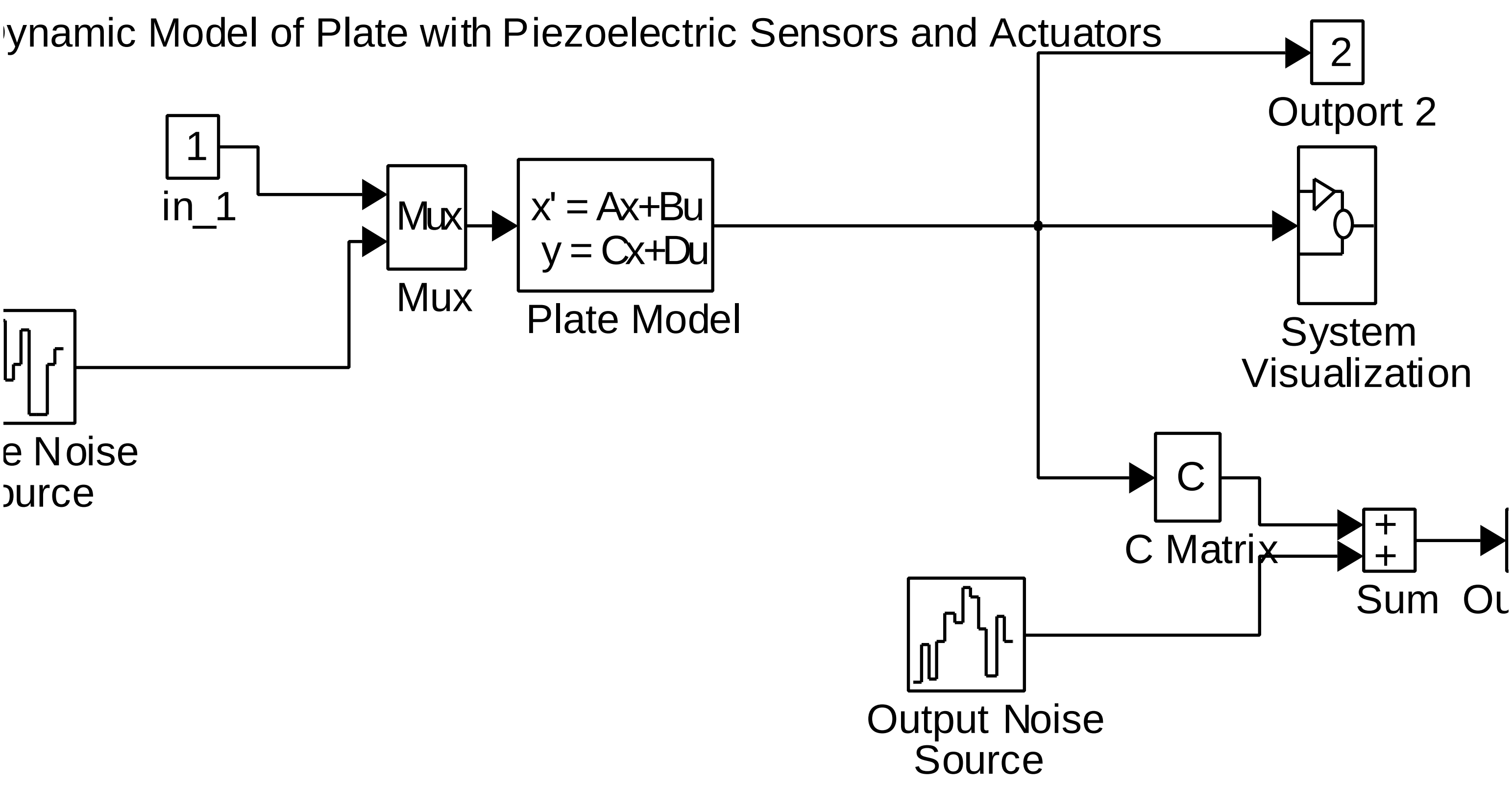
#1 #2 #3 #4 #5



Critical Aerodynamic Parameter

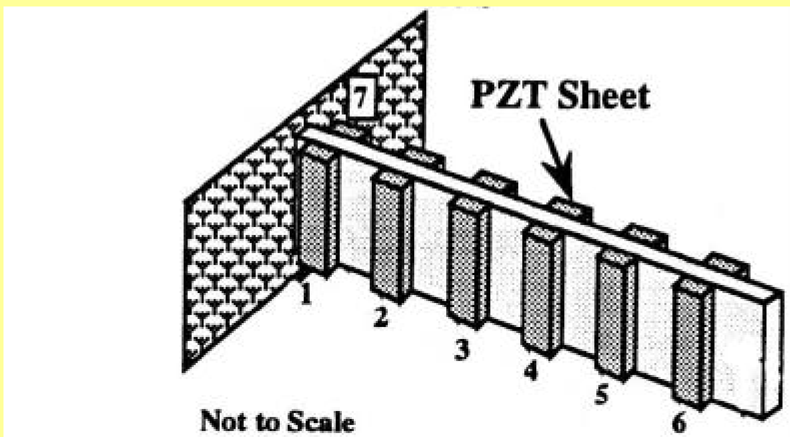
$l_{critical} = 36.8$

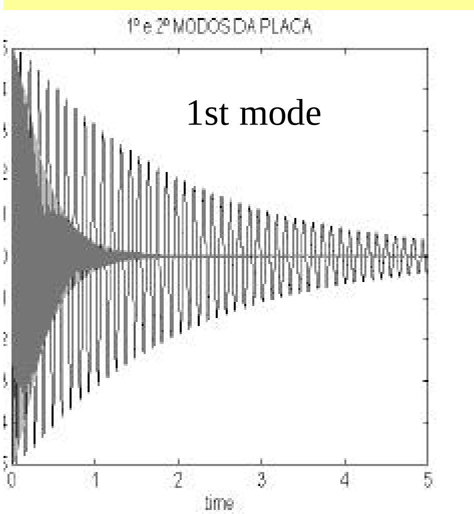
	Configuration				
	#1	#2	#3	#4	#5
0 V	46.9	70.5	88.5	91.8	63.9
400 V	66.7	93.5	92.5	99.2	76.5
$l_{critical}$	+42%	+32%	+5%	+8%	+20%
Mass	+17%	+69%	+86%	+69%	+52%

