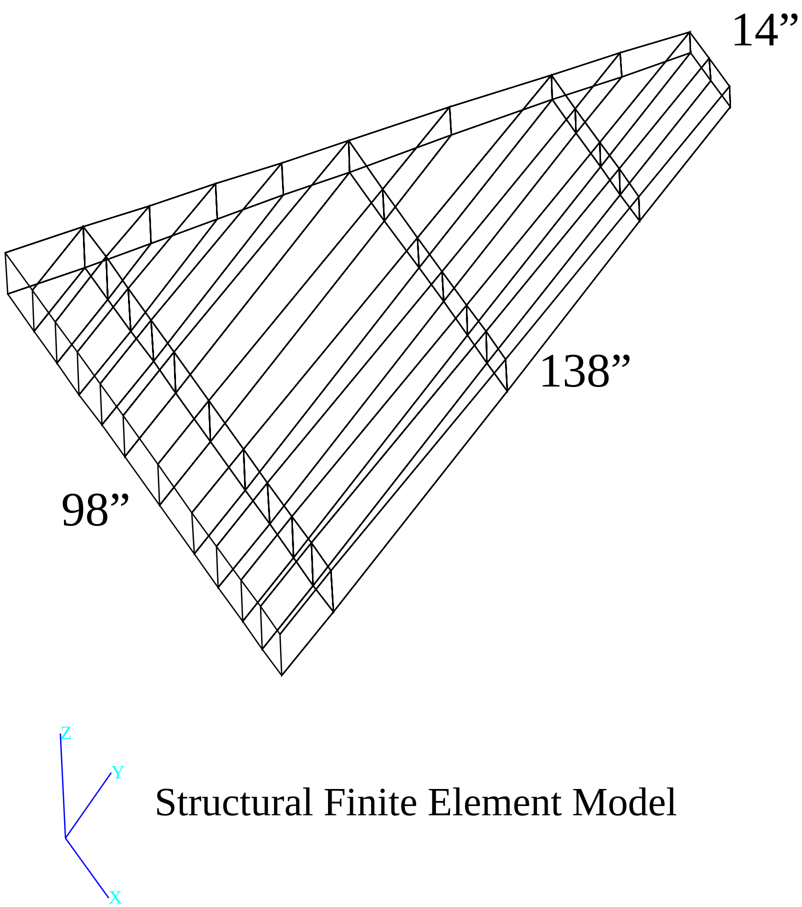
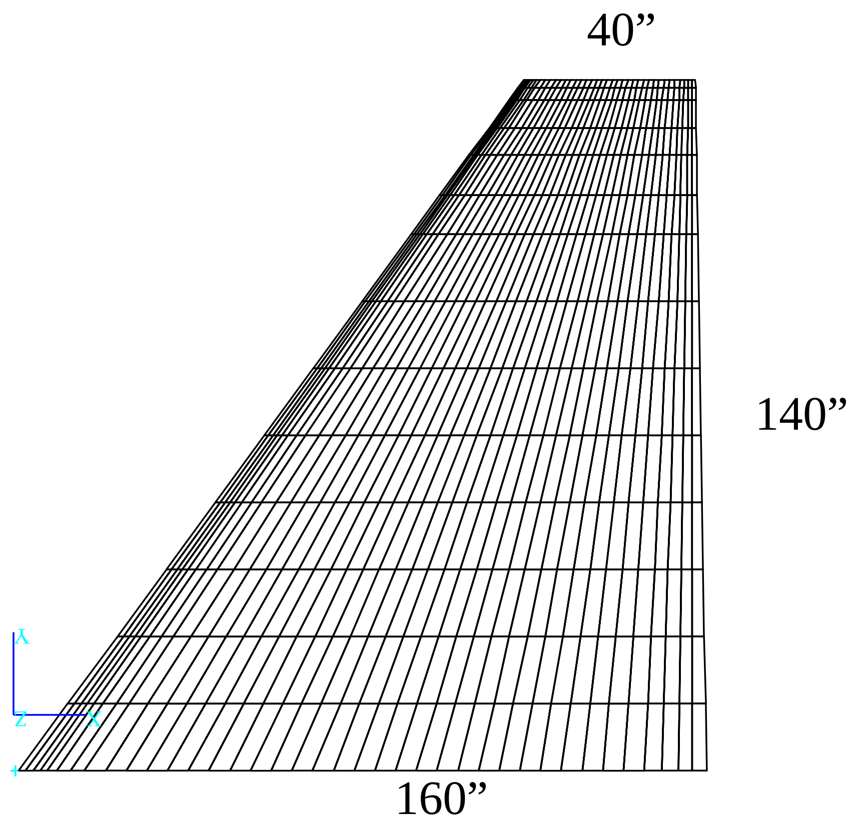
Nonlinear Unsteady Aeroelastic Analysis

Fighter Wing Example



Structural Finite Element Model

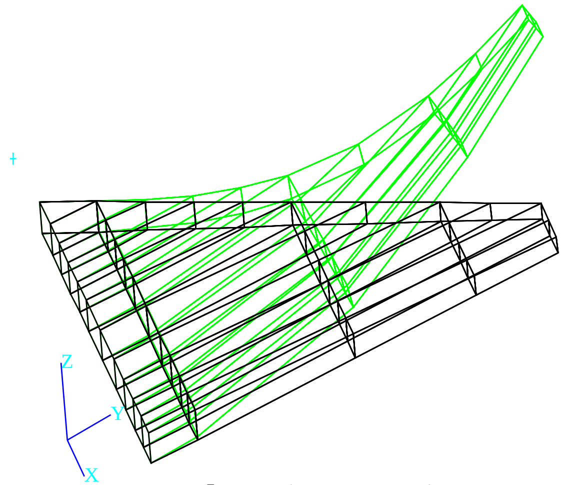


Small Disturbance Aerodynamic Model

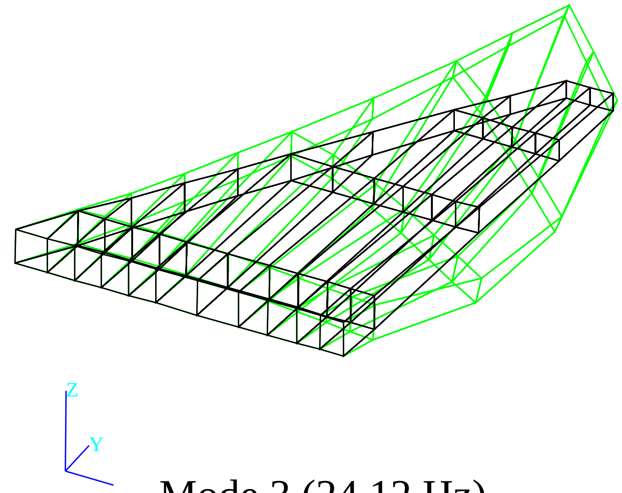
Nonlinear Unsteady Aeroelastic Analysis

Fighter Wing Example

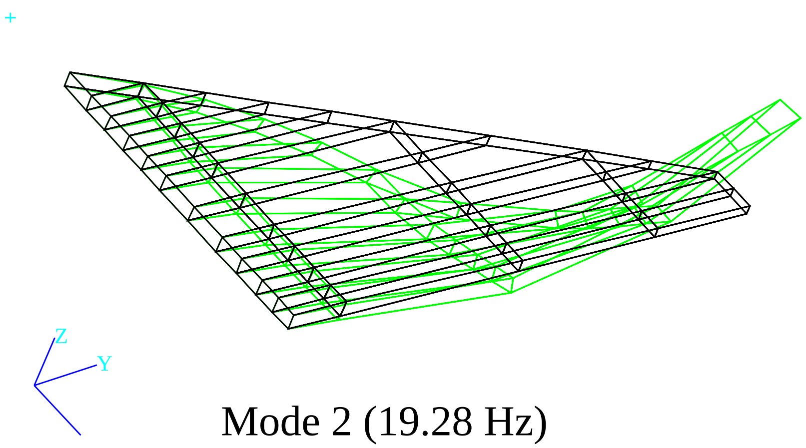
Mode Shapes and Frequencies



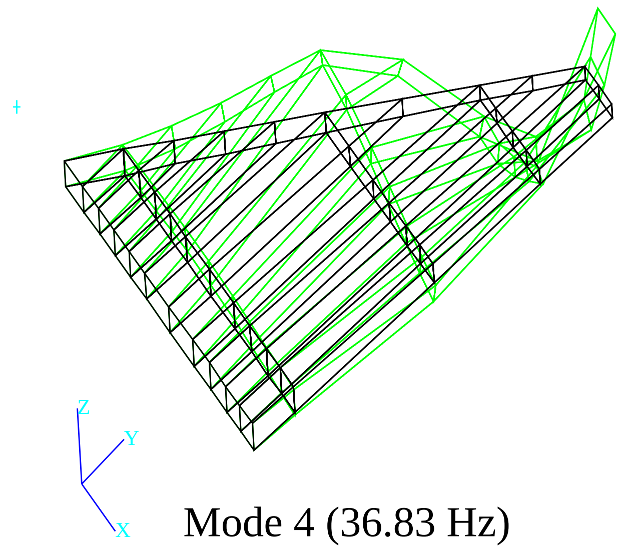
Mode 1 (5.62 Hz)



Mode 3 (24.12 Hz)



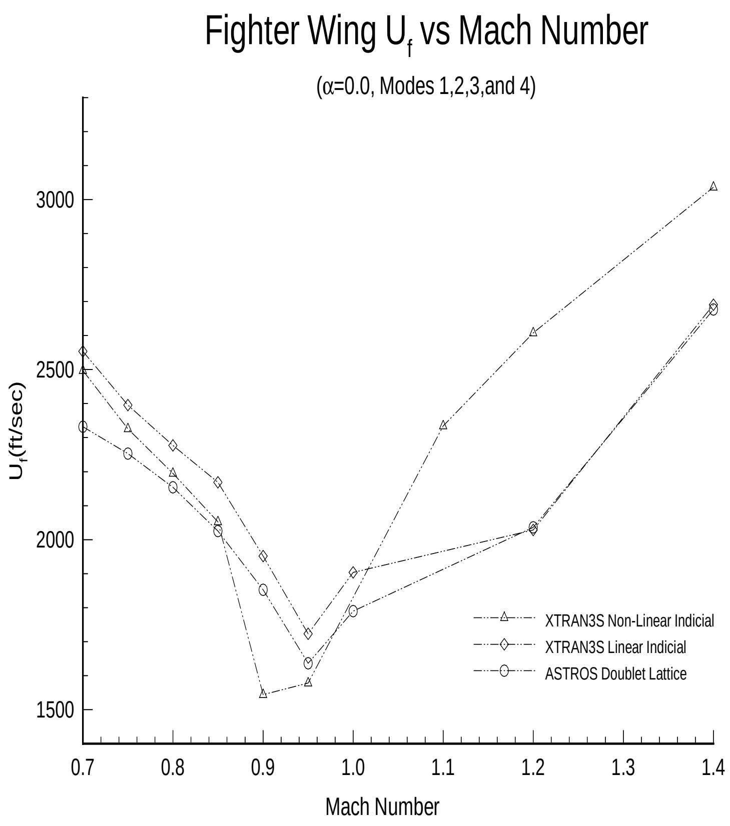
Mode 2 (19.28 Hz)

