CRD

Nonlinear Aeroelastic Optimization

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Presentation Outline

- Introduction
- Nonlinear Unsteady Aerodynamic Approximations
- Nonlinear Unsteady Aeroelastic Analysis for Design
- Nonlinear Unsteady Aeroelastic Sensitivity Analysis
- Nonlinear Unsteady Aeroelastic Optimization (Transonic)
- Nonlinear Static Aeroelasticity Analysis for Design (Transonic)

Introduction

Goal of Computational Aeroelastic Design Methods

- To accurately ***predict*** static and dynamic response/stability at a ***reasonable computational cost***
- Methods need to be suitable for incorporation into MDA and MDO Environments

Introduction

Discretization of EOM

- Structures K, B, M - Typically, although not necessarily, represented by Finite Elements in either physical or generalized coordinates. Derived in a *Lagrangian* frame of reference.

- External Loads $F(u, \dot{u}, t)$ - Aerodynamic loads. Representations range from Prandtl's lifting line theory to full Navier-Stokes with turbulence modeling. Represented in physical and generalized coordinates in a (usually) *Eulerian* frame of reference.

Introduction

Fluid-Structural Coupling Requirements

- Must ensure spatial compatibility - proper energy exchange across the fluid-structural boundary
- Time marching solutions require proper time synchronization between fluid and structural systems
- For moving CFD meshes GCL [1] must be satisfied

If coupling requirements for time-accurate aeroelastic simulation are not met then dynamical equivalence cannot be achieved. That is, regardless of the fineness of the CFD/CSM meshes and the reduction of time step to 0, the scheme may converge to the "wrong" equilibrium/instability point.[2]

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Introduction

Nonlinear Aerodynamics

- Assume a continuum (Conservation of Mass, Momentum, Energy)

Decreasing Degree of Approximation	**Equations**
Complete	Navier-Stokes Equations
No Viscosity	Euler Equations
Irrotational, Isentropic	Full Potential Equations
Small Perturbations	Transonic Small Disturbance
Linearize	Linear Lifting Surface Theory

Introduction

Nonlinear Flow Conditions

- High angles of attack
- Large control surface deflections
- Transonic Speeds
- . . .

Nonlinear Flow Characteristics

- Attached flows with shocks
- Mixed attached and separated flows
- Mixed attached and separated flows with shocks
- Fully separated flows
- Vortex flows
- . . .

Introduction

General Modeling Comments

- Use appropriate theory to capture desired phenomena

  - Fluids - Navier-Stokes vs. Prandtl's' lifting line theory
  - Structures - Nonlinear FEM vs. Euler beam theory

- Model the fluid and structure with a consistent fidelity

  - For a wing do not model the fluid with NS and the structure with beam theory

- For design methods remember:
  - Total Load = Weight*Nz
  - Stability is determined by a perturbation about a steady state

Introduction

Aeroelastic Phenomena

Static Aeroelastic Phenomena

- Lift Effectiveness
- Divergence
- Control Surface Effective-ness/Reversal
- Aileron Effectiveness/ Reversal

Dynamic Aeroelastic Phenomena

- Flutter
- Gust Response
- Buffet
- Limit Cycle Oscillations (LCO)
- Panel Flutter
- Transient Maneuvers
- Control Surface Buzz

Introduction

Equations of Motion

The equations of motion can be expressed in generalized coordinates as

$$[\bar{M}]\{\ddot{q}\} + [\bar{B}]\{\dot{q}\} + [\bar{K}]\{q\} = \{\bar{F}\} \qquad (1)$$

Where $[\bar{M}]$, $[\bar{K}]$, $[\bar{B}]$, $\{\bar{F}\}$ are generalized mass, stiffness, damping, and aerodynamic forces respectively.

$\{q\}$ - set of independent generalized coordinates defined by $\{u(t)\} = [\Phi]\{q(t)\}$.

$\{u(t)\}$ - spatially discretized structural dof.

$[\Phi]$ - Modal transformation matrix

And the generalized aerodynamic forces can be represented as

$$\bar{F}_i(t) = \frac{\rho_\infty U_\infty^2}{2} C_R^2 \iint_{\hat{S}} \left[C_{p_L}(x, y, t) - C_{P_U}(x, y, t) \right] \Phi_i(x, y) dS \qquad (2)$$

Some Approximation Methods

- Reduced Order Unsteady Aerodynamics
- Pulse Transfer Function (Linear Impulse Function)
- Indicial Response Method
- Discrete-Time Linear and Nonlinear Aerodynamic Impulses (Volterra Theory)

Unsteady Aerodynamic Approximations

Reduced Order Unsteady Aerodynamics [3]

- Use Karhunen-Loeve (K-L) Modes as basis for fluid system

K-L eigenvectors $[V]$ are determined from the eigenvalue problem $[\Phi]\{v\} = \lambda\{v\}$

$$\Phi_{j,k} = \left\{\tilde{q}_c^j\right\}^T \left\{\tilde{q}_c^k\right\}$$

$\left\{\tilde{q}_c^j\right\} \equiv \{\tilde{q}^j\} - \{\tilde{q}'\}$ - the deviation of each instantaneous flow field from the mean flow.