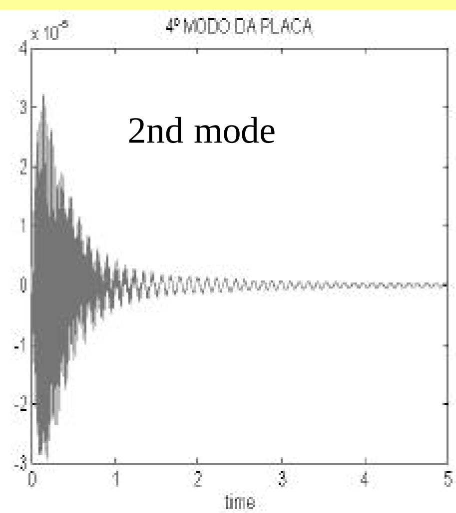

ynamic Model of Plate with Piezoelectric Sensors and Actuators
1
in_1
Mux
Mux
x' = Ax+Bu
y = Cx+Du
Plate Model
2
Outport 2
System
Visualization
e Noise
ource
C
C Matrix
+
+
Sum
Ou
Output Noise
Source

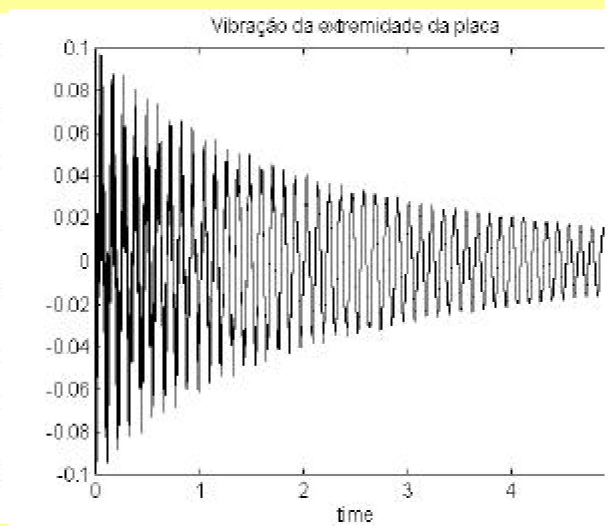
ADAPTIVE STRUCTURES

ACTIVE CONTROL

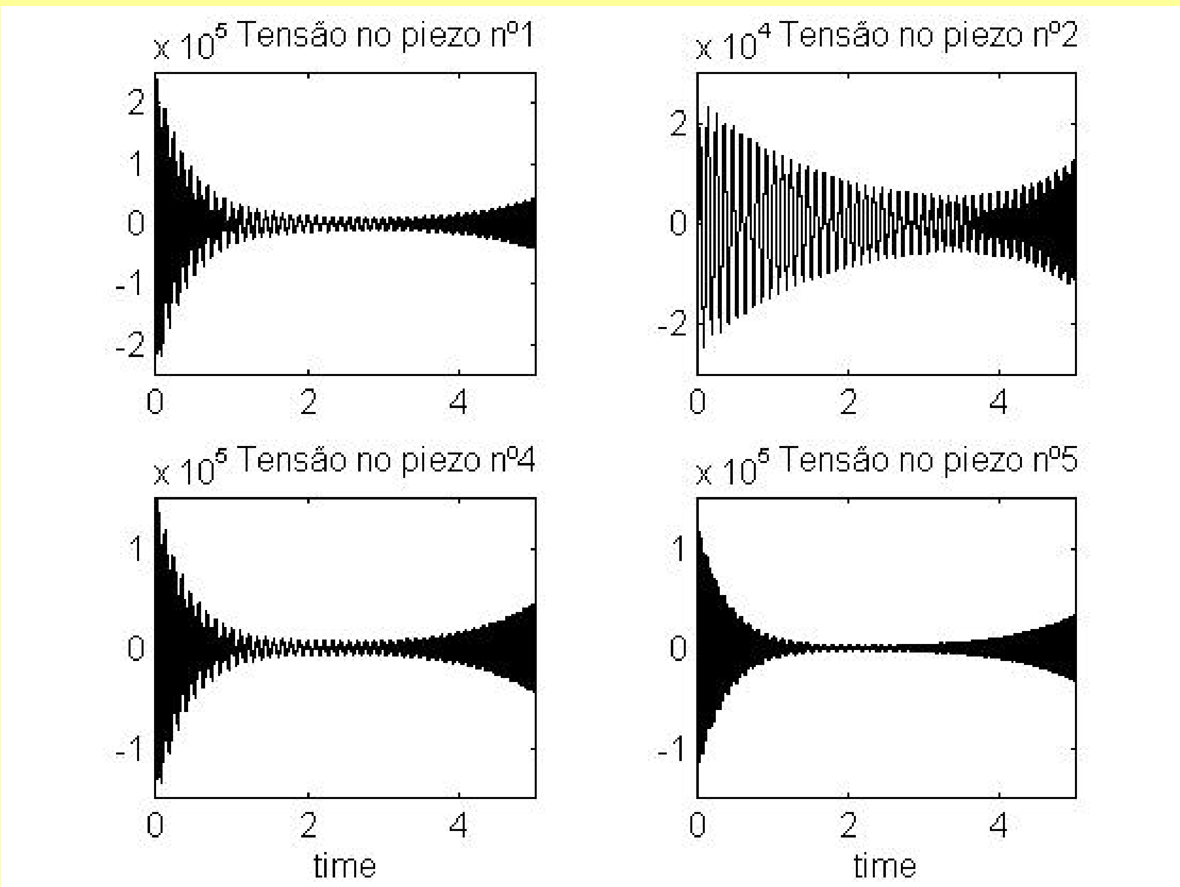








ACTIVE CONTROL



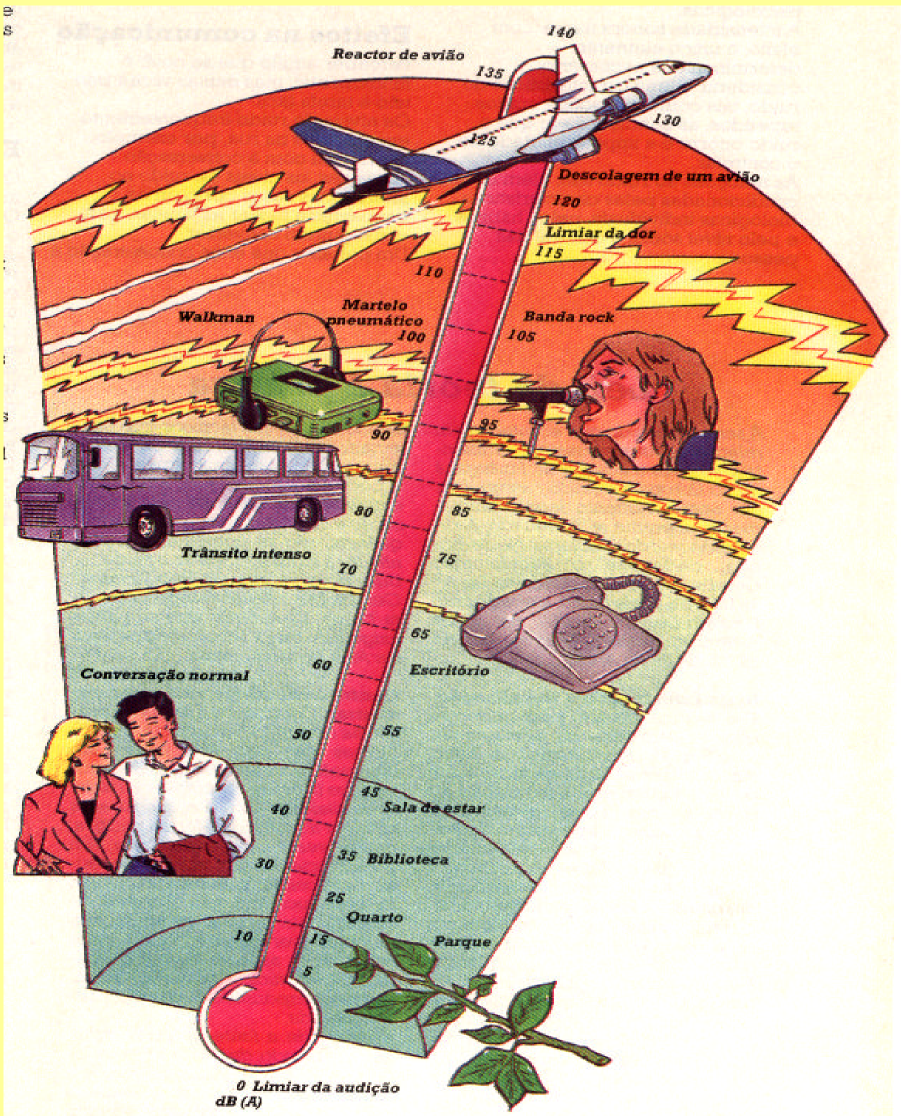
ADAPTIVE STRUCTURES ω_n