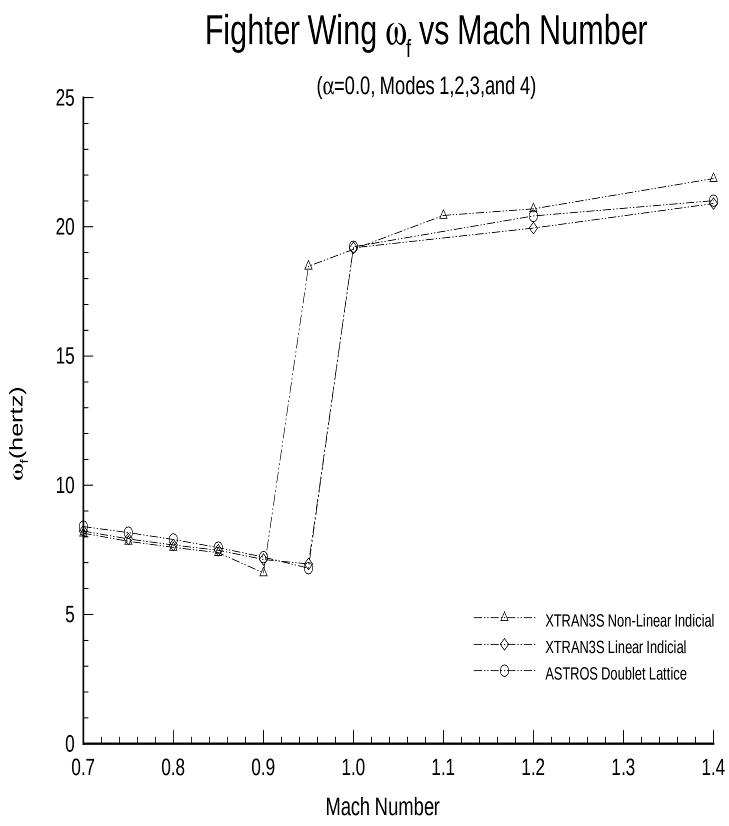


Mode 4 (36.83 Hz)

Fighter Wing Example

Flutter Velocity and Flutter Frequency vs. Mach Number



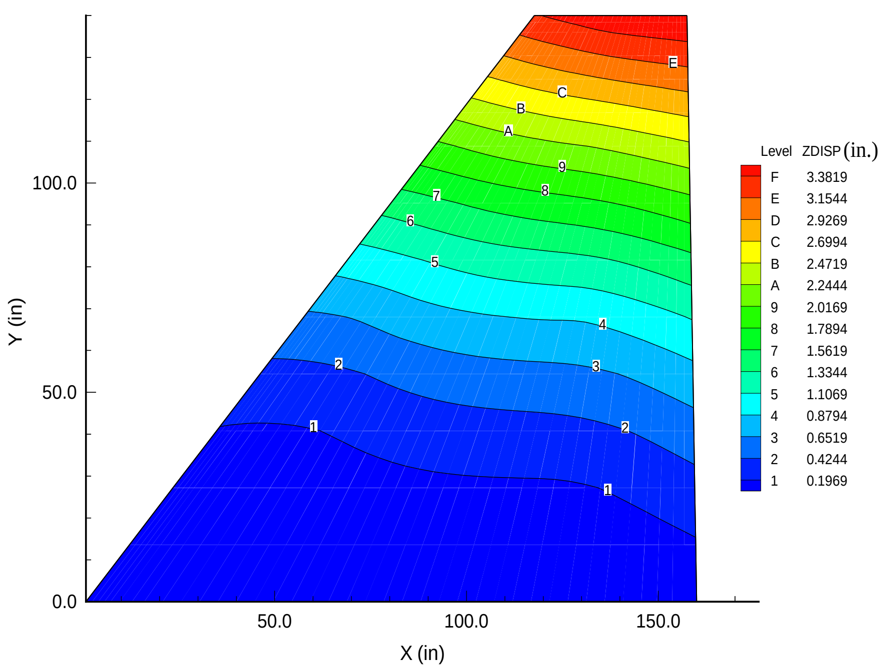


Nonlinear Unsteady Aeroelastic Analysis

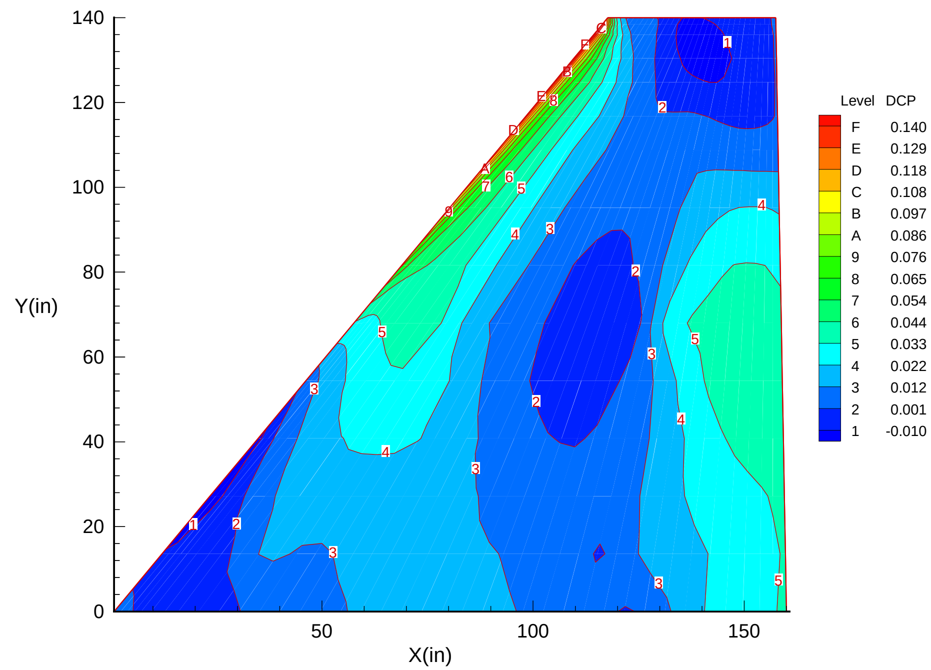
Fighter Wing Example

Static Aeroelastic Effects on IRM Flutter

$$M_\infty = 1.1, \alpha_0 = 1.0°, q_\infty = 50.\ \text{psi}, U_\infty = 14736\ \text{in/sec}$$



Static Aeroelastic Deflections



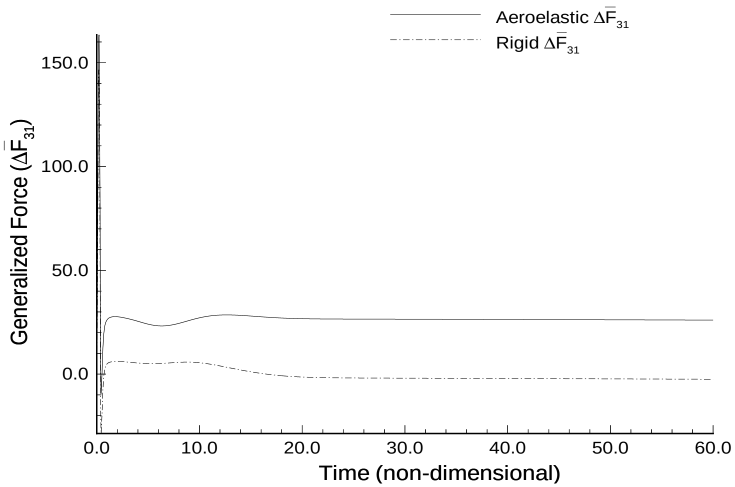
$\Delta C_{p\ \text{rigid}} - \Delta C_{p\ \text{aeroelastic}}$

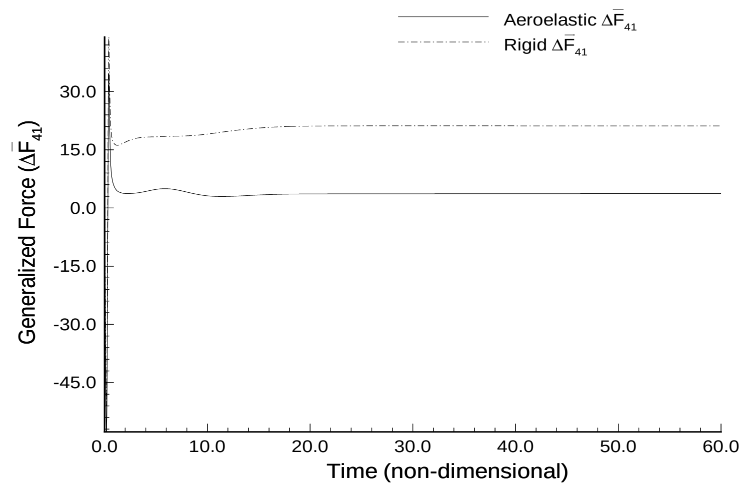
Nonlinear Unsteady Aeroelastic Analysis

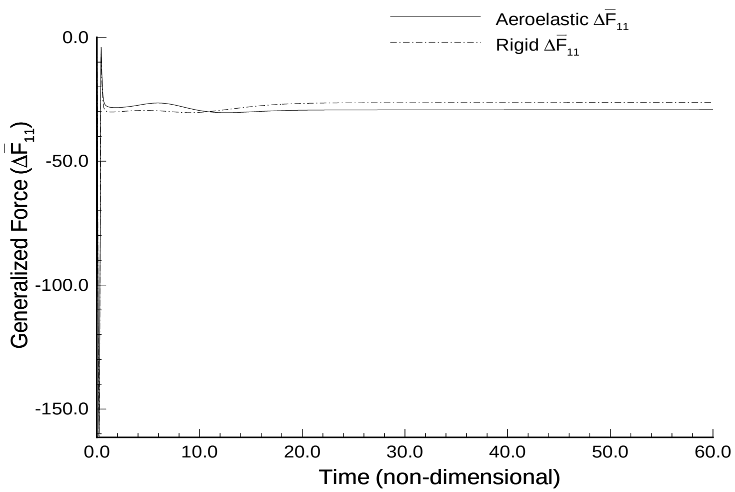
Fighter Wing Example

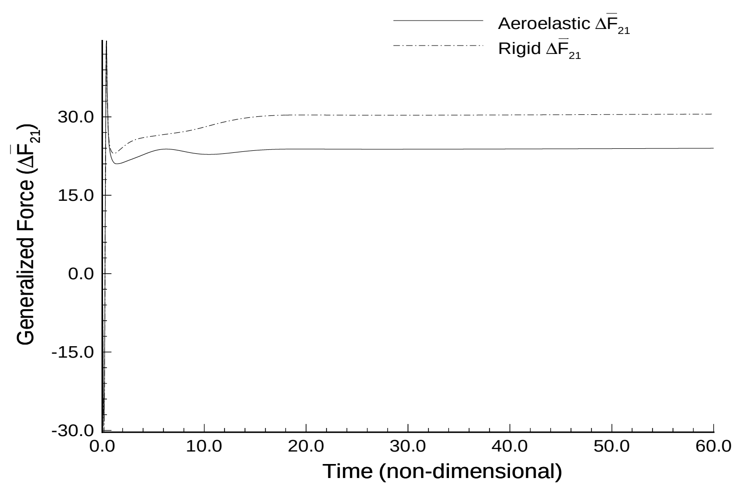
Static Aeroelastic Effects on IRM Flutter

$M_\infty = 1.1, \alpha_0 = 1.0°, q_\infty = 50.$ psi , $U_\infty = 14736$ in/sec







