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STATE SPACE REPRESENTATION OF SMART STRUCTURES UNDER UNSTEADY AERODYNAMIC LOADING

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ABSTRACT

This study presents a technique for the state space representation of the aeroelastic models of smart structures. It was based on a rational approximation of the unsteady aerodynamic loads in the Laplace domain, which yielded state-space matrix equations of motion with constant coefficients. In this study, an aluminum plate-like structure with twenty-four surface bonded piezoelectric patches was considered. The unsteady aerodynamic loads acting on the structure were calculated for a range of reduced frequencies and a Mach number by using a linear two-dimensional Doublet-Lattice Method available in MSC/NASTRAN®. Those discrete air loads were approximated as rational functions of the Laplace variable by using one of the aerodynamic approximation schemes, Roger's approximation, with least-squares method. Then the state space representation of the aeroelastic model was constructed by using the approximated air loads together with structural matrices. In order to verify the state space approach, the flutter characteristics of the plate-like structure were investigated and the results were compared to those obtained by MSC/NASTRAN® analysis.

INTRODUCTION

Aeroservoelasticity theory represents the combination of aerodynamics, control systems and aeroelasticity disciplines regarding different aspects of aircraft dynamics. The interactions between the flexible structure, the aerodynamic forces and the control laws acting on the structure can cause instabilities at any time inside the flight envelope, and these aeroservoelastic interactions on aircraft are very complex problems to solve.

One main aspect of aroservoelasticity is the construction of aeroelastic model in state-space form which has apparent advantages for the aeroelastic analysis, for instance the *V-g* method for flutter analysis may be replaced by a simple root-locus [5,8] or for feedback control utilization, for instance in the use of modern control techniques [1,2].

Recent development of smart structures, the structures which can sense the external disturbance and respond to that with active control in real time to maintain the mission requirements, has encouraged many researchers to work in aeroelastic control. Raja et al. [3] developed an active flutter velocity enhancement scheme employing linear quadratic Gaussian (LQG) based MIMO controller with piezoelectric actuators and sensors. An active smart material control system, using distributed piezoelectric actuators, was presented for buffet alleviation by Sheta et al. [4].

The main difficulty in modeling an aeroelastic system in state-space form for control design lies in the representation of the unsteady aerodynamic loads. The air loads in the frequency domain have to be approximated by rational functions of Laplace variable *s* (namely fraction of polynomials of *s*). There are several methods in the literature in order to approximate the unsteady generalized forces by rational functions from frequency domain to the Laplace domain. Karpel et al. [10] presented minimum-state formulation for rational function approximations of aerodynamic forces to an active flexible wing wind-tunnel

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model. Matrix Pade approximant technique was introduced by Vepa [14] and modified by Edwards [12]. Also, Roger [15] introduced a formulation in the Laplace domain for the aerodynamic forces by using common denominator roots. In this paper, Roger’s formulation with the least-squares method [13] was used for aerodynamic approximation. By using these approximated air loads the state space representation of the aeroelastic model was constructed for the smart structure. Also, a root-locus analysis was performed in order to obtain the flutter characteristics of the structure by using the state-space model.

THEORY

Generalized equations of motion

The basic assumption of the modal approach to structural dynamics is that the structural displacements can be adequately expressed as a linear combination of some baseline modes and modal displacement vectors [9]:

$$\{x\} = [\phi]\{\zeta\} \tag{1}$$

where $\{x\}$ is the structural displacements vector, $\{\zeta\}$ is the modal displacements vector and $[\phi]$ is the baseline modes.

By assuming no damping, one can obtain the following generalized equations of motion:

$$[M_s]\{\ddot{\zeta}(t)\} + [K_S]\{\zeta(t)\} = \{F_S(t)\} \tag{2}$$

where $[M_S]$ is the modal mass matrix calculated as $[M_S] = [\phi]^T [M][\phi]$ by using the discrete mass matrix $[M]$, $[K_S]$ is the modal stiffness matrix and determined as $[K_S] = \omega_i^2 [M_S]$ where ω_i is the natural frequency and $\{F_S\}$ is the generalized forces calculated by $\{F_S\} = [\phi]^T \{F(t)\}$ where $\{F(t)\}$ is the force vector in discrete coordinates.