$\{\tilde{q}^j\}$ - an instantaneous flow field vector retained at J discrete times. $(j = 1, 2, 3\ldots, J)$

$\{\tilde{q}'\}$ - mean flow field.

- Use $[V]$ to reduce the aerodynamic system from NxN to RxR where $R << N$.
- Demonstrated on 2-D Euler.

Pulse Transfer Function [4]

- Determine the response due to a smoothly varying pulse (exponential pulse) for each structural mode participating in the analysis.
- Unsteady generalized aerodynamic forces in *frequency* domain are determined by dividing the Fast Fourier Transform (FFT) of the time domain generalized displacements with the FFT of the time domain generalized aerodynamic forces (Transfer Function)
- Requires one time domain solution for each mode in analysis
- Assumes dynamic response linear about a nonlinear steady state
- Demonstrated on 3-D Euler and Navier-Stokes

Discrete-Time Linear and Nonlinear Aerodynamic Impulses (Volterra Theory) [5]

- Volterra theory of nonlinear systems - Any nonlinear system can be modeled as an infinite sum of multidimensional convolution integrals of increasing order.

$$y(t) = h_o + \int_0^\infty h_1(t-\tau)u(\tau)d\tau + \int_0^\infty\int_0^\infty h_2(t-\tau_1)u(\tau_1)u(\tau_2)d\tau_1 d\tau_2 + \dots$$

$$+ \int_0^\infty \dots \int_0^\infty h_n(t-\tau_1, \dots, t-\tau_n)u(\tau_1)\dots u(\tau_n)d\tau_1 \dots d\tau_n + \dots$$

$y(t)$ - Response of the nonlinear system to $u(t)$

$u(t)$ - An arbitrary input

h_o - Steady state value about which the response is computed

h_1 - First-order kernel (linear unit impulse response)

h_2 - Second-order kernel

h_n - *nth* order kernel

Unsteady Aerodynamic Approximations

- Kernel Identification
  - h_1 - Impulse response
  - h_n - Responses of the nonlinear system to multiple unit impulses, with the number of impulses applied to the system equal to the order of the kernel of interest
  - h_2 - Response to two unit impulses applied at times t_1 and t_2.
- Requires more time integrations to determine higher order kernels
- Generalization of Impulse Function Technique
- Approximation *can capture nonlinear effects* that occur during perturbation

Scope of Remaining Discussions

- To develop a transonic unsteady aeroelastic design methodology for *preliminary design* at a reasonable computational cost.

- Such a methodology could be incorporated into the multidisciplinary analysis and design optimization environment (MDAO)

Definition of Preliminary Design

- Geometry assumed fixed (wing planform, airfoil shape, #spars, #ribs, spar spacing etc.)
- Designed components consist of structural elements (thicknesses of skins, ribs and spars, cross sectional areas of posts, spar and rib caps, and concentrated masses etc.)

Critical Issues/Requirements

- Transonic aerodynamics are nonlinear
- Optimization problem is nonlinear
- Coupling two nonlinear problems is not realistic

Preliminary Design MDDA Requirements

- Structural and mass distribution down to substructure level
- F.E.M 50k to 100k d.o.f maximum
- Aerodynamic loads normal to lifting surface
- Small disturbance theory (location and strength of weak shocks)

Solution Approach

- Formulate as nonlinear mathematical programming problem
- Solve by gradient based optimization
- Analysis
  - F.E.M
  - TSD with Indicial Response Method to approximate the unsteady aerodynamic forces
    - * Modal based *p*-Method for flutter analysis
- Constraint/Constraint gradients
  - Constraint on modal damping rather than on flutter velocity
  - Semi-analytic gradients (some assumptions can produce fully analytic gradients)
- Redesign
  - Use First Order Taylor Series Approximations for functional values and constraints
  - Solve approximate problem by Method of Modified Feasible Directions