ACTIVE CONTROL

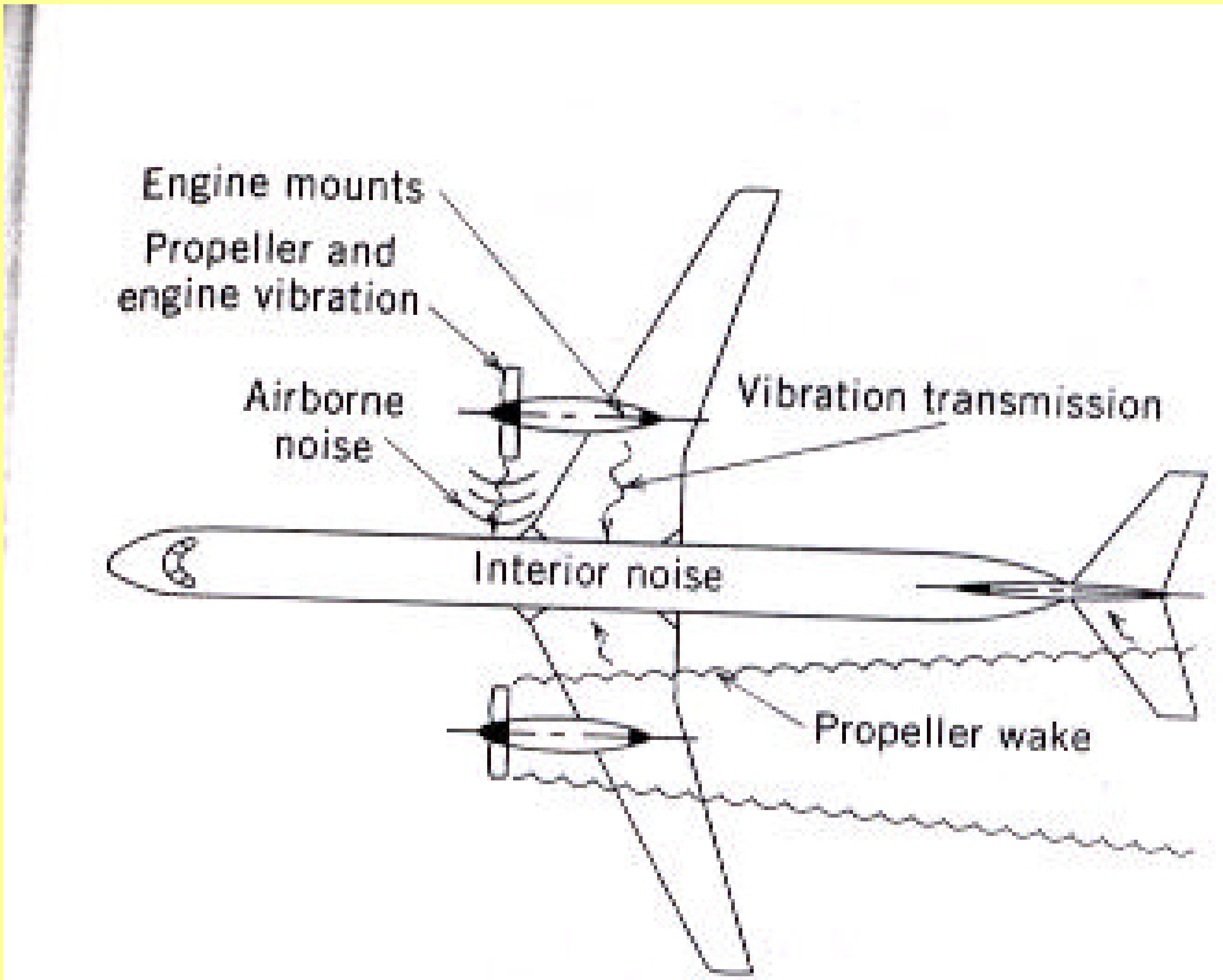
de	OPEN LOOP	CLOSED LOOP		
	(rad/s)	(rad/s)	Damping	Comment
	57.30249	**57.30438561**	**0.009520978**	**Structure**
	365.4731	**70.25461695**	**0.927711305**	**Controller**
	1057.714	**365.4493213**	**0.007791086**	**Structure**
	1133.858	**567.3243875**	**0.52316159**	**Controller**
	1193.705	**1056.67806**	**0.000688374**	**Structure**
	2178.078	**1133.858129**	**0**	**Structure**

