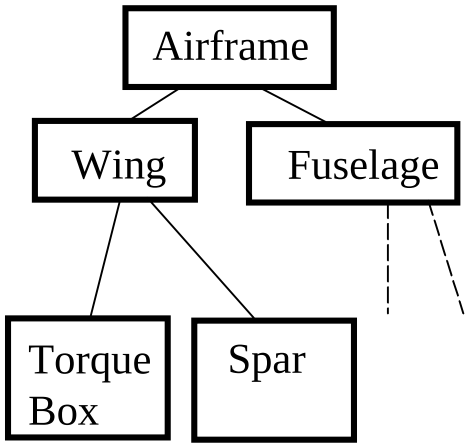
Decomposition Methods in MDO

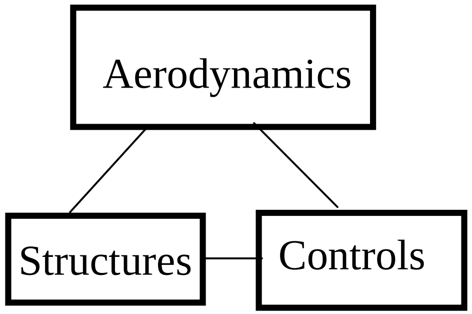
Decomposition Methods in Design

- Decomposition strategies have been used in the design of multidisciplinary systems
  - hierarchic decomposition
  - non-hierarchic decomposition
  - hybrid decomposition
- Elements of decomposition
  - partitioning
  - co-ordination