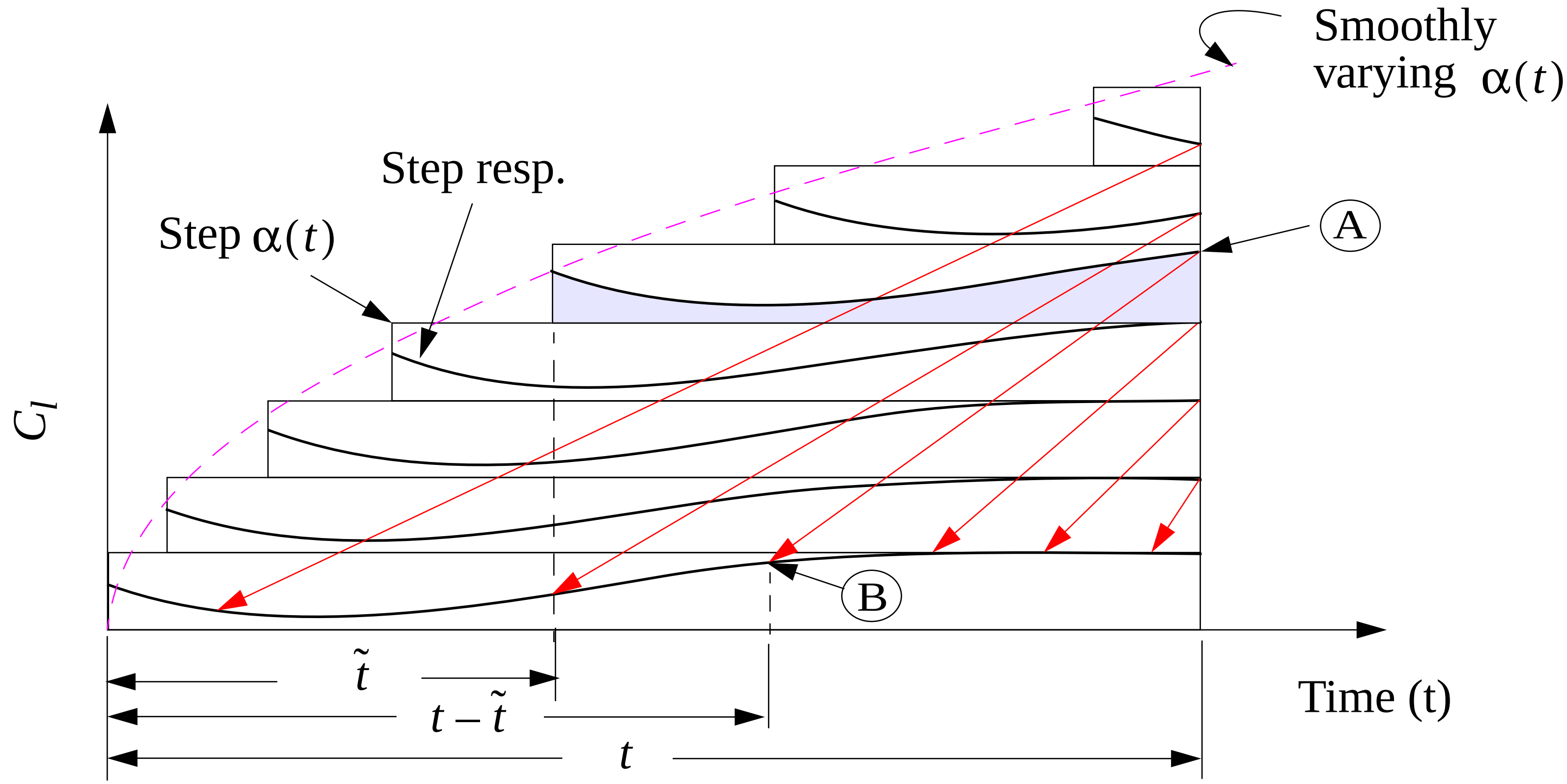
Unsteady Aerodynamic Approximations

Indicial Response Method [6],[10]

- Assume the unsteady aerodynamic forces are linear about some non-linear static aeroelastic solution (valid for small perturbations).
- Allows use of an *Indicial Response Method* (IRM) to approximate unsteady aerodynamic forces (limited linear superposition).
- IRM requires one nonlinear solution for each mode that participates in flutter analysis for a given set of initial conditions (Mach number, initial angle of attack, Reynolds #, etc.).
- Small perturbations - can neither create, destroy nor significantly move a shock.
- Impulse Function and IRM related (Impulse response derivative of Step response)

Unsteady Aerodynamic Approximations

Indicial Response Method (IRM)



$$C_l(t) = \sum_{i=0}^{n} C_{l_{\alpha\tilde{t}_i}}(t)\Delta\alpha(\tilde{t}_i) \text{ or } C_l(t) = \sum_{0}^{t} C_{l_{\alpha t_0}}(t-\tilde{t})\Delta\alpha(\tilde{t}) \quad (3)$$

Unsteady Aerodynamic Approximations (IRM)

In the limit $\Delta t_i \to 0$ and also with a change of variable $\tau = t - \tilde{t}$ Eq (3) becomes

$$C_l(t) = C_{l_{\alpha t_0}}(t)\alpha(0) + \int_o^t C_{l_{\alpha t_0}}(\tau)\dot{\alpha}(t-\tau)d\tau \tag{4}$$

Eq (4) is a form of Duhamel's formula. It enables the calculation of the response of a system to an arbitrary input by knowing only the indicial response of the system. It assumes that linear superposition applies.