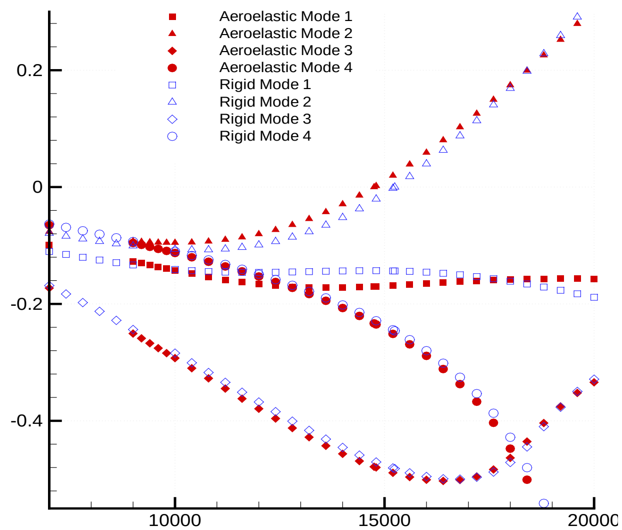


Nonlinear Unsteady Aeroelastic Analysis

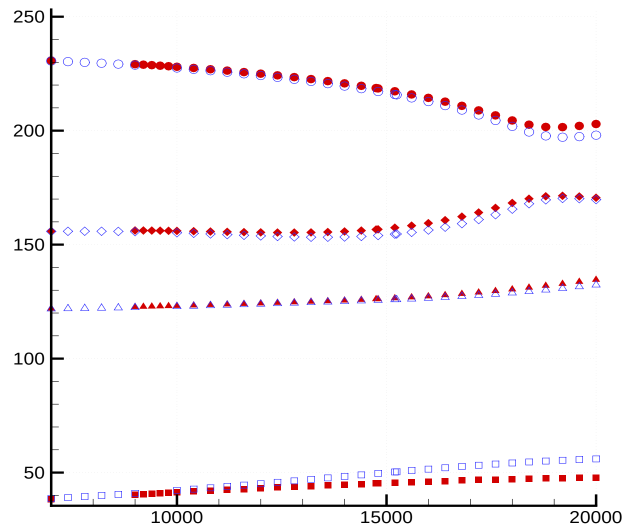
Fighter Wing Example

Static Aeroelastic Effects on IRM Flutter

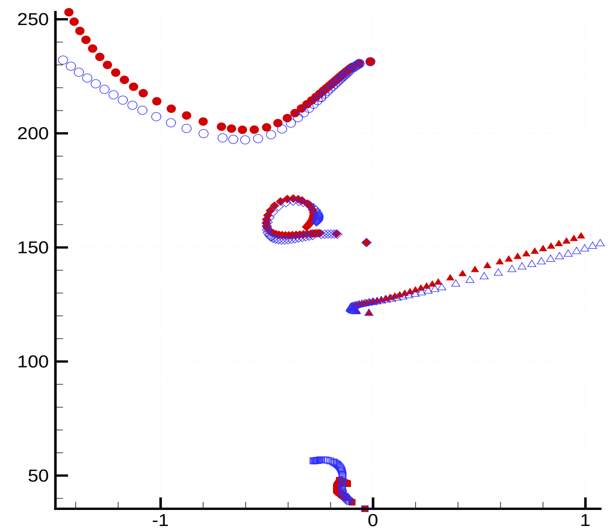
$V-g$



$V-\omega$



$g-\omega$

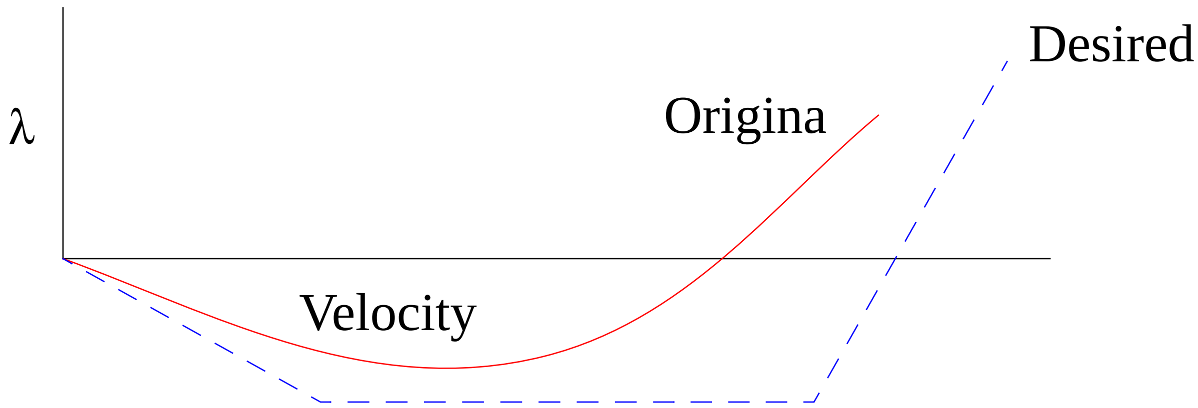


Method	M_∞	α_0 (deg.)	ρ_∞ (slinches/in^3)	q_f (psi)	U_f (in/sec)	ω_f (Hz)
Time Integration	1.1	1.0	4.9272x10^{-7}	53.50	14736.48	20.1
IRM Rigid	1.1	1.0	4.9272x10^{-7}	57.24	15243.28	20.38
IRM Aeroelastic	1.1	1.0	4.9272x10^{-7}	53.61	14751.37	20.07

7% Difference in Flutter q

Sensitivity Analysis

- Flutter Constraint Definition



$$\gamma_{lj} \le \gamma_{jREQ} \qquad \begin{array}{l} j \;=\; 1, 2, \ldots, \text{number of velocities} \\ l \;=\; 1, 2, \ldots, \text{number of modes} \end{array}$$

or

$$Z_j = \frac{\gamma_{lj} - \gamma_{jREQ}}{\text{GFACT}} \le 0 \qquad (10)$$

Nonlinear Unsteady Aeroelastic Sensitivities

Differentiating w.r.t. v_s gives

$$\frac{\partial Z_j}{\partial v_s} = \frac{1}{\mathrm{GFACT}}\frac{\partial \gamma_{lj}}{\partial v_s} \tag{11}$$

Recalling the definition for $p \equiv k(\gamma + i)$, $\frac{\partial \gamma}{\partial v_s}$ (dropping subscript) can be represented as

(Note: since γ is real $\frac{\partial \gamma}{\partial v_s}$ is real)

$$\frac{\partial \gamma}{\partial v_s} = \frac{1}{k}\left[\frac{\partial}{\partial v_s}\Re(p) - \gamma\frac{\partial}{\partial v_s}\Im(p)\right] \tag{12}$$

The sensitivity of p can be found by differentiating Eq. (9).
Restating Eq. (9) as

$$[\Phi]^T[W][\Phi]\{q\} = 0 \tag{13}$$

with the adjoint relation

$$\{y\}^T[\Phi]^T[W][\Phi] = 0 \tag{14}$$

With $\{y\}^T$ the left had eigenvectors and $[W]$ the system matrices in physical coordinates

Nonlinear Unsteady Aeroelastic Sensitivities

Differentiating Eq. (13) w.r.t. v_s, pre-multiplying by $\{y\}^T$ and employing Eq (14) gives

$$\{y\}^T \frac{\partial}{\partial v_s}[\Phi]^T[W][\Phi]\{q\} + \{y\}^T[\Phi]^T \frac{\partial}{\partial v_s}[W][\Phi]\{q\} + \{y_h\}^T[\Phi]^T[W]\frac{\partial}{\partial v_s}[\Phi]\{q\} = 0 \quad (15)$$

Assuming $[\Phi]$ is a basis for the system[13],[14] $\frac{\partial}{\partial v_s}[\Phi], \frac{\partial[\Phi]^T}{\partial v_s}$ are 0 (valid for small moves of v_s) results in

$$\{y\}^T\left[2p\left(\frac{U_\infty}{b}\right)^2\frac{\partial p}{\partial v_s}[\bar{M}] + \left(\frac{U_\infty}{b}\right)^2 p^2\frac{\partial}{\partial v_s}[\bar{M}] + \left(\frac{U_\infty}{b}\right)\frac{\partial p}{\partial v_s}[\bar{B}] + p\left(\frac{U_\infty}{b}\right)\frac{\partial}{\partial v_s}[\bar{B}] + \right. \quad (16)$$

$$\left.\frac{\partial}{\partial v_s}[\bar{K}] - \frac{1}{2}\rho_\infty U_\infty^2\frac{\partial}{\partial v_s}[\bar{Q}(p)]\right]\{q\} = 0$$

- For Doublet Lattice $\bar{Q}(p)$ depends on only M, k.
- For non-linear unsteady $\bar{Q}(p)$ depends on initial conditions and the static aeroelastic equilibrium position

Nonlinear Unsteady Aeroelastic Sensitivities

Let $\bar{Q}(p) = C_R^2 \sum_{r=1}^{n} 2p\left(\frac{C_{r_{ij}}}{2p + b_{r_{ij}}}\right)$ then

$$\frac{\partial}{\partial v_s}[\bar{Q}(p)] = C_R^2\left[\frac{\partial}{\partial p}[\bar{Q}(p)]\left(\frac{\partial p}{\partial v_s}\right) + \sum_{l=1}^{m}\left(\frac{\partial}{\partial \omega_l}[\bar{Q}(p)]\right)\left(\frac{\partial \omega_l}{\partial v_s}\right)\right] \qquad (17)$$

Substituting (17) into (16) and solving for $\frac{\partial p}{\partial v_s}$ yields

$$\frac{\partial p}{\partial v_s} = \left[-p^2\left(\frac{U_\infty}{b}\right)^2\{y\}^T\frac{\partial}{\partial v_s}[\bar{M}]\{q\} - p\left(\frac{U_\infty}{b}\right)\{y\}^T\frac{\partial}{\partial v_s}[\bar{B}]\{q\}\right. \qquad (18)$$

$$\left. - \{y\}^T\frac{\partial}{\partial v_s}[\bar{K}]\{q\} + \frac{1}{2}\rho_\infty U_\infty^2 C_R^2\{y\}^T\left[\sum_{l=1}^{m}\left(\frac{\partial}{\partial \omega_l}\bar{Q}[(p)]\right)\left(\frac{\partial \omega_l}{\partial v_s}\right)\right]\{q\}\right]$$

$$/\left[2p\left(\frac{U_\infty}{b}\right)^2\{y\}^T[\bar{M}]\{q\} + \left(\frac{U_\infty}{b}\right)\{y\}^T[\bar{B}]\{q\} - \frac{1}{2}\rho_\infty U_\infty^2 C_R^2\{y\}^T\frac{\partial}{\partial p}[\bar{Q}(p)]\{q\}\right]$$

$\frac{\partial p}{\partial v_s}$ - in general is a complex quantity which can be used to complete the evaluation of Eq. (12)

Sensitivity of Structural Matrices $\frac{\partial}{\partial v_s}[\bar{K}], \frac{\partial}{\partial v_s}[\bar{M}], \frac{\partial}{\partial v_s}[\bar{B}]$

The left and right had eigenvectors can be expressed in physical coordinates as

$$\begin{aligned} \{y_g\} &= [\Phi]\{y\} \\ \{q_g\} &= [\Phi]\{q\} \end{aligned} \qquad (19)$$

Using (19) the moves the derivatives of the mass, stiffness, and damping matrices into the *g-set*

- Reference Eq. 30-32 of Aeroelastic Optimization Notes for $[M], [K], [B]$ sensitivities