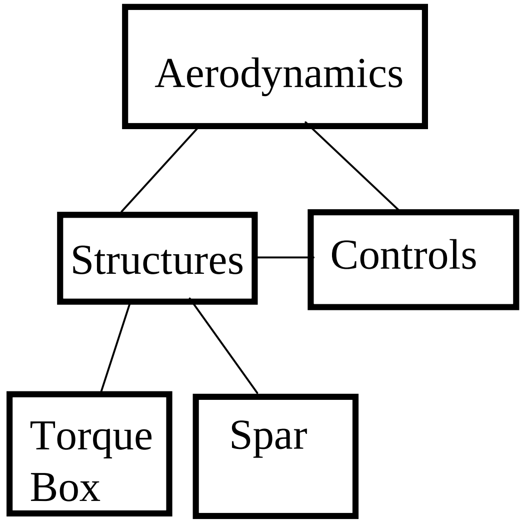
Decomposition Methods in Design



Hierarchic

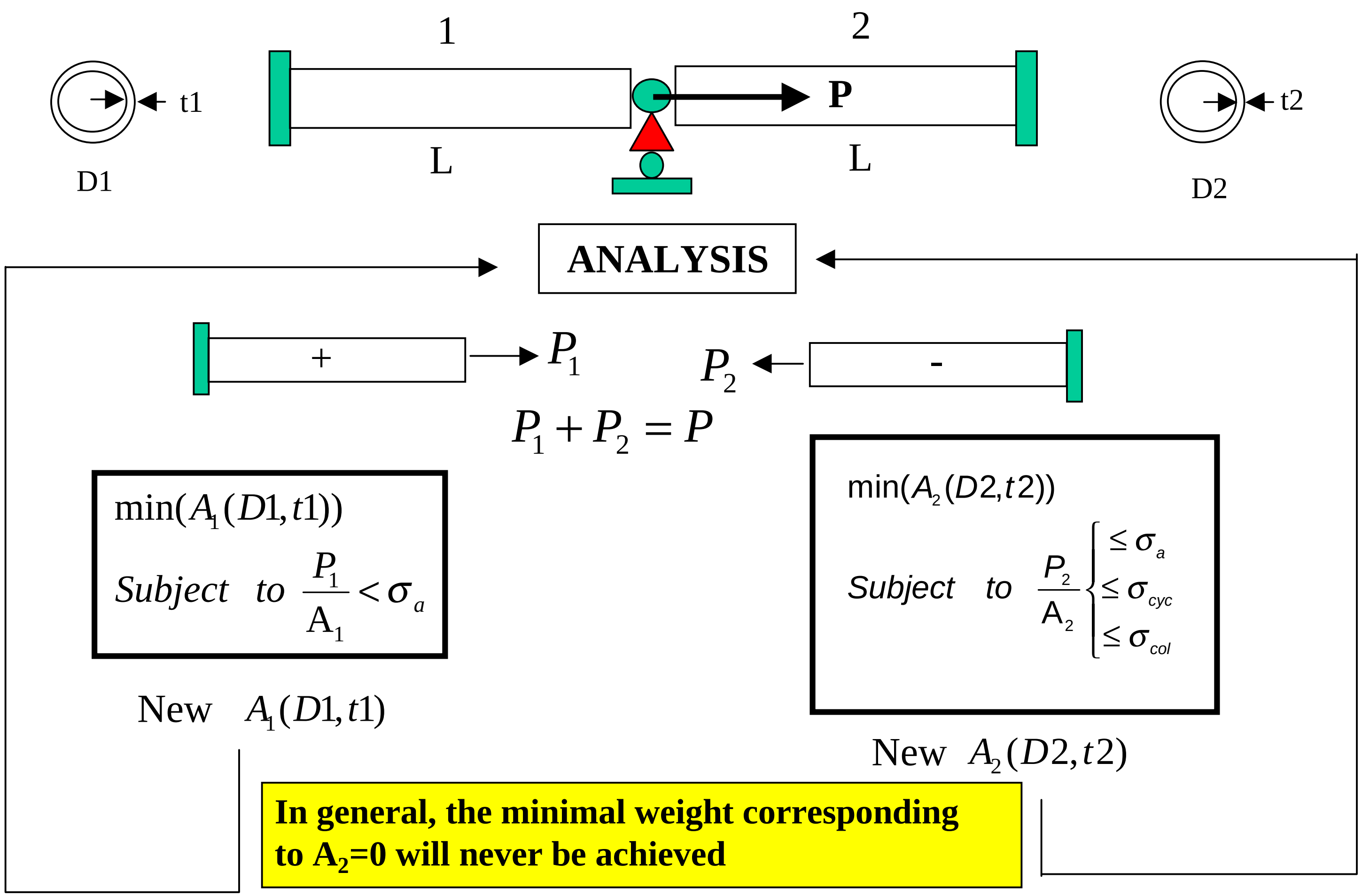


Non-Hierarchic



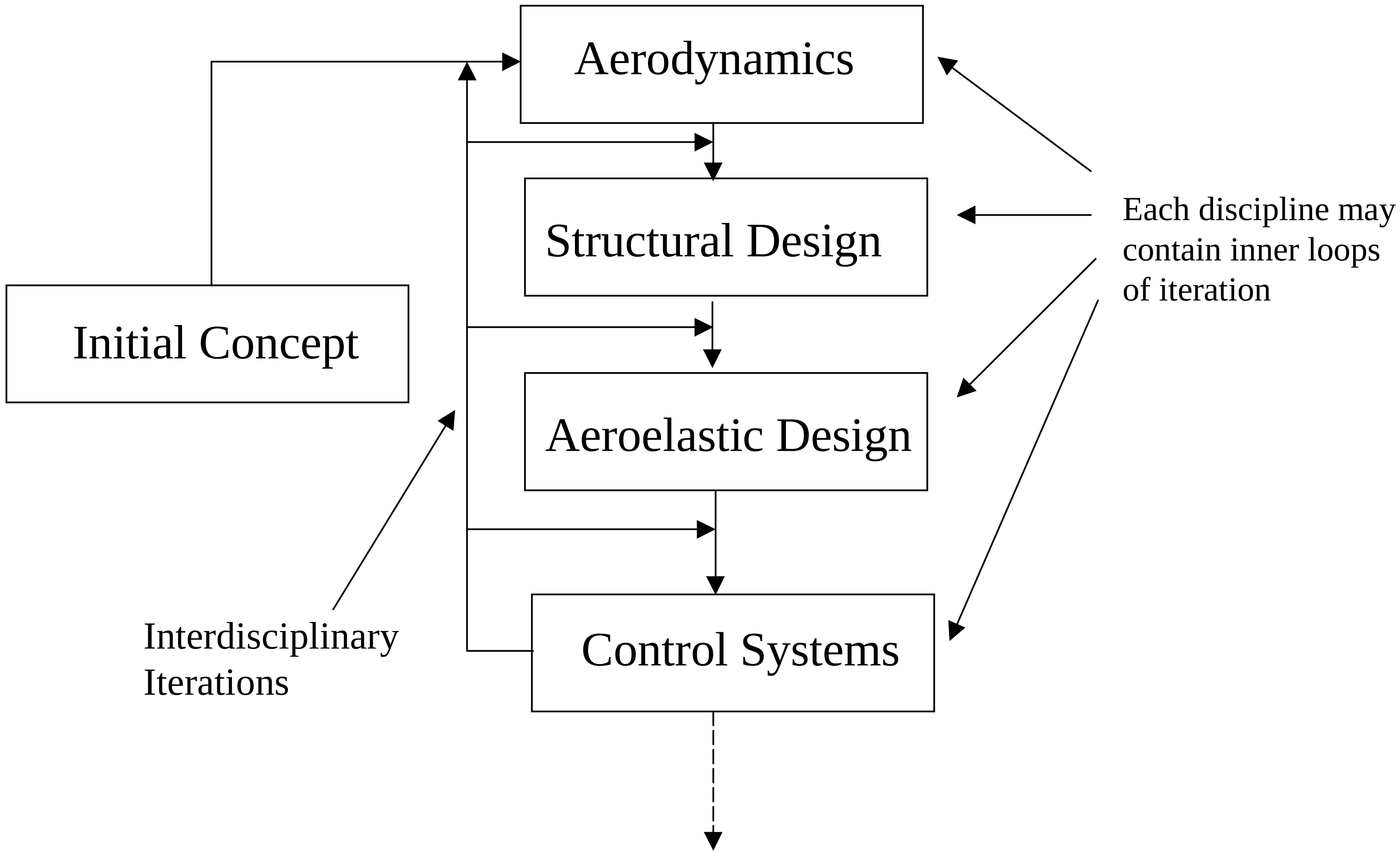
Hybrid

Partitioned Design W/O Coordination



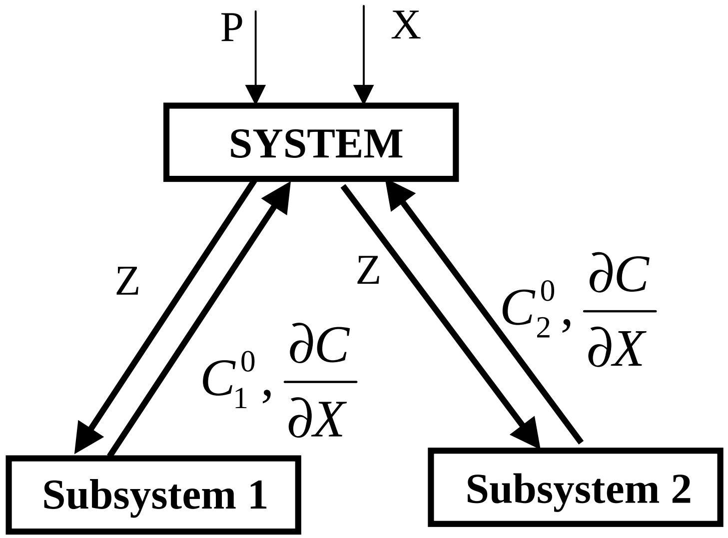
In general, the minimal weight corresponding to A_2=0 will never be achieved

Sequential Approach



Hierarchic Decomposition Based Design

- System level
  - P - problem parameters
  - X - design variables
  - Q - behavior variables
  - Analysis F(X,P,Q)=0
    - Z passes as input to subsystem Z=f(X,Q)
- Subsystem level
  - x - local design variables
  - q - local behavior variables
  - analysis f(Z,q,x)=0



No lateral interaction

Hierarchic Decomposition Based Design

- Subsystem level optimization
  - g=g(q(Z(X),x) ≤ 0 and h=h(Z(X),x)=0 are the inequality and equality constraints respectively
  - C is a cumulative constraint representation of all inequality constraints
- Optimization problem statement

$$\underset{x}{Minimize}\ \ C \text{ to obtain } C_i^0$$

$$Subject\ to\ \ h = 0$$

- Pass back $C_1^0, \dfrac{\partial C}{\partial X}$ and $C_2^0, \dfrac{\partial C}{\partial X}$ to the system level