Now for a generalized force $\bar{F}_i(t)$, step $q_j(t)$ $Q_{ij}(t)$ Eq can be written as

$$\frac{\bar{F}_i(t)}{\frac{1}{2}\rho_\infty U_\infty^2 C_R^2} = Q_{ij}(t)q_j(0) + \int_0^t Q_{ij}(\tau)\dot{q}_j(t-\tau)d\tau \tag{5}$$

Transforming from the time domain to the Laplace domain via the convolution integral gives

$$\bar{F}_i(s) = \frac{1}{2}\rho_\infty U_\infty C_R^2 s Q_{ij}(s) q_j(s) \tag{6}$$

Eq (6) represents generalized unsteady aerodynamics forces in the *s-domain*

Unsteady Aerodynamic Approximations (IRM)

Defining $[\bar{Q}(s)] = C_R^2 s[Q(s)]$ and transforming the rest of Eq. (1) into the *s-domain* yields

$$\left[[\bar{M}]s^2 + [\bar{K}] + [\bar{B}]s - \frac{1}{2}\rho_\infty U_\infty^2[\bar{Q}(s)]\right]\{q(s)\} = 0 \qquad (7)$$

The Laplace variable s is in general complex and is defined as $s = \sigma + i\omega$

σ - exponential damping

ω - oscillatory frequency of the system

- Solutions to Eq (7) for flutter can be found by *p-k*, *p*, root-locus etc. Provided $\bar{Q}(s)$ is a continuous function of *s* but need not be harmonic.

Unsteady Aerodynamic Approximations (IRM)

- Assume

$$\bar{Q}_{ij}(s) = sC_R^2 \sum_{r=1}^{n} \frac{C_{r_{ij}}}{s + b_{r_{ij}}} \qquad (8)$$

- Determine the coefficients $C_{r_{ij}}, b_{r_{ij}}$ in the following manner

  - Determine the static aeroelastic response for a given flight condition
  - Determine $\Delta\bar{F}_{ij}(t)_N$ in the generalized force $\bar{F}_i(t)$ due to a Laplace transformable step input $q_{S_j}(t)$ for the jth mode.
  - Once $\Delta\bar{F}_{ij}(t)_N$ is obtained $C_{r_{ij}}, b_{r_{ij}}$ are selected to produce the "best" approximating curve fit between $\Delta\bar{F}_{ij}(t)_N$ and $\Delta\bar{F}_{ij}(t)_A$ where $\Delta\bar{F}_{ij}(t)_A = \Im^{-1}\left\{\bar{Q}_{ij}(s)q_{S_j}(s)\right\}$
  - with $q_j(s) = A_j\Phi_j\left\{\int_0^a\left(10\left(\frac{t}{a}\right)^3 e^{-st} - 15\left(\frac{t}{a}\right)^4 e^{-st} + 6\left(\frac{t}{a}\right)^5 e^{-st}\right)dt + \int_a^\infty e^{-st}\right\}$

Unsteady Aerodynamic Approximations (IRM)

Let $s = \omega(\gamma + i)$, define $p \equiv k(\gamma + i)$, $k = (\omega C_R)/(2U_\infty)$, $b = \frac{C_R}{2}$ Eq. (7) becomes

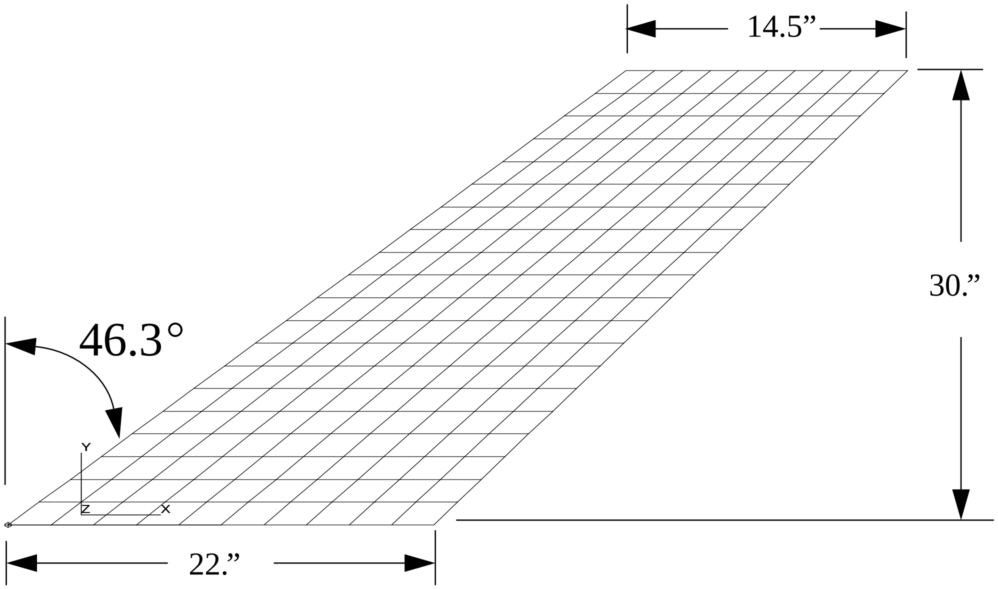
$$\left[\left(\frac{U_\infty}{b}\right)^2 p^2[\bar{M}] + \left(\frac{U_\infty}{b}\right)p[\bar{B}] + [\bar{K}] - \frac{1}{2}\rho_\infty U_\infty^2[\bar{Q}(p)]\right]\{q(p)\} = 0 \quad (9)$$