Sensitivity of Structural Natural Frequencies $\frac{\partial \omega_l}{\partial v_s}$

Let $\omega_l^2 = \lambda_l$, where λ_l lth undamped eigenvalue of the system excluding external loads.

$$\frac{\partial \omega_l}{\partial v_s} = \left(\frac{\partial \omega_l}{\partial \lambda_l}\right)\left(\frac{\partial \lambda_l}{\partial v_s}\right) = \frac{1}{2\omega_l}\left(\frac{\partial \lambda_l}{\partial v_s}\right) \tag{20}$$

Where $\frac{\partial \lambda_l}{\partial v_s}$ is determined by differentiating $[[K] - \lambda_l[M]]\{\varphi_l\} = 0$ w.r.t. v_s yielding

$$\left[\frac{\partial}{\partial v_s}[K] - \frac{\partial \lambda_l}{\partial v_s}[M] - \lambda_l \frac{\partial}{\partial v_s}[M]\right]\{\varphi_l\} + [[K] - \lambda_l[M]]\frac{\partial}{\partial v_s}\{\varphi_l\} = 0 \tag{21}$$

Pre-multiplying by $\{\varphi\}^T$ and taking advantage of the self-adjoint nature $\{\varphi_l\}^T[[K] - \lambda_l[M]] = 0$

$$\frac{\partial \lambda_l}{\partial v_s} = \frac{\{\varphi_l\}^T\left[\frac{\partial}{\partial v_s}[K] - \lambda_l \frac{\partial}{\partial v_s}[M]\right]\{\varphi_l\}}{\{\varphi_l\}^T[M]\{\varphi_l\}} \tag{22}$$

Sensitivity of Aerodynamic Terms

$$\frac{\partial}{\partial p}[\bar{Q}(p)],\ \partial[\bar{Q}(p)]/\partial\omega_l$$

$$\frac{\partial}{\partial p}[\bar{Q}(p)] = 2\sum_{r=1}^{n}\frac{C_{r_{ij}}}{2p + b_{r_{ij}}} - \sum_{r=1}^{n}\frac{4pC_{r_{ij}}}{(2p + b_{r_{ij}})^2} \qquad (23)$$

- $\partial[\bar{Q}(p)]/\partial\omega_l$ can be found by one of the following methods

  - Differentiating the modal static aeroelastic equation of motion w.r.t. ω_l and solving for $\partial[\bar{Q}(p)]/\partial\omega_l$

  - Finite Difference

Computational Cost of Sensitivity for IRM Analysis

- Retaining m structural modes
- $m+1$ CFD Time integrations to get unsteady aerodynamics into Laplace domain
- Additional time integrations for $\partial[\bar{Q}(p)]/\partial\omega_l$ calculations

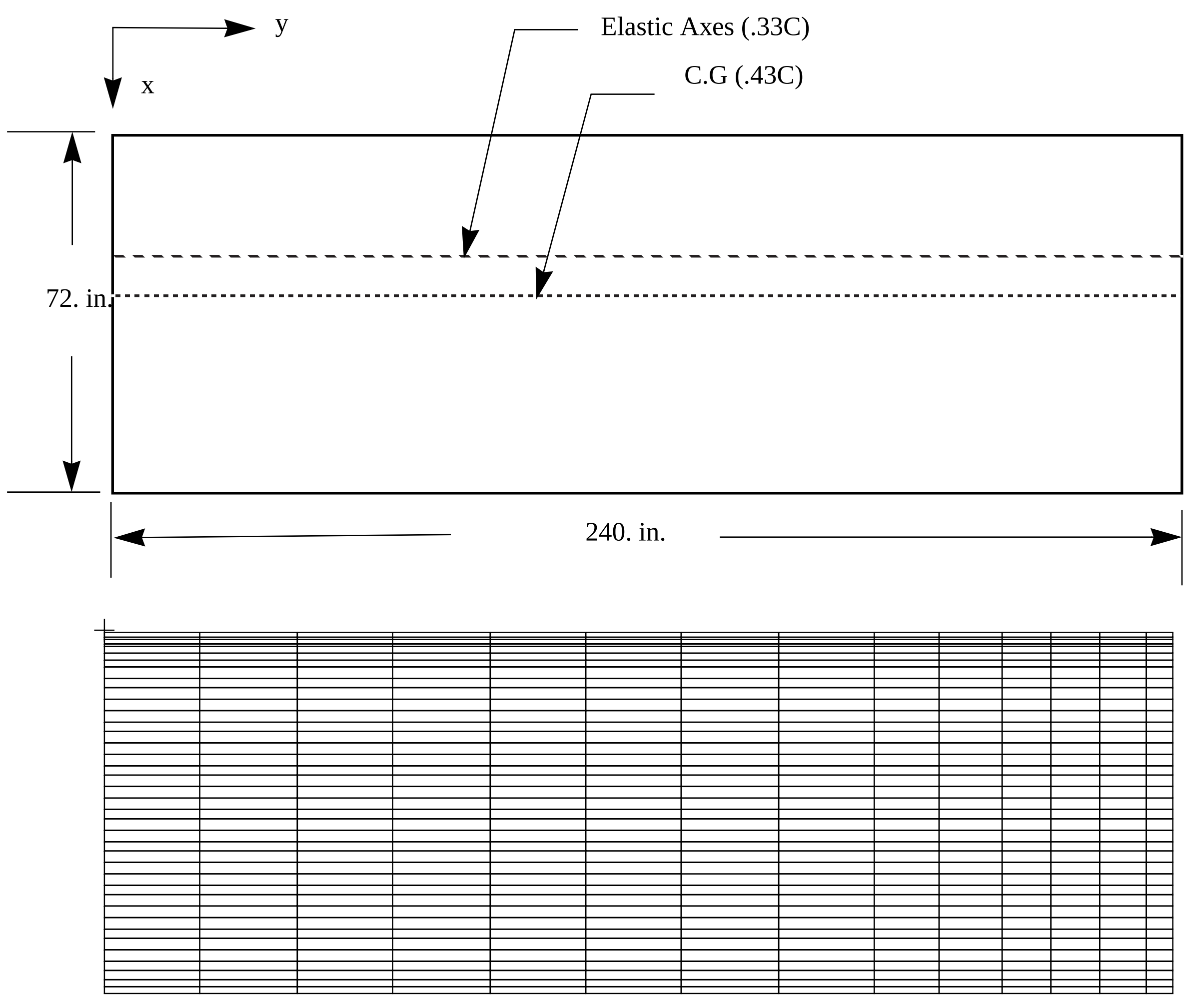
  - Finite Difference $m(m+1)$
    - m - perturbations of each ω_l to determine static aeroelastic equilibrium
    - $m \times m$ time integrations to perform the indicial response about these perturbed static aeroelastic states.
  - Analytically if possible m

If static equilibrium is a rigid solution instead of static aeroelastic

$$\partial[\bar{Q}(p)]/\partial\omega_l \equiv 0$$

Sensitivity Analysis Example

Moderate Aspect Ratio Wing (Goland Wing)



- 6% parabolic airfoil

Modes

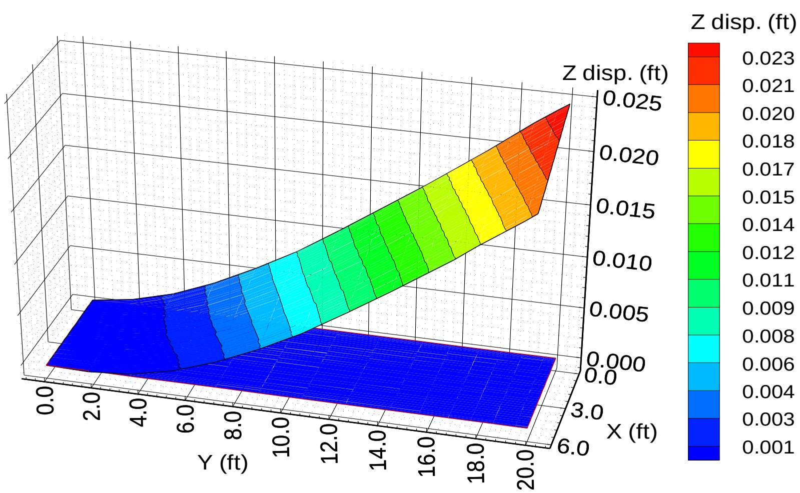
- 1.789 hz 1st Bnd.
- 3.922 hz 1st Trsn.

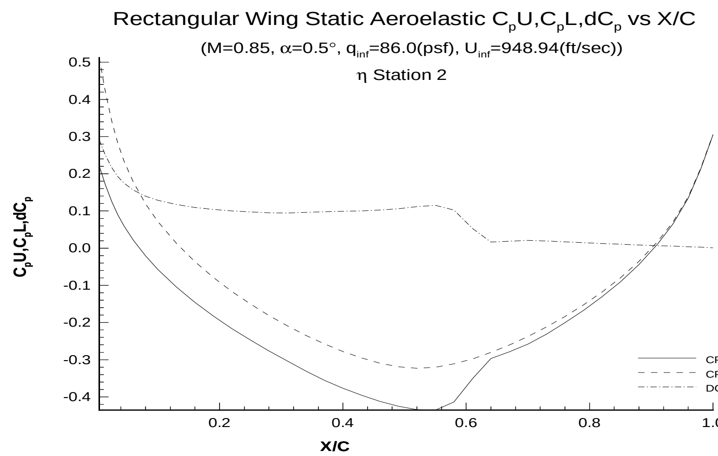
Sensitivity Analysis Example

Rectangular Wing Sensitivity Analysis

$M_\infty = 0.85, \alpha_0 = 0.5°$

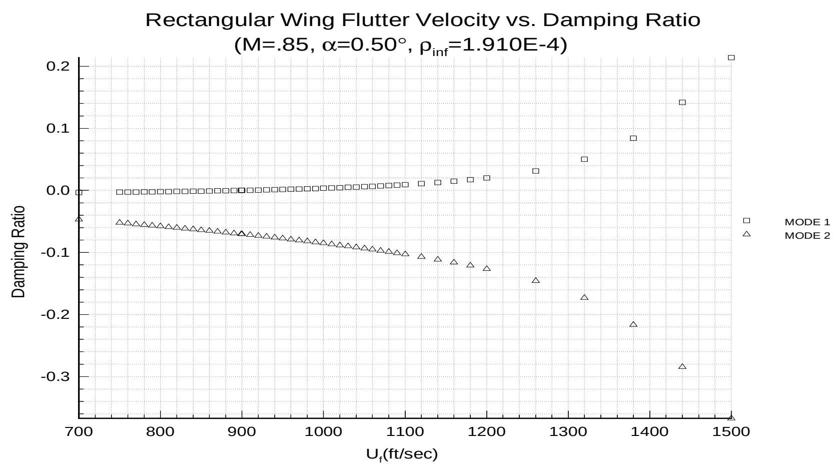
Static Aeroelastic Equilibrium





Rectangular Wing Constraint Values

Mode #	800. (fps)	850. (fps)	1000. (fps)	1150. (fps)
1	-0.01035	-0.00576	0.01742	0.06940
2	-0.28870	-0.31910	-0.42490	-0.56820



Sensitivity Analysis Example

Rectangular Wing Sensitivity Analysis

Rectangular Wing Gradient Terms ($M_\infty = 0.85, \alpha_0 = 0.5°$)