Hierarchic Decomposition Based Design

- System level optimization
  - F is system objective F=F(Q(X))
  - G are system constraints G=G(Q(X))
- Mathematical problem statement

$$\underset{X}{Minimize}\ \ F(X)$$

$$Subject\ to\ G_j \le 0; C_i \le 0 \text{ for all i subsystems}$$

- Here Ci are obtained as a linear extrapolation

$$C_i = C_i^0 + \frac{dC}{dX}\Delta X \qquad \frac{dC}{dX} = \frac{\partial C}{\partial Z}\frac{\partial Z}{\partial X}$$

$\frac{\partial Z}{\partial X}$ ← System sensitivity

$\frac{\partial C}{\partial Z}$ ↑ Subsystem optimal sensitivity

Hierarchic Decomposition Based Design

- Errors in extrapolation due to active constraint switching
- Data management gets involved in realistic, large-scale MDO problems
- Handling of equality constraints is required as these provide system to subsystem coordination - proves to be problematic!

Non-Hierarchic Decomposition Based Design - CSSO

- Concurrent Subspace Optimization - Overview

$$\begin{aligned} &Minimize \quad f(x^k) \\ &Subject\ to: \quad C^p \le C^{po}[s^p(1-r_k^p)+(1-s^p)t_k^p] \quad p=1,nss \\ &\qquad\qquad\quad x_L^k \le x^k \le x_U^k \end{aligned}$$

- C^p is a measure of all constraints in subspace p, super and subscript p and k denote the influence of subspace p on subspace k, r's are the responsibility coefficients, t's are the trade-off coefficients, and s are the switch parameters

CSSO Overview

- Optimum in each subspace is a function of r and t coefficients, and a second level problem needs to be solved - COP

$$Minimize \quad F = f^{o} + \sum_{p}\sum_{k}\frac{df}{dr_k}\Delta r_k^p + \sum_{p}\sum_{k}\frac{df}{dt_k}\Delta t_k^p$$

$$Subject\ to: \quad \sum_{k} r_k^p = 1 \quad p,k = 1,nss$$

$$\sum_{k} t_k^p = 0 \quad 0 \le r_k^p \le 1$$

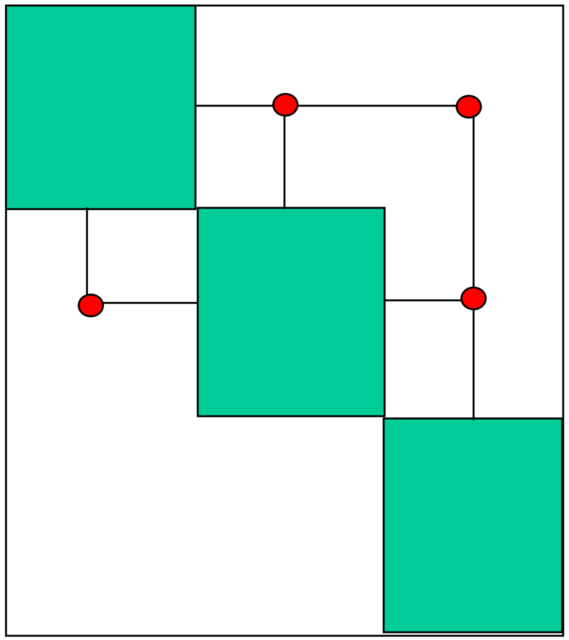
$$r_{kL}^p \le r_k^p \le r_{kU}^p \quad t_{kL}^p \le t_k^p \le t_{kU}^p$$