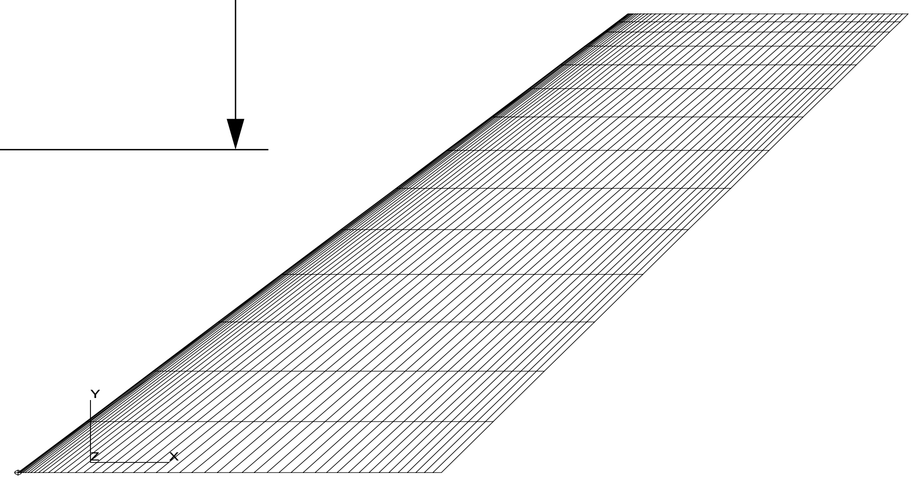
- Solve Eq. (9) by the *p-method*
- *p-method* γ damping valid away from axis (nice for flutter constraints)

Nonlinear Unsteady Aeroelastic Analysis

Indicial Response Flutter Analysis

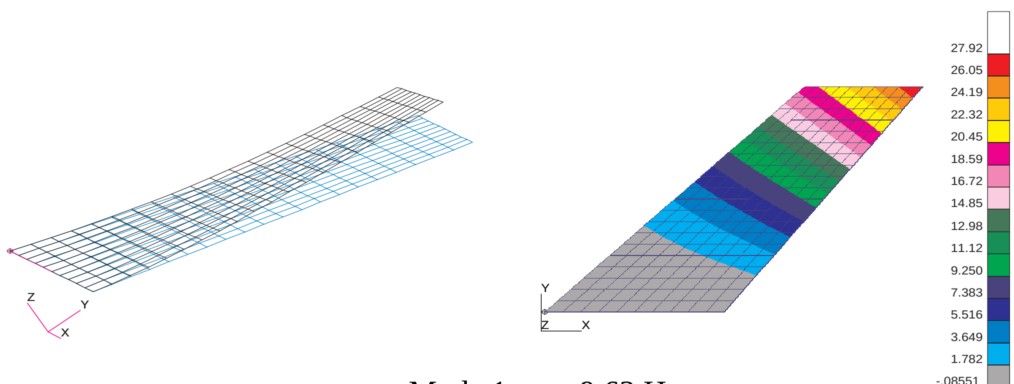
AGARD 445.6 Wing (Weak Model 3) - TSD Aerodynamics (CAP-TSD)