$\frac{\partial Z_j}{\partial v_s}$	Analytical	Finite Difference	Analytical	Finite Difference	Finite Difference
	Includes $\frac{\partial[\bar{Q}]}{\partial v_s}$ No $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$		No $\frac{\partial[\bar{Q}]}{\partial v_s}$ No $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$		Includes $\frac{\partial[\bar{Q}]}{\partial v_s}$ Includes $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$
$\frac{\partial Z}{\partial v_1}$ $U_\infty = 800.$	-0.5442	-0.5453	-0.5653	-0.5648	-0.5423
$\frac{\partial Z}{\partial v_1}$ $U_\infty = 850.$	-0.7262	-0.7267	-0.7455	-0.7445	-0.7220
$\frac{\partial Z}{\partial v_1}$ $U_\infty = 1000.$	-1.7099	-1.7091	-1.7345	-1.7332	-1.7104
$\frac{\partial Z}{\partial v_1}$ $U_\infty = 1150.$	-4.4704	-4.4656	-4.5144	-4.5110	-4.4948
$\frac{\partial Z}{\partial v_2}$ $U_\infty = 800.$	0.0156	0.0149	0.0191	0.0188	0.0166
$\frac{\partial Z}{\partial v_2}$ $U_\infty = 850.$	0.0136	0.0130	0.0183	0.0183	0.0146
$\frac{\partial Z}{\partial v_2}$ $U_\infty = 1000.$	0.0000	-0.0011	0.0069	0.0069	0.0027
$\frac{\partial Z}{\partial v_2}$ $U_\infty = 1150.$	-0.0532	-0.0549	-0.0456	-0.0465	-0.0431

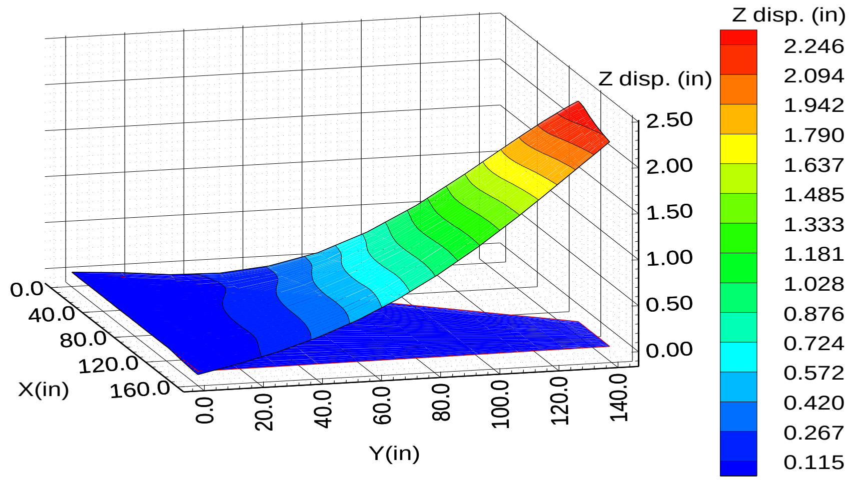
Nonlinear Unsteady Aeroelastic Sensitivities

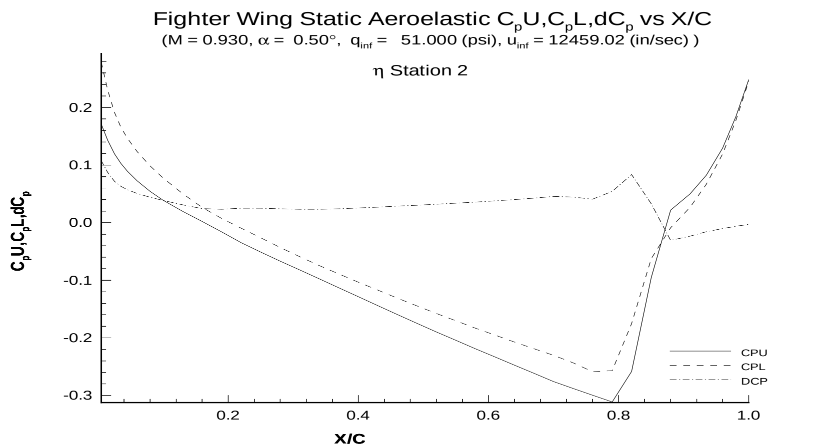
Sensitivity Analysis Example

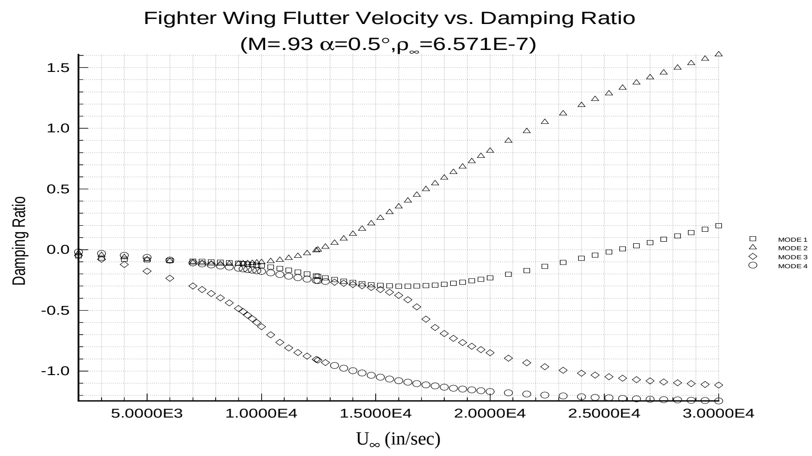
Fighter Wing Sensitivity Analysis

$M_\infty = 0.93, \alpha_0 = 0.5°$

Static Aeroelastic Equilibrium







Fighter Wing Constraint Values

Mode #	11000. (in/sec)	12000. (in/sec)	13500. (in/sec)	15000. (in/sec)
1	-0.5966	-0.8035	-1.1640	-1.4500
2	-0.2821	-0.4002	0.4950	1.2485
3	-2.3970	-2.5530	-2.5250	-1.9500
4	-0.8538	-0.9871	-1.4050	-2.3170

Sensitivity Analysis Example

Fighter Wing Sensitivity Analysis

Fighter wing gradient terms ($M_\infty = 0.93$, $\alpha_0 = 0.5^\circ$)

$\frac{\partial Z_j}{\partial v_s}$	Analytical	Finite Difference	Analytical	Finite Difference	Finite Difference
	Includes $\frac{\partial[\bar{Q}]}{\partial v_s}$ No $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$		No $\frac{\partial[\bar{Q}]}{\partial v_s}$ No $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$		Includes $\frac{\partial[\bar{Q}]}{\partial v_s}$ Includes $\frac{\partial[\bar{Q}]}{\partial[\Phi]}, \frac{\partial[\Phi]}{\partial v_s}$
$\frac{\partial Z_{U_\infty = 11000.}}{\partial v_{16}}$	-0.1742	-0.182	-0.1665	-0.1875	-0.1725
$\frac{\partial Z_{U_\infty = 12000.}}{\partial v_{16}}$	-0.3170	-0.3424	-0.3191	-0.3390	-0.320
$\frac{\partial Z_{U_\infty = 13500.}}{\partial v_{16}}$	-0.5675	-0.5895	-0.5654	-0.5960	-0.550
$\frac{\partial Z_{U_\infty = 15000.}}{\partial v_{16}}$	-0.8418	-0.8600	-0.8380	-0.8600	-0.8150

For cases tested Non-linear term in sensitivity found negligible

Rectangular Wing Design Example

$M_\infty = 0.85$, $\alpha_0 = 0.5°$, $U_\infty = 948.$ ft/sec, 20% Increase in U_f

Linear

Initial Design

$U_f = 987.$ ft/sec

$\omega_f = 2.10$ Hz

$\rho_\infty = 2.465E\text{-}4$ slugs/ft^3

$v_1 = 0.2590$ ft, $v_2 = 0.9092$ ft

Final Design (7 iterations 0.71 hr YMP)

$U_f = 1184.$ ft/sec

$\omega_f = 2.27$ Hz

$v_1 = 0.2711$ ft, $v_2 = 0.9275$ ft

(objective 6.7% greater than initial)

Nonlinear

Initial Design

$U_f = 899.$ ft/sec

$\omega_f = 2.07$ Hz

$\rho_\infty = 1.910E\text{-}4$ slugs/ft^3

$v_1 = 0.2590$ ft, $v_2 = 0.9092$ ft

Final Design (3 iterations 0.36 hr YMP)

$U_f = 1078.$ ft/sec

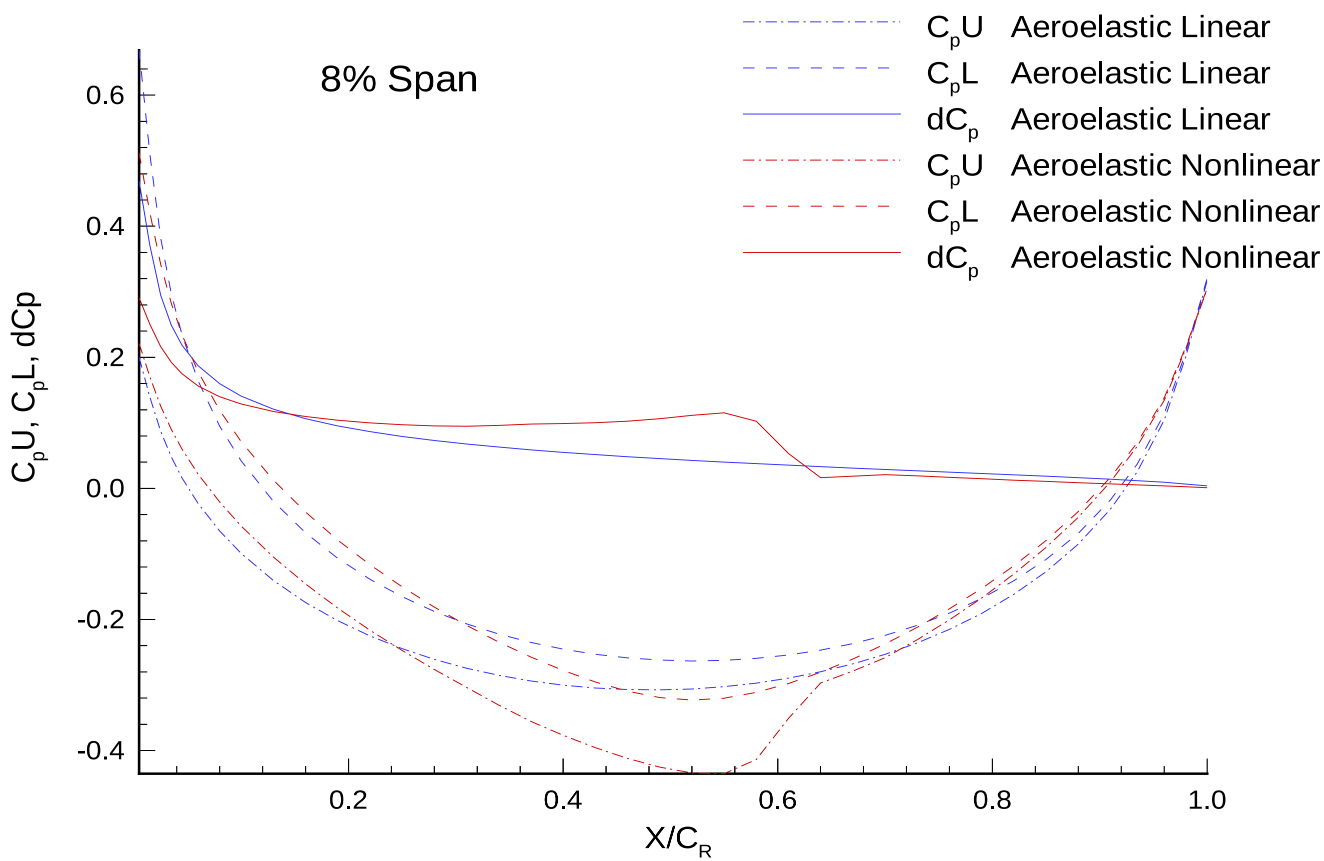
$\omega_f = 2.17$ Hz

$v_1 = 0.2642$ ft, $v_2 = 0.9161$ ft

(objective 2.7% greater than initial)