ADAPTIVE STRUCTURES

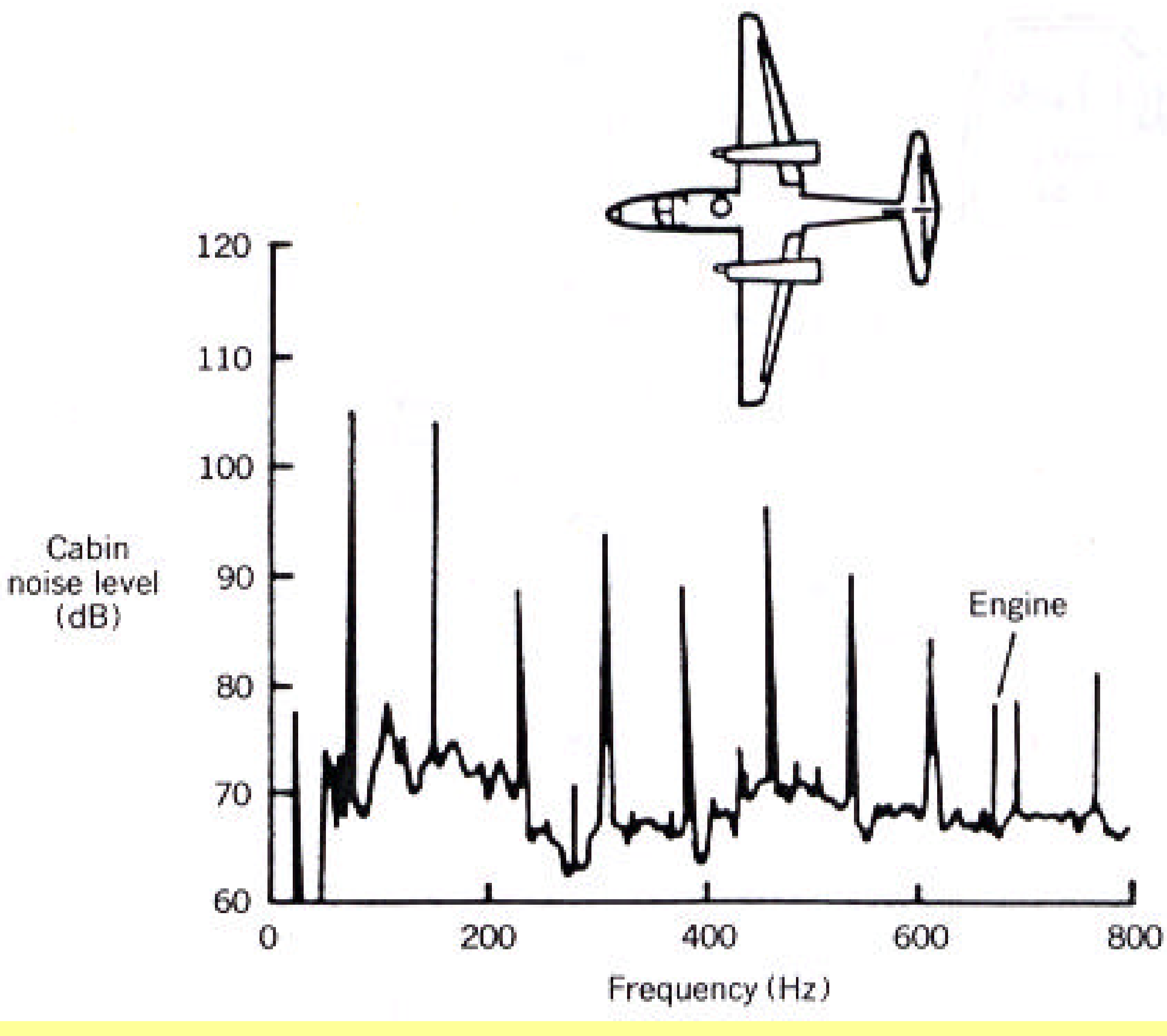
NOISE SUPPRESSION



STRUCTURE BORNE NOISE



SPECTRUM OF CABIN NOISE



COMPOSITE SHELL

A composite shell element with electromechanical properti and with principal radii of curvature R_x and R_y has be formulated and implemented.

This 8-noded isoparametric finite element has five degre of freedom at each node, which includes three displacemen and two rotations .

To derive the equations of motion for the laminat composite shell, in an acoustic field with piezoelectrical coupled electromechanical properties, we use t generalized form of Hamilton's principle

$$d\int_{t_1}^{t_2}\left[T - \Pi + W_e - W_p\right]dt = 0$$

ACOUSTIC CAVITY MODEL

•To derive the equations of motion for the acoustic cavity, we use the generalized form of Hamilton's principle $d\int_{t_1}^{t_2} \left[T_p - \Pi_p + W_p\right] dt = 0$

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

With the following boundary conditions:

$$\frac{\partial p}{\partial n} = 0 \qquad \text{at a rigid boundary}$$

$$\frac{\partial p}{\partial n} = -r_a \frac{\partial^2 w}{\partial t^2} \qquad \text{at a vibrating boundary}$$

EQUATIONS OF MOTION