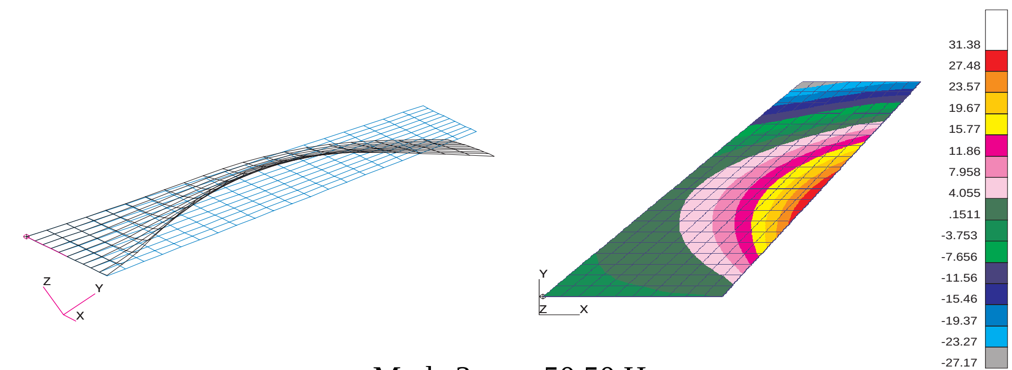


445.6 Flutter Analysis

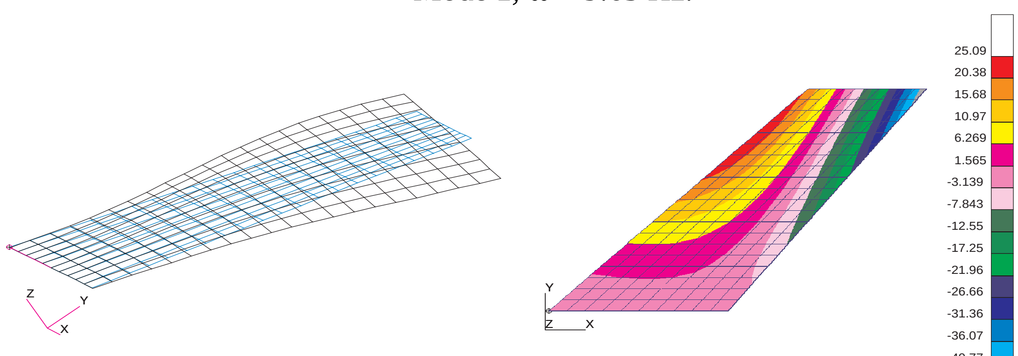
Mode Shapes and frequencies



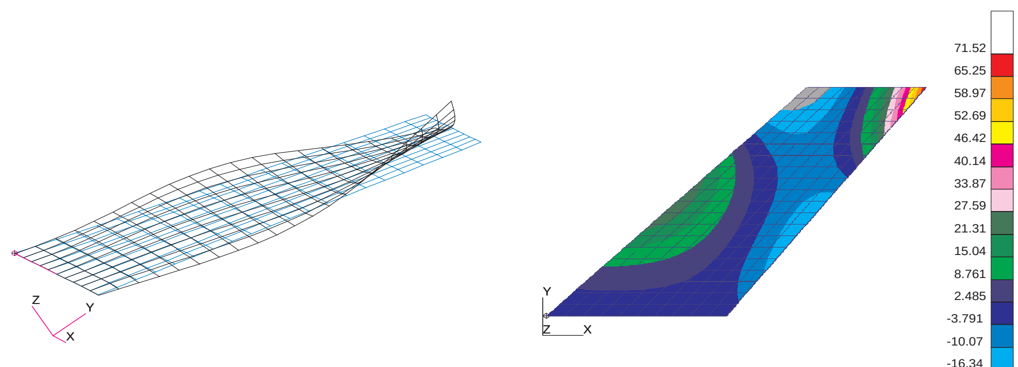
Mode 1, ω = 9.63 Hz.



Mode 3, ω = 50.50 Hz.