Rectangular Wing Design Example

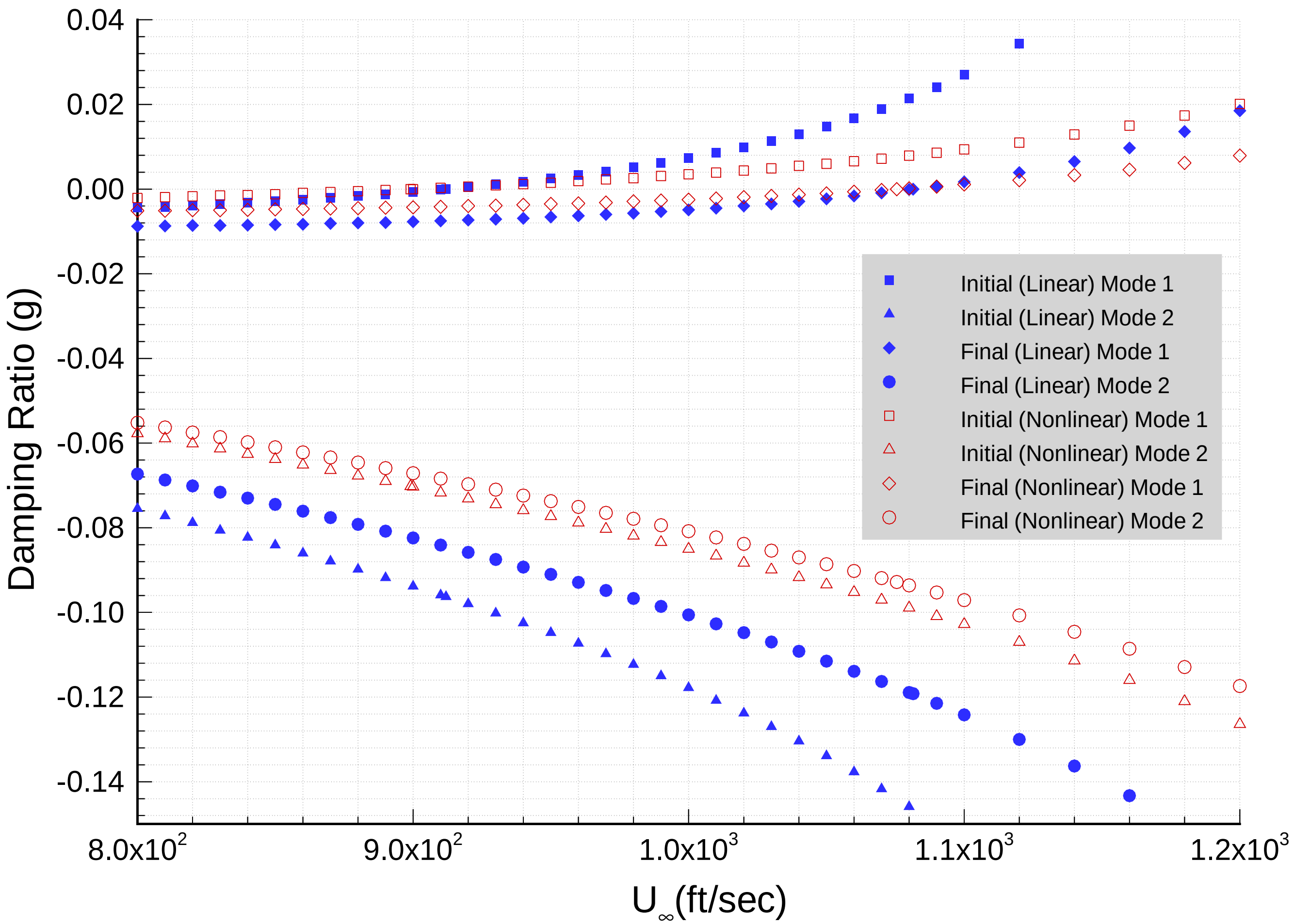
$M_\infty = 0.85$, $\alpha_0 = 0.5°$, 20% Increase in U_f