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \Theta^T & \mathbf{M}_{pp} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\overline{U}}_s \\ \ddot{\overline{U}}_p \\ \ddot{\overline{U}}_e \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & -\Theta & \mathbf{K}_{se} \\ \mathbf{0} & \mathbf{K}_{pp} & \mathbf{0} \\ \mathbf{K}_{es} & \mathbf{0} & \mathbf{K}_{ee} \end{bmatrix} \begin{Bmatrix} \overline{U}_s \\ \overline{U}_p \\ \overline{U}_e \end{Bmatrix} = \begin{Bmatrix} \overline{F}_p \\ \overline{0} \\ \overline{0} \end{Bmatrix}$$

$\mathbf{M}_{pp} = \frac{1}{r_a c^2} \int_V \mathbf{N}_p^T \mathbf{N}_p \, dV$ is the acoustic "mass" matrix;

$\mathbf{K}_{pp} = \frac{1}{r_a} \int_V \mathbf{b}_p^T \mathbf{b}_p \, dV$ is the acoustic "stiffness" matrix;

$\Theta = \int_S \mathbf{N}_s^T \mathbf{N}_p \, dS$ is the structural-acoustic coupling matrix

and

$$\mathbf{b}_i^p = \begin{bmatrix} \frac{\partial \mathbf{N}_i^p}{\partial x} & \frac{\partial \mathbf{N}_i^p}{\partial y} & \frac{\partial \mathbf{N}_i^p}{\partial z} \end{bmatrix}.$$

ASSUMED PRESSURE DISTRIBUTION

Axial distribution:

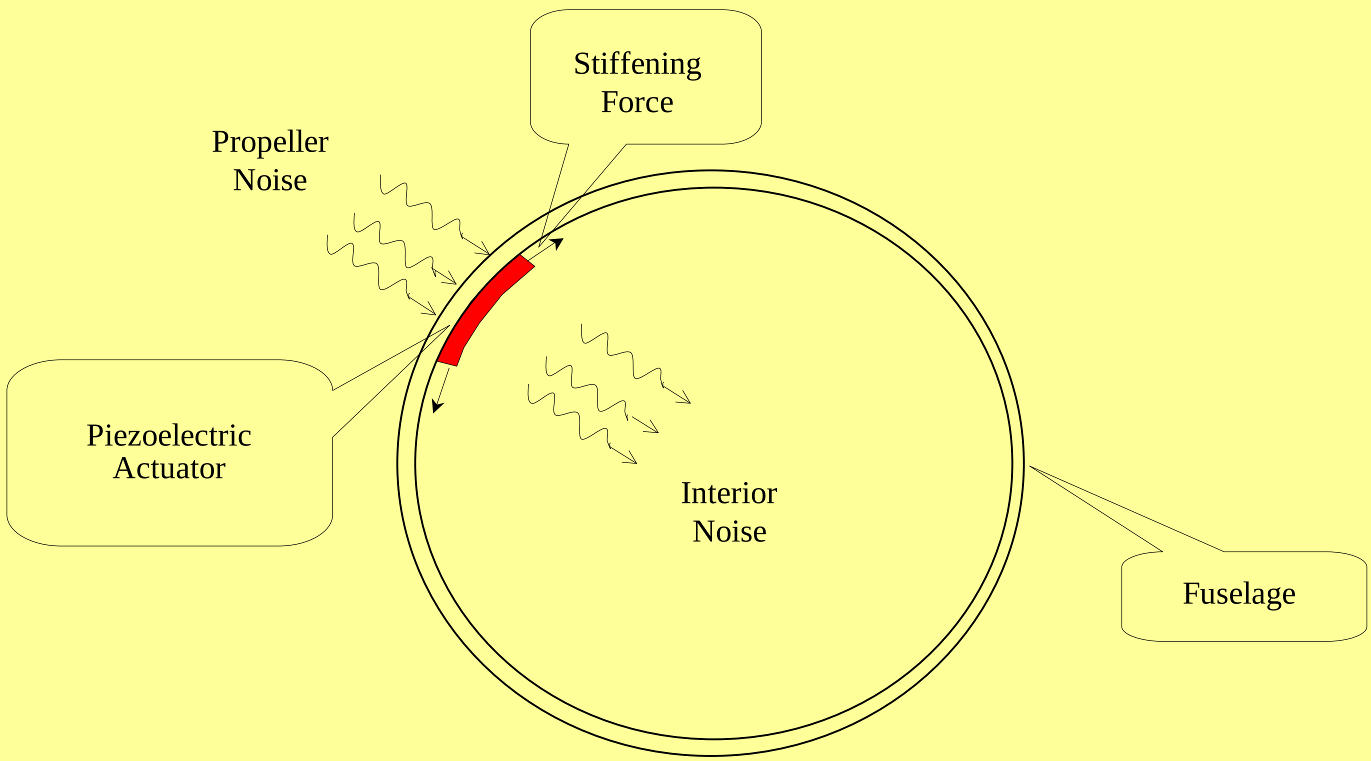
$$P_f\left(z\right)=\begin{cases} P\,e^{-d\left(z_o-z\right)} & \text{for } z<z_o \\ P\,e^{-d\left(z-z_o\right)} & \text{for } z>z_o \end{cases}$$