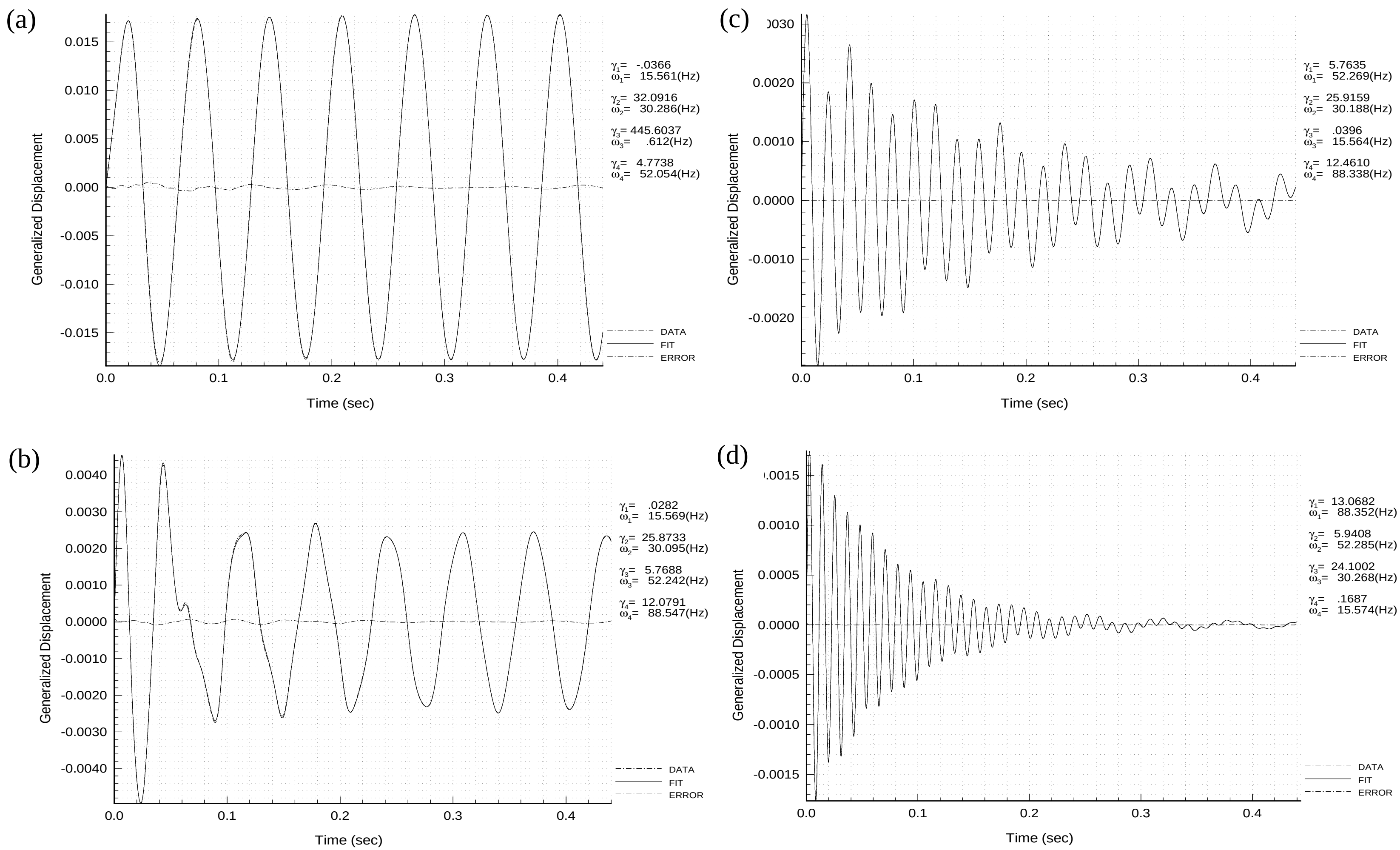
Mode 2, ω = 37.12 Hz.



Mode 4, ω = 89.94 Hz.

445.6 Time Integration Response

$q_f \approx 0.667$ psi, $U_f \approx 11,971$ in/sec , $\omega_f \approx 15.45$ Hz