Nonlinear Unsteady Aeroelastic Optimization

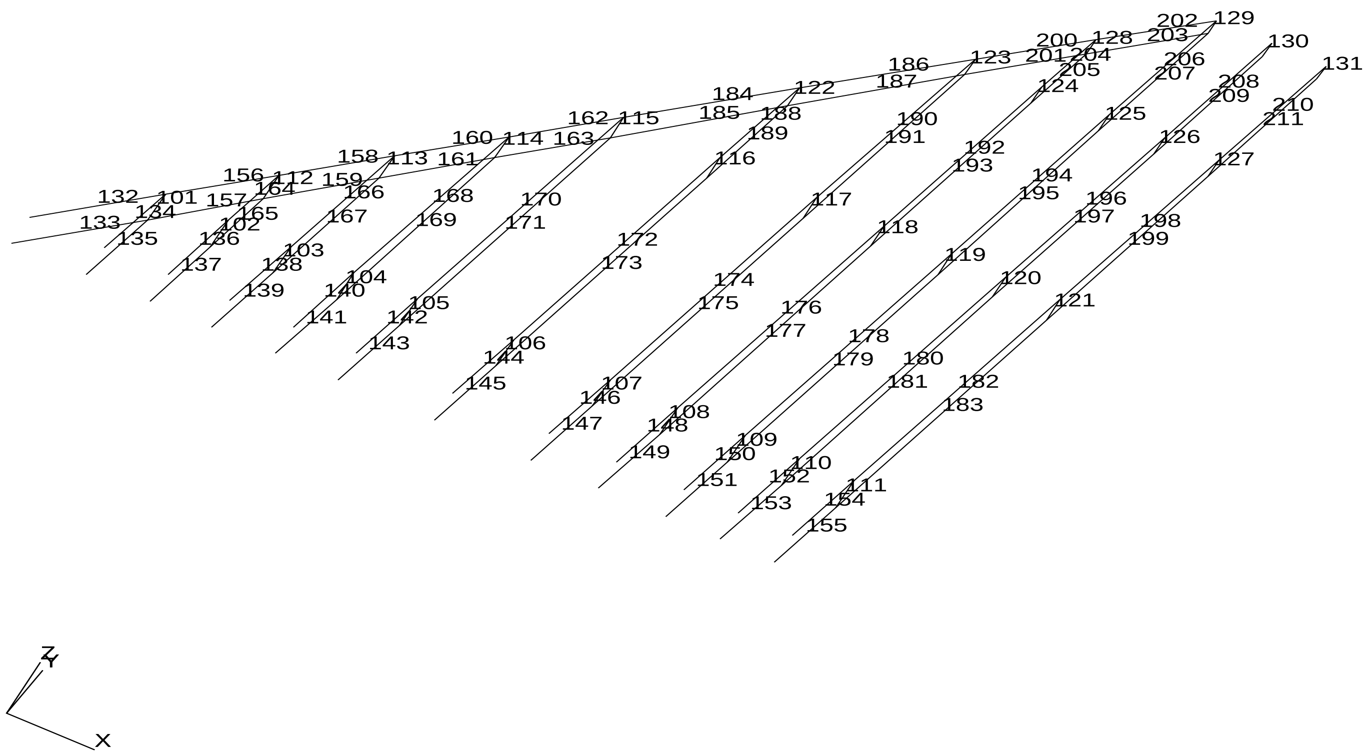
Rectangular Wing Design Example

$M_\infty = 0.85$, $\alpha_0 = 0.5°$, 20% Increase in U_f



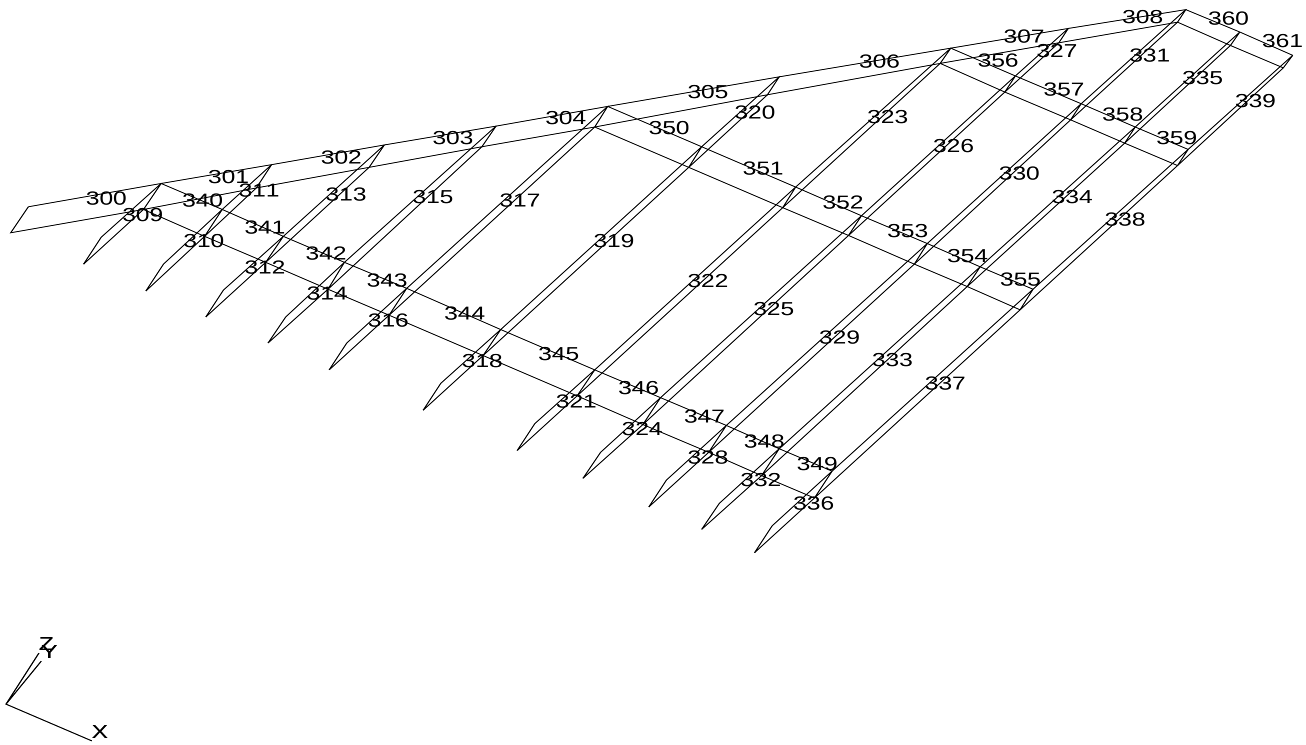
Fighter Wing Design Example

Fighter Wing Post Element IDs



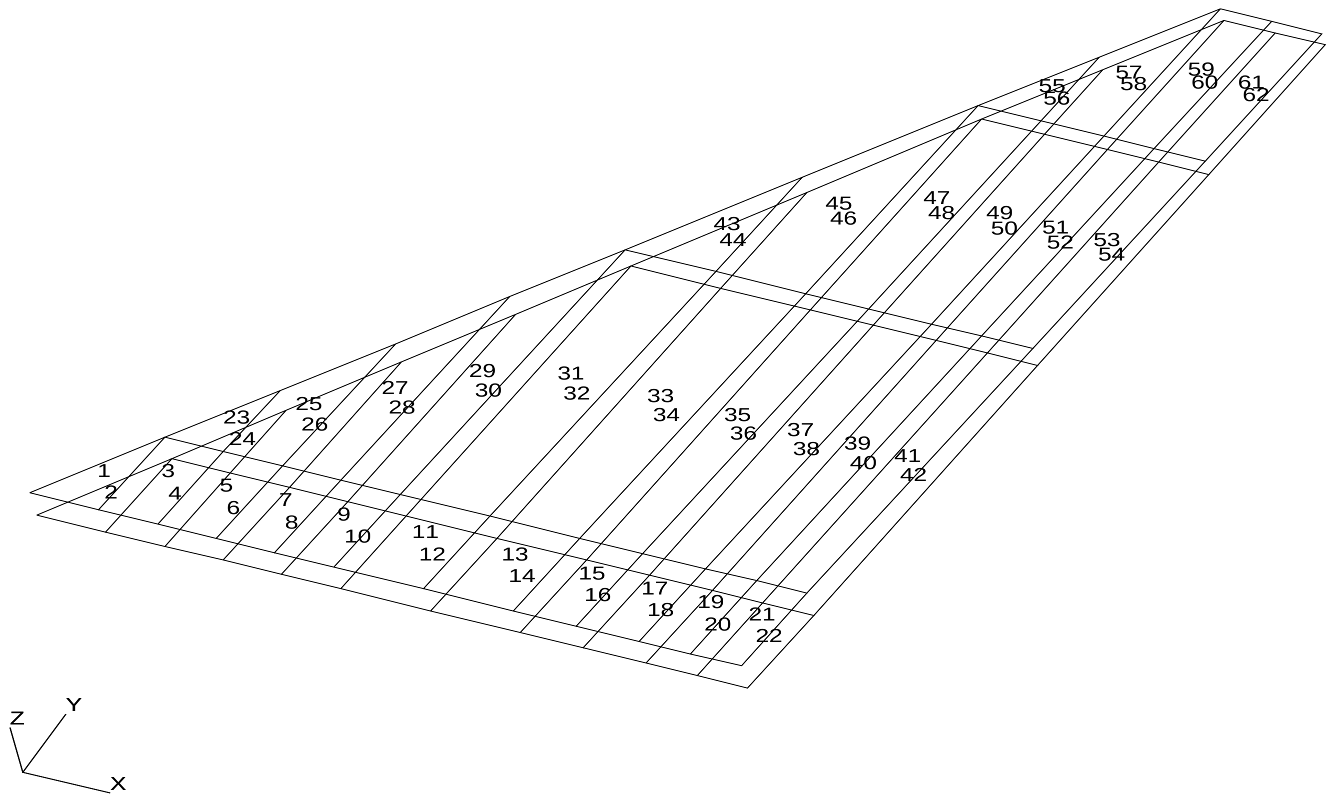
Fighter Wing Design Example

Fighter Wing Shear Panel Element IDs



Fighter Wing Design Example

Fighter Wing Skin Element IDs



Fighter Wing Design Variables

Global Design Variable	Local Design Variable	Initial Property Value	Initial Weight (lbs)	Initial Global Value	Min. Global Value	Max. Global Value
1	CROD 101 - 131	0.05 in^2	0.737	1.00	0.10	1.00
2	CSHEAR 309, 310, 312, 314, 316, 318, 321, 324, 328, 332	0.075 in	7.11	1.00	0.25	1.50
3	CSHEAR 311, 313, 315, 317, 319, 322, 325, 329, 333	0.065 in	13.909	1.00	0.25	1.50
4	CSHEAR 320, 323, 326, 330, 334	0.05 in	3.785	1.00	0.25	1.50
5	CSHEAR 327, 331, 335	0.03 in	0.702	1.00	0.25	1.50
6	CROD 134 - 153	1.00 in^2	31.618	1.00	0.10	1.25
7	CROD 164 - 181	0.75 in^2	60.565	1.00	0.10	1.25
8	CROD 188 - 197	0.60 in^2	22.226	1.00	0.10	1.25
9	CROD 204 - 209	0.50 in^2	6.983	1.00	0.10	1.25
10	CSHEAR 340 - 361	0.08 in	7.532	1.00	0.25	1.50
11	CTRMEM 1, 2	0.25 in	3.755	1.00	0.10	.150
12	CTRMEM 23, 24	0.188 in	2.046	1.00	0.10	1.50
13	CTRMEM 43, 44	0.08 in	2.036	1.00	0.10	1.50
14	CTRMEM 55, 56	0.04 in	0.482	1.00	0.10	1.50
15	CQDMEM1 3 - 22	0.25 in	69.839	1.00	0.10	1.50
16	CQDMEM1 25 - 42	0.188 in	144.050	1.00	0.10	1.50
17	CQDMEM1 45 - 54	0.08 in	26.701	1.00	0.10	1.50
18	CQDMEM1 57 - 62	0.04 in	4.578	1.00	0.10	1.50
19	CSHEAR 300, 336	0.135 in	2.833	1.00	0.25	1.50
20	CSHEAR 301 - 304, 337	0.12 in	7.405	1.00	0.25	1.50
21	CSHEAR 305, 306, 338	0.09 in	3.329	1.00	0.25	1.50
22	CSHEAR 307, 308, 339	0.05 in	1.0309	1.00	0.25	1.50
23	CROD 132, 133, 154, 155	1.75 in^2	12.240	1.00	0.10	1.25
24	CROD 156 - 163, 182, 183	1.35 in^2	32.152	1.00	0.10	1.25
25	CROD 184 - 187, 198, 199	1.05 in^2	19.119	1.00	0.10	1.25
26	CROD 200 - 203, 210, 211	0.88 in^2	10.924	1.00	0.10	1.25

Fighter Wing Design Example 1

$M_\infty = 0.93$, $\alpha_0 = 0.5°$, $U_\infty = 12459.$ in/sec, 20% Increase in U_f

Linear

Initial Design

$U_f = 12492.07$ in/sec

$\omega_f = 19.72$ Hz

$\rho_\infty = 6.764E\text{-}7$ slinches/in^3

designed weight = 497.69 lbs

Final Design (15 iterations)

$U_f = 14990.$ in/sec

$\omega_f = 19.94$ Hz

designed weight = 506.98 lbs

(1.87% greater than initial)

Nonlinear

Initial Design

$U_f = 12137.04$ in/sec

$\omega_f = 19.64$ Hz

$\rho_\infty = 6.571E\text{-}7$ slinches/in^3

designed weight = 497.69 lbs

Final Design (15 iterations 4.8 hr YMP)

$U_f = 14565.46$ in/sec

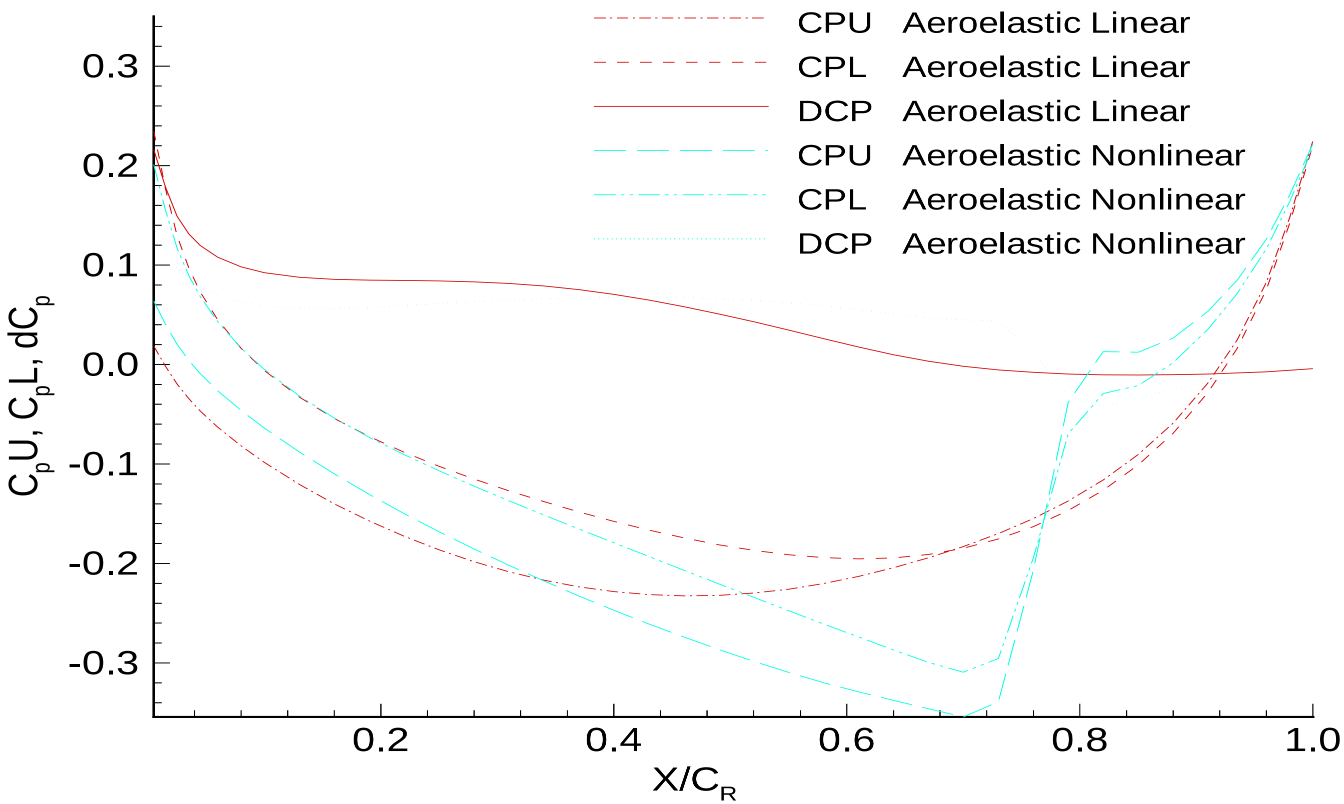
$\omega_f = 17.64$ Hz

designed weight = 426.91

(14.22% decrease from initial)