Circumferential Distribution:

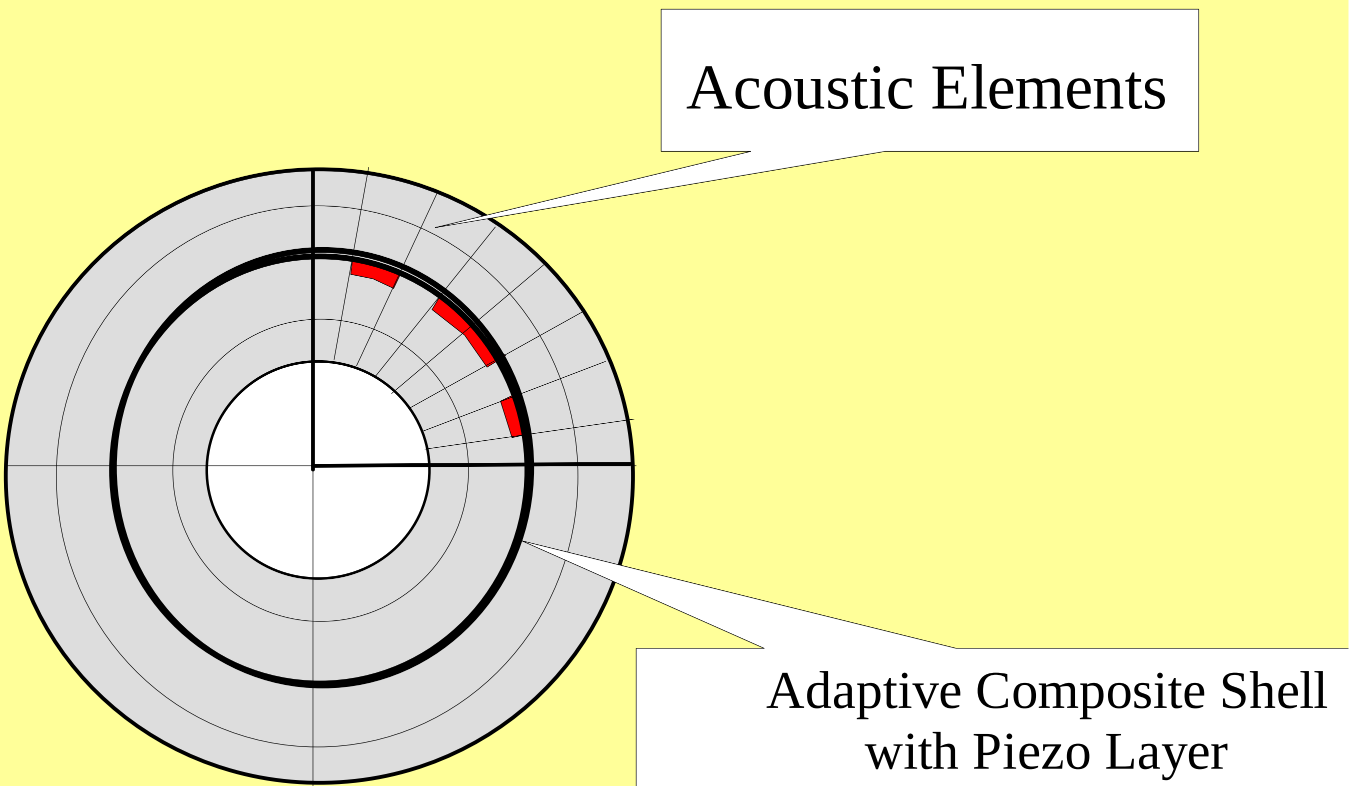
$$P_f\left(q\right)=\begin{cases} P\,\dfrac{q-q_1}{q_o-q_1}\,e^{jkq} & \text{for } q<q_o \\ P\,\dfrac{q-q_2}{q_2-q_o}\,e^{jkq} & \text{for } q>q \end{cases}$$