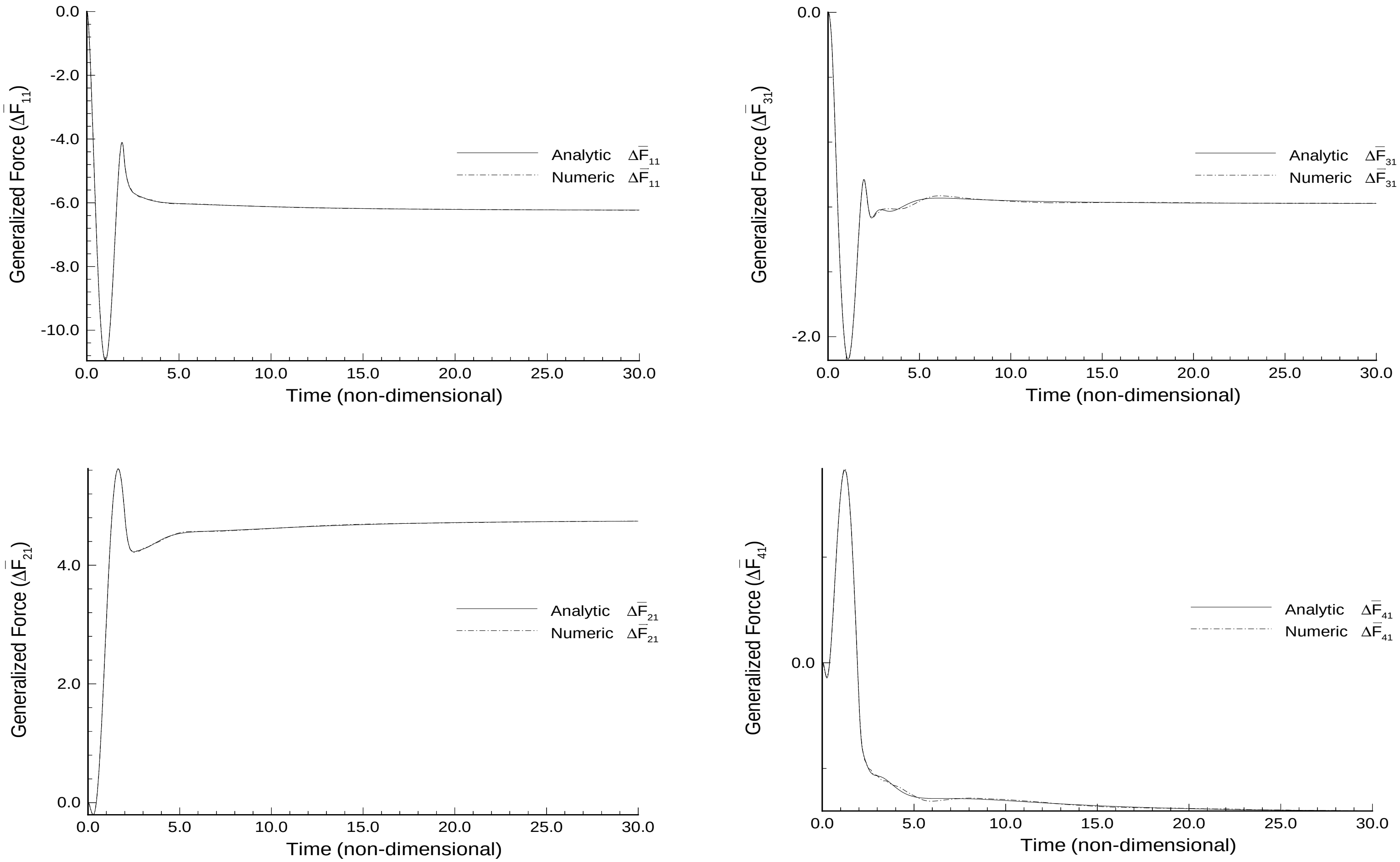


Time response for 445.6 wing (a) mode 1 (b) mode 2 (c) mode 3 (d) mode 4 (M_∞ = 0.901, α_0 = 0.0˚, q_∞ = 0.67 psi, U_∞ = 11998.75 in/sec)

Nonlinear Unsteady Aeroelastic Analysis

445.6 Indicial Responses $\Delta\bar{F}_{j1}$

$M_\infty = 0.901,\ \alpha_0 = 0.0^\circ$

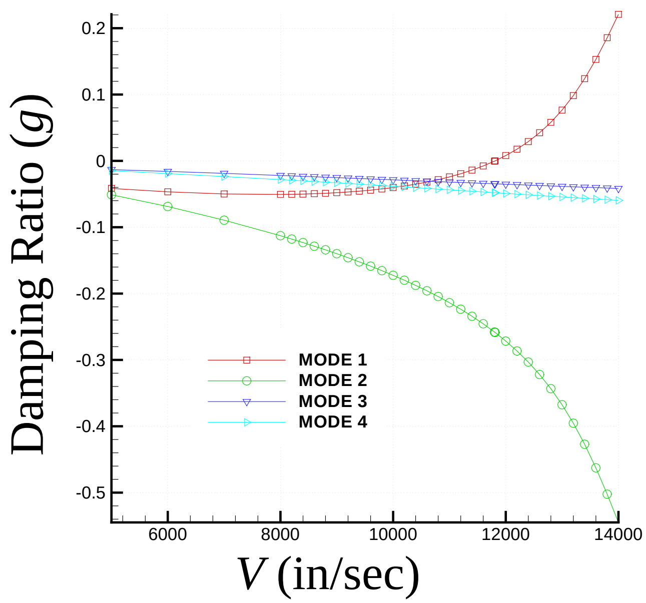


Nonlinear Unsteady Aeroelastic Analysis

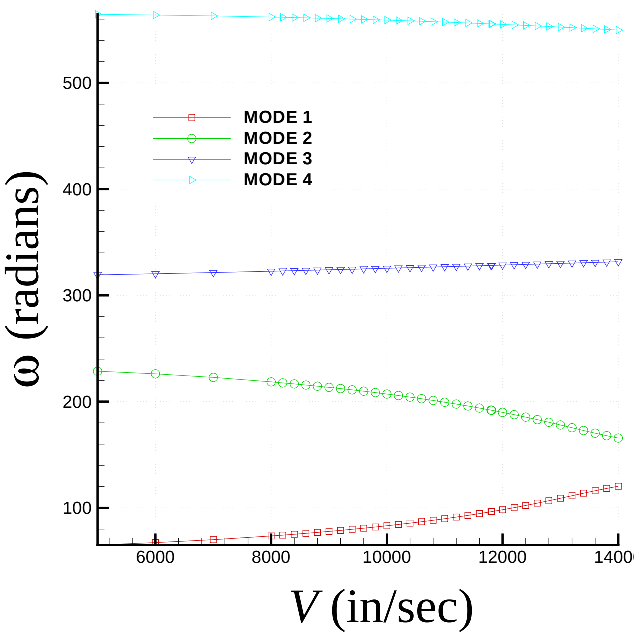
445.6 Indicial Response Flutter Analysis

$q_f \approx 0.650$ psi, $U_f \approx 11,810$ in/sec , $\omega_f \approx 15.35$ Hz

V vs. *g*



V vs. ω



g vs. ω