Fighter Wing Design Example 1

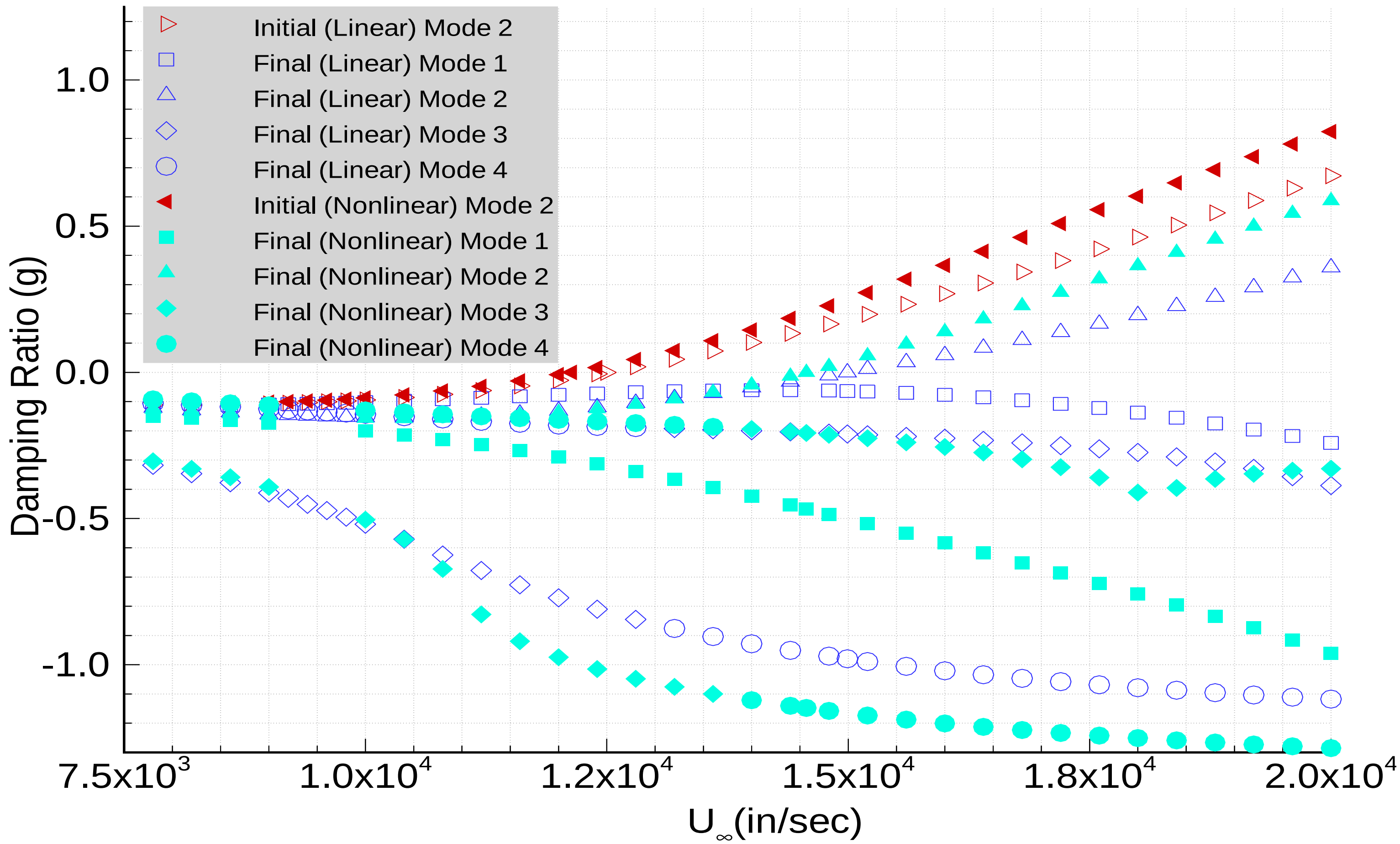
$M_\infty = 0.93$, $\alpha_0 = 0.5°$, 20% Increase in U_f



- $\rho_{f\ Linear}$= 6.764E-7 slinches/in^3
- $\rho_{f\ Nonlinear}$= 6.571E-7 slinches/in^3

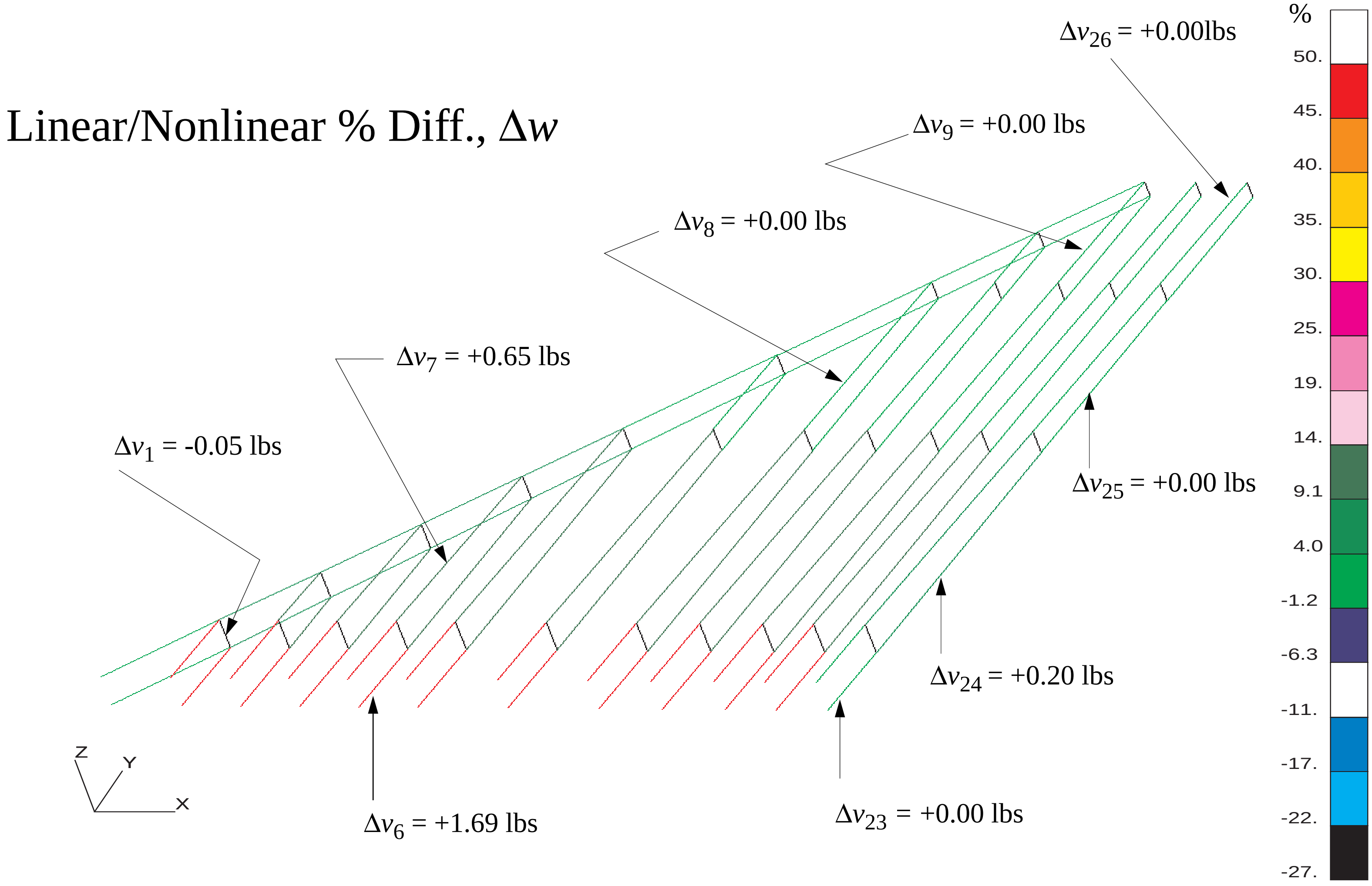
Fighter Wing Design Example 1

M_∞ = 0.93, α_0=0.5°, 20% Increase in U_f



Fighter Wing Design Example 1

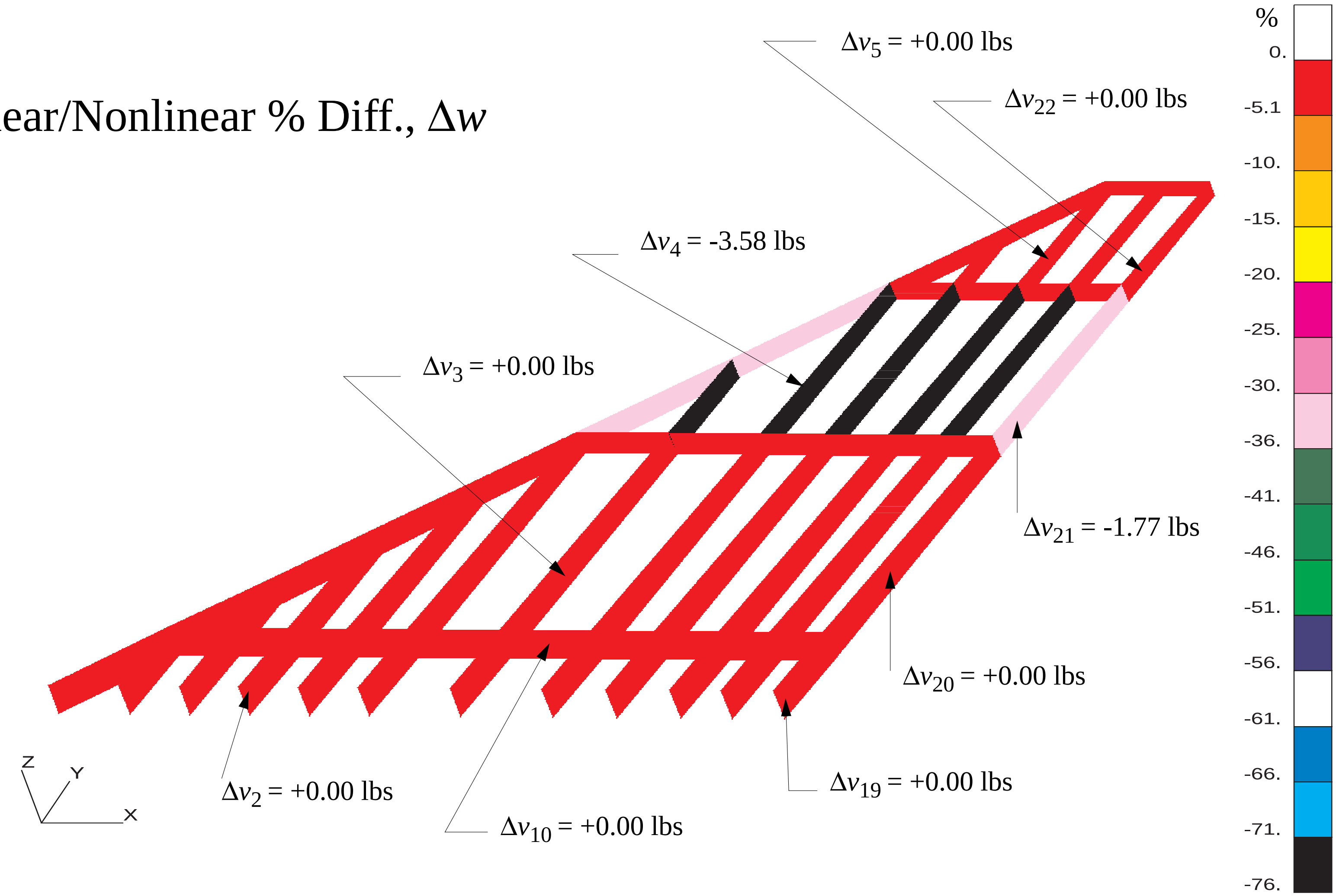
$M_\infty = 0.93$, $\alpha_0 = 0.5°$, 20% Increase in U_f



Fighter Wing Design Example 1

$M_\infty = 0.93$, $\alpha_0 = 0.5°$, 20% Increase in U_f

Linear/Nonlinear % Diff., Δw



Fighter Wing Design Example 1

$M_\infty = 0.93$, $\alpha_0 = 0.5°$, 20% Increase in U_f

Linear/Nonlinear % Diff., Δw