- COP yields a new set of r’s and t’s to be used in next round of SSO’s

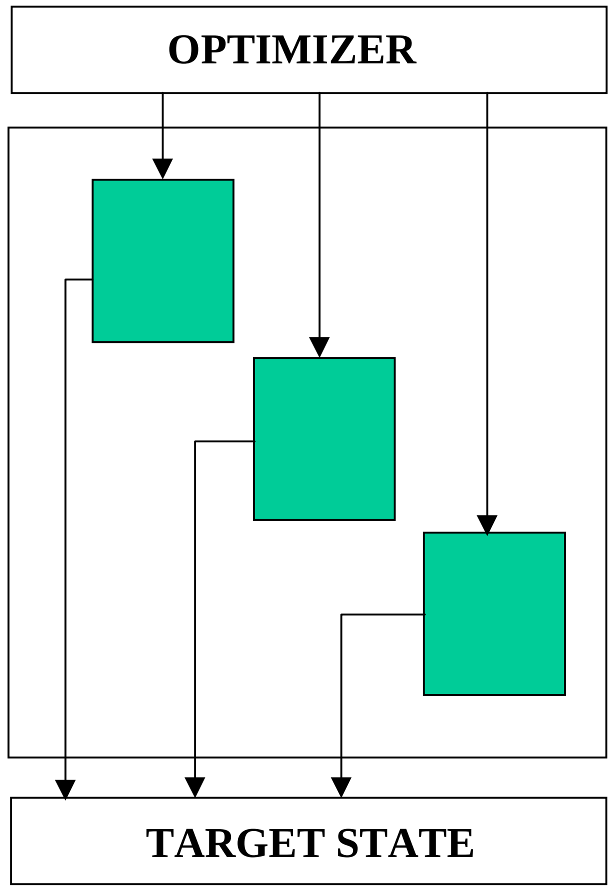
Deficiencies in CSSO

- Coupling in CSSO is resolved through the use of linear or higher-order approximations. These require move limits to be placed on the design variables.
- The formulation of the coordination problem is based on an optimal sensitivity analysis procedure that cannot be regarded as robust.
- The use of heuristics in solving the coordination problem have proven to be of limited benefit, often introducing cycling and convergence problems.
- Sensitivity information is unavailable when dealing with discrete and integer design variables.
- Solution of GSE for computing global sensitivity information is problematic in terms of the required computational effort, and numerical problems such as singularity and ill-conditioning.