ADAPTIVE STRUCTURES

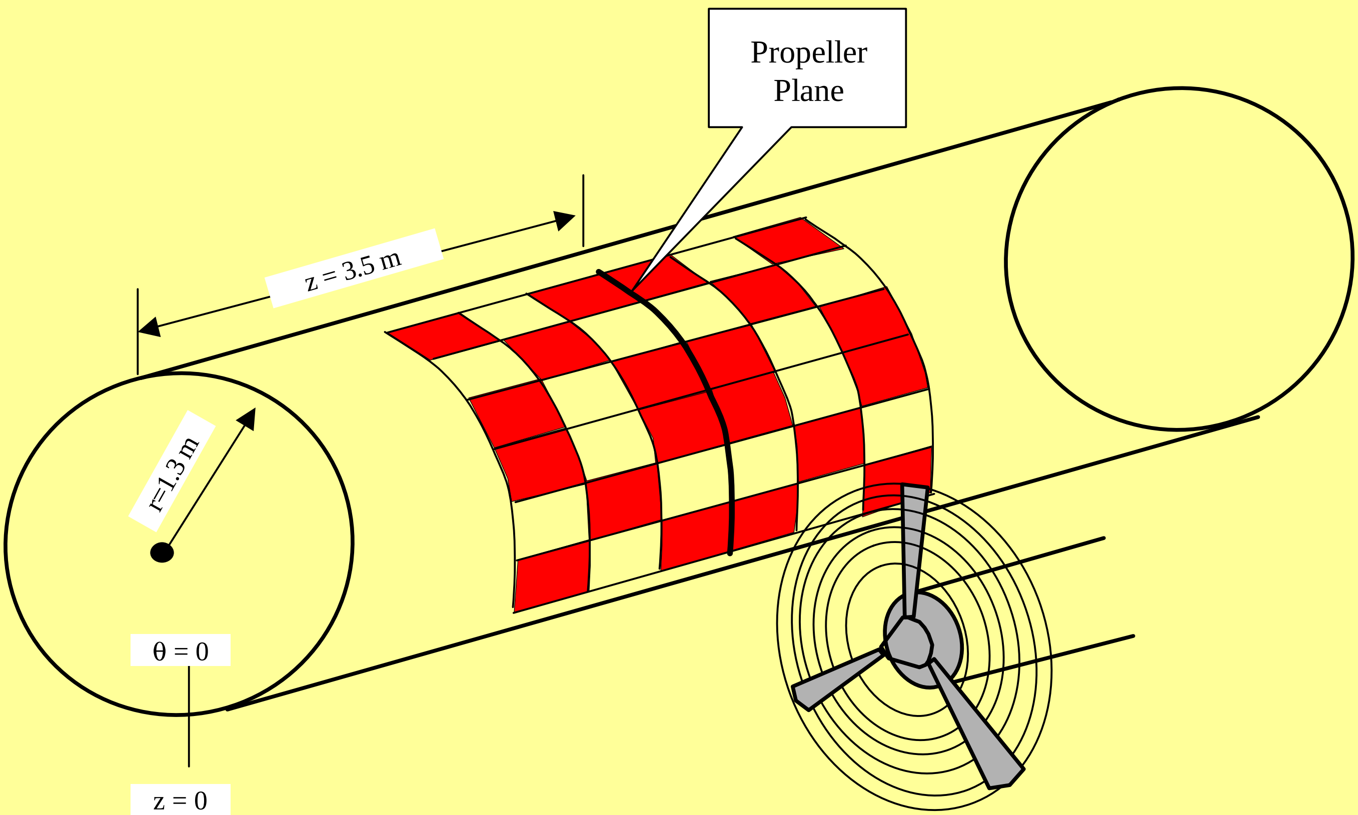
PASSIVE ACTUATION MECHANISM



FINITE ELEMENT MESH

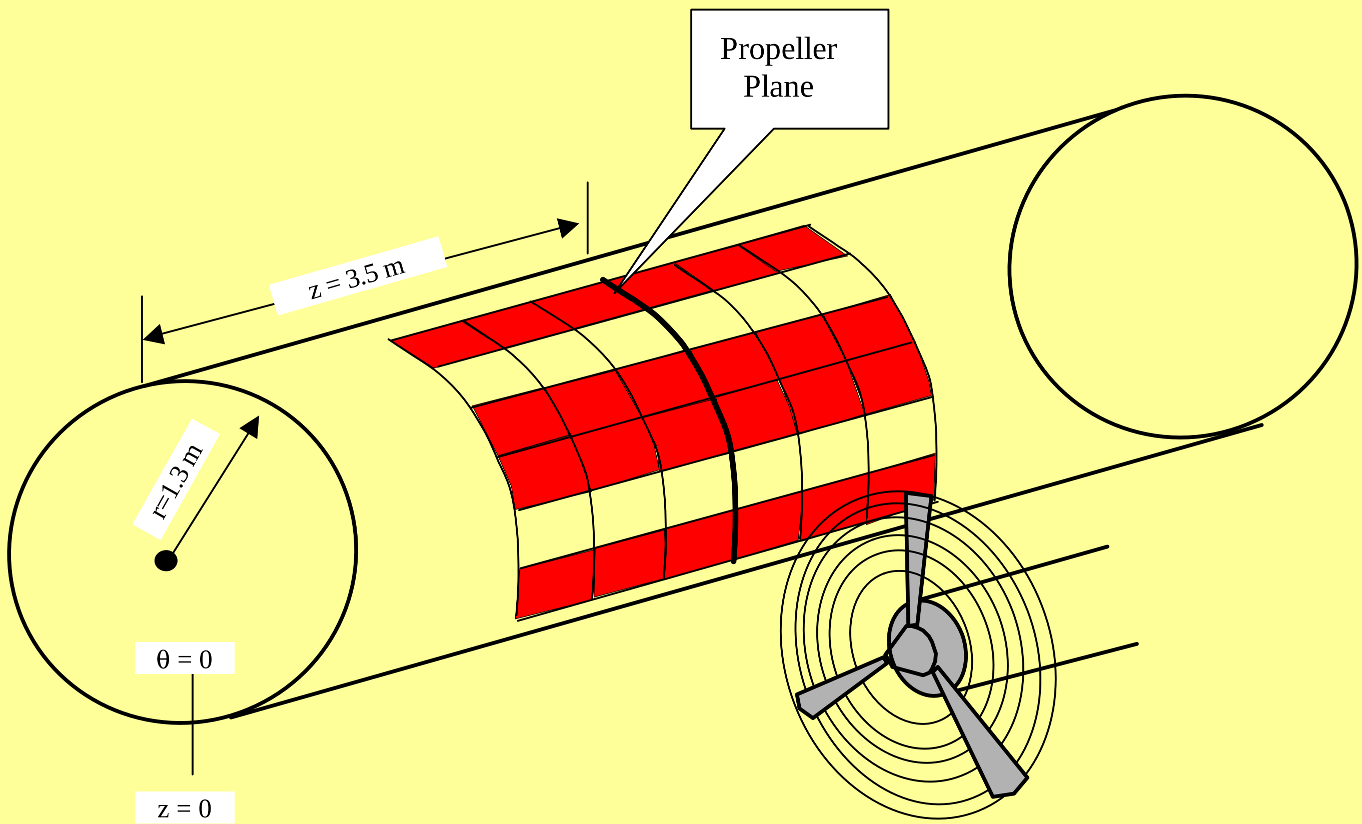


ACTUATOR CONFIGURATION



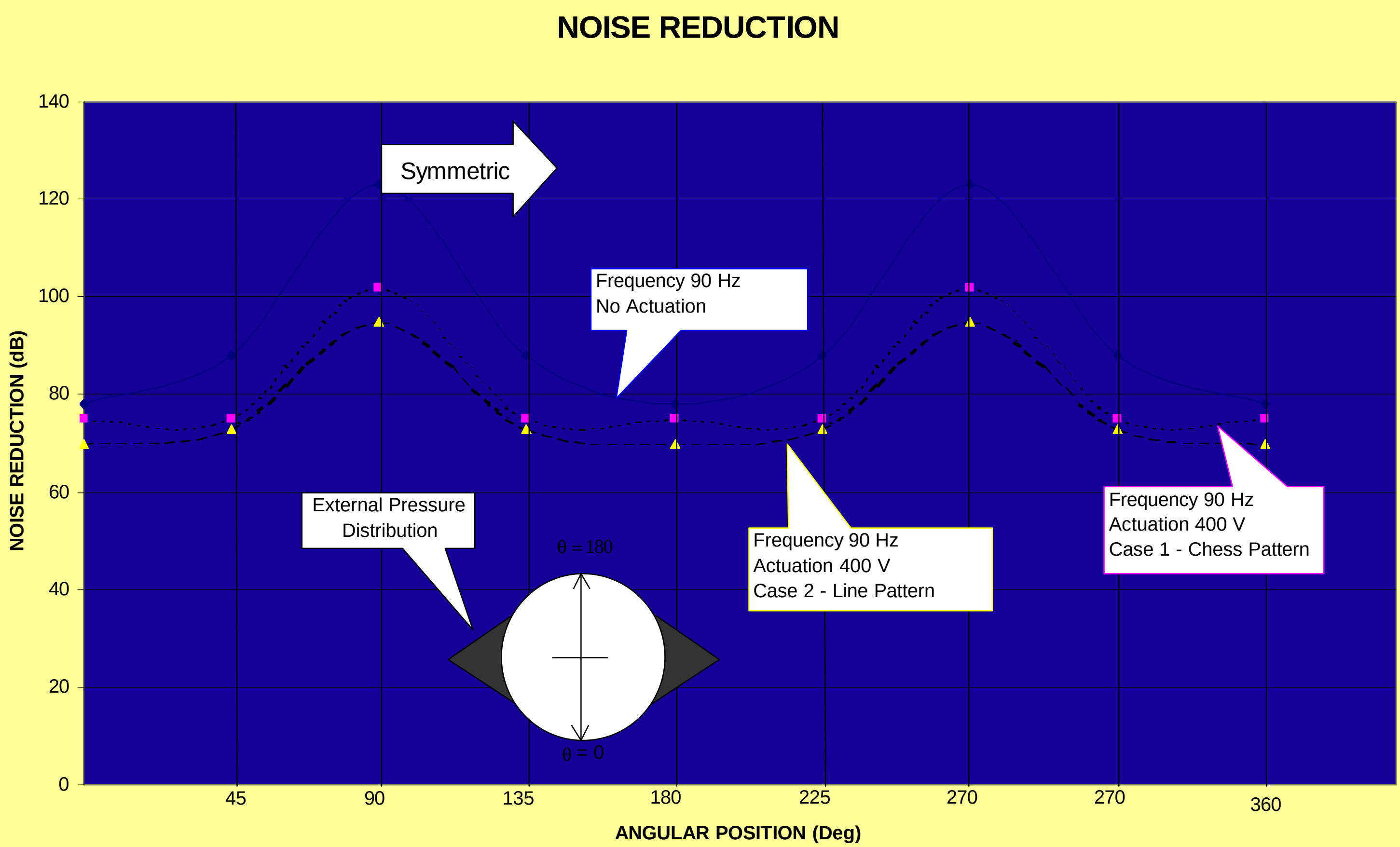
ADAPTIVE STRUCTURES

ACTUATOR CONFIGURATION



ADAPTIVE STRUCTURES

RESULTS



ADAPTIVE STRUCTURES

CONCLUSIONS

- Analytical and finite element models with ectromechanical properties have been presented.
- Application of piezoelectric patches to control par utter has been demonstrated.
- Internal noise reduction using a stiffened fuselage ith piezo pacthes achieved considerable reduction ir oise levels.