Collaborative Optimization

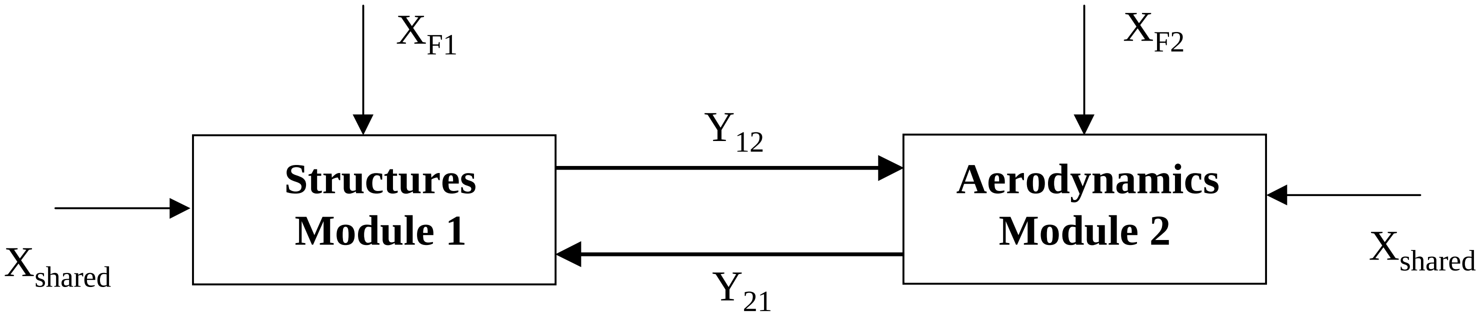


Coupled



Uncoupled

Collaborative Optimization - Example



X_{F1}- cross sectional dimensions of stringers and skins

X_M - Y_{21} and X_{shared}

g(X) - constraints on stress and displacements

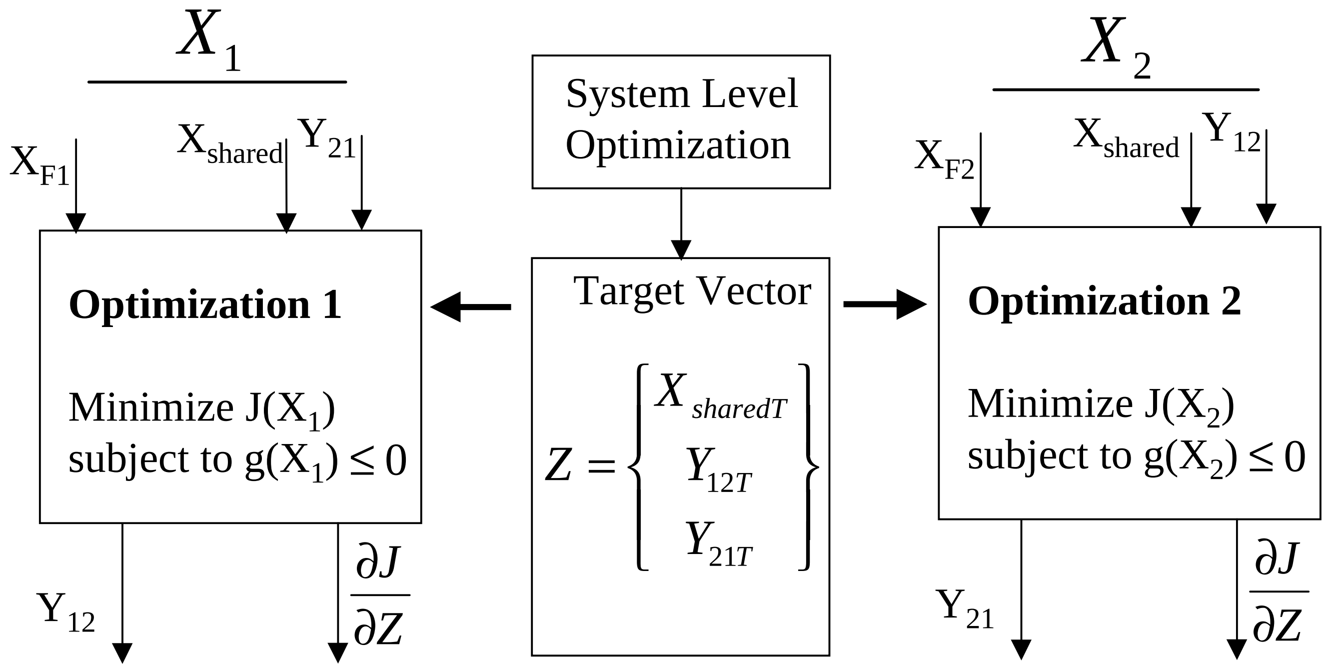
X_{F2}- airfoil leading edge radius and camber

X_M - Y_{12} and X_{shared}

g(X) - constraints on pressure distribution gradients

X_{shared} - wing taper, wing sweep, wing aspect ratio

Collaborative Optimization - Procedure



$$J = \begin{Bmatrix} Y_{21} - Y_{21T} \\ X_{shared} - X_{sharedT} \\ Y_{12} - Y_{12T} \end{Bmatrix}^T \begin{Bmatrix} Y_{21} - Y_{21T} \\ X_{shared} - X_{sharedT} \\ Y_{12} - Y_{12T} \end{Bmatrix}$$

$$J = \begin{Bmatrix} Y_{12} - Y_{12T} \\ X_{shared} - X_{sharedT} \\ Y_{21} - Y_{21T} \end{Bmatrix}^T \begin{Bmatrix} Y_{12} - Y_{12T} \\ X_{shared} - X_{sharedT} \\ Y_{21} - Y_{21T} \end{Bmatrix}$$

Collaborative Optimization
System Level Optimization

- Find Z (system level variables shared by different modules) to minimize F(Z) (system level objective)
- Satisfy J_i=0 for all modules I
  - here J_i is obtained from extrapolation based on optimal problem parameter sensitivity from each module

$$J_i = (J_i)_{old} + \left\{ \frac{\partial J_i}{\partial Z} \right\}^T \{\Delta Z\}$$

Collaborative Optimization Summary

- Each module optimized separately - can be done concurrently
- System level coordination optimization is generally with small number of variables
- Each discipline is allowed to function autonomously
- There is no explicit system analysis - instead we have a situation akin to simultaneous analysis and design