Most of the commercial unsteady aerodynamic routines assume that the structure undergoes harmonic oscillations. Thus, Equation 2 can be written in the frequency domain as [9]:

$$(-\omega^2 [M_S] + [K_S])\{\zeta(i\omega)\} = \{F_S(i\omega)\} \tag{3}$$

where ω is the frequency of the oscillations.

The generalized unsteady aerodynamic forces acting on the structural modes of a linear aeroelastic system is also expressed in the frequency domain as [16]:

$$\{F_S(i\omega)\} = q[Q(ik)]\{\zeta(i\omega)\} \tag{4}$$

where q is the dynamic pressure given by $q = \frac{1}{2}\rho V^2$, ρ is the air density and V is the air velocity . k is called the reduced frequency and is known to be $k = \omega b / 2V$, where b is a reference chord length. The generalized aerodynamic influence coefficient matrix $[Q(ik)]$ is a complex function of reduced frequency and flight conditions .

By substituting Equation 4 into Equation 3 and writing in the Laplace domain results in the generalized equations of motion for aeroelastic motion:

$$(s^2 [M_S] + [K_S] - q[Q(s)])\{\zeta(s)\} = 0 \tag{5}$$

Aerodynamic load approximation

Aerodynamic influence coefficient matrices $[Q(ik)]$ can be calculated at several discrete reduced frequencies by MSC®/NASTRAN [7]. In order to obtain an aeroelastic system, with a finite-order state form matrix equation for linear stability analysis, the aerodynamic influence coefficients have to be approximated by rational functions of Laplace variable *s* [10].

The unsteady aerodynamic forces can be approximated by Roger's approximation [15] as:

$$Q(k) = A_0 + (jk)A_1 + (jk)^2 A_2 + \sum_{m=3}^{M} \frac{(jk)A_m}{(jk + \beta_{m-2})} \tag{6}$$

One can, like most of the applications ([10] and [13]), separate the real and imaginary parts of *Q(k)* by considering only four denominator roots. This leads to *M=6* . Then it becomes that;

$$Q_R(k) = A_0 - A_2 k^2 + \frac{k^2 A_3}{k^2+\beta_1^2} + \frac{k^2 A_4}{k^2+\beta_2^2} + \frac{k^2 A_5}{k^2+\beta_3^2} + \frac{k^2 A_6}{k^2+\beta_4^2} \quad \text{(Real part)}$$

$$Q_I(k) = A_1 k + \frac{\beta_1 k A_3}{k^2+\beta_1^2} + \frac{\beta_2 k A_4}{k^2+\beta_2^2} + \frac{\beta_3 k A_5}{k^2+\beta_3^2} + \frac{\beta_4 k A_6}{k^2+\beta_4^2} \quad \text{(Imaginary part)} \tag{7}$$

Q is calculated at discrete values of the reduced frequency *k*. At each value of the reduced frequency real and imaginary error functions are determined from Equation 7 as:

$$E_{R,i} = Q_{R,i} + \lfloor B_{R,i} \rfloor \{C\}$$

$$E_{I,i} = Q_{I,i} + \lfloor B_{I,i} \rfloor \{C\} \tag{8}$$

where

$$\lfloor B_{R,i} \rfloor = \left[-1 \quad 0 \quad k_i^2 \quad \frac{-k_i^2}{k_i^2+\beta_1^2} \quad \frac{-k_i^2}{k_i^2+\beta_2^2} \quad \frac{-k_i^2}{k_i^2+\beta_3^2} \quad \frac{-k_i^2}{k_i^2+\beta_4^2} \right]$$

$$\lfloor B_{I,i} \rfloor = \left[0 \quad -k_i \quad 0 \quad \frac{-\beta_1 k_i}{k_i^2+\beta_1^2} \quad \frac{-\beta_2 k_i}{k_i^2+\beta_2^2} \quad \frac{-\beta_3 k_i}{k_i^2+\beta_3^2} \quad \frac{-\beta_4 k_i}{k_i^2+\beta_4^2} \right]$$

$$\{C\} = \begin{Bmatrix} A_0 \\ A_1 \\ . \\ . \\ . \\ A_6 \end{Bmatrix}$$

i refers to a particular reduced frequency k_i at which *Q* is calculated. Defining a complex error function [13]:

$$E_i = E_{R,i} + jE_{I,i} \tag{9}$$

A least square fit can be passed through the N data points by setting

$$\frac{\partial}{\partial C} \sum_{i=1}^{N} (E_i \times E_i^{'}) = 0 \tag{10}$$

where $E_i^{'}$ is the complex conjugate of E_i. By solving Equation 10 for the coefficients of the fit, $\{C\} = [A_0 \quad A_1 \quad . \quad . \quad . \quad A_6]^T$, one obtains the following:

$$\{C\} = -\left[\sum_{i=1}^{N}([B_{R,i}]^T[B_{R,i}] + [B_{I,i}]^T[B_{I,i}])\right]^{-1}\sum_{i=1}^{N}(Q_{R,i}[B_{R,i}]^T + Q_{I,i}[B_{I,i}]^T) \tag{11}$$

Substituting $k = \omega b / 2V$ into Equation 6, one obtains the approximated aerodynamic forces in the Laplace domain as:

$$Q(s) = A_0 + A_1(\frac{b}{2V})s + A_2(\frac{b}{2V})^2 s^2 + \sum_{m=3}^{M}\frac{A_m s}{(s + \frac{2V}{b}\beta_{m-2})} \tag{12}$$

Coefficients $A_0, A_1, ..., A_6$ are determined from Equation 11. The values of β are selected to be in the reduced frequency range of interest.

State-space equations of motion

Determination of an aeroelastic model in state-space form allows the use of control algorithms, and facilitates the root-locus analysis and optimization methods. However in the present study, a state space approach is only used to perform an open-loop flutter analysis.

Defining a state vector and an augmented state vector as [11]:

$$\{x\} = \begin{Bmatrix}\zeta \\ \dot{\zeta}\end{Bmatrix}, \quad \{\zeta_{ai}(s)\} = \frac{s}{s + \frac{2V}{b}\beta_{i-2}}\{\zeta(s)\}$$

and substituting approximated aerodynamic forces, Equation 12, into aeroelastic equation of motion, Equation 5, a state-space matrix equation of motion is formed:

$$\{\dot{x}\} = [A]\{x\}$$

$$\begin{Bmatrix}\dot{\zeta} \\ \ddot{\zeta} \\ \dot{\zeta}_{a3} \\ . \\ . \\ \dot{\zeta}_{a6}\end{Bmatrix} = \begin{bmatrix} 0 & [I] & 0 & 0 & 0 & 0 \\ -[M]^{-1}[K] & -[M]^{-1}[B] & q[M]^{-1}[A_3] & q[M]^{-1}[A_4] & q[M]^{-1}[A_5] & q[M]^{-1}[A_6] \\ 0 & [I] & -\left(\frac{2V}{b}\right)\beta_1[I] & 0 & 0 & 0 \\ 0 & [I] & 0 & -\left(\frac{2V}{b}\right)\beta_2[I] & 0 & 0 \\ 0 & [I] & 0 & 0 & -\left(\frac{2V}{b}\right)\beta_3[I] & 0 \\ 0 & [I] & 0 & 0 & & -\left(\frac{2V}{b}\right)\beta_4[I] \end{bmatrix}\begin{Bmatrix}\zeta \\ \dot{\zeta} \\ \zeta_{a3} \\ . \\ . \\ \zeta_{a6}\end{Bmatrix} \tag{13}$$

where

$$[M] = [M_S] - \frac{1}{2}\rho(\frac{b}{2})^2[A_2]$$

$$[B] = -\frac{1}{2}\rho(\frac{b}{2})V[A_1]$$

$$[K]=[K_S]-\frac{1}{2}\rho V^2[A_0]$$

The state-space model is of the order of *6xn,* where *n* is the number of modes.

NUMERICAL RESULTS

Smart Structure Model

The physical model considered in this study is a smart aluminum plate-like structure called as a smart fin which is constructed by symmetrically attaching twenty-four PZT patches (25mm x 25mm x 0.5mm, Sensortech BM500 type) as actuators on a passive aluminum plate-like structure. In the analysis, the smart fin is considered as being in clamped-free configuration. Geometrical model of the smart fin is shown in Figure 1. Material properties of the structural components including the aluminum fin and piezoelectric patches are given in Table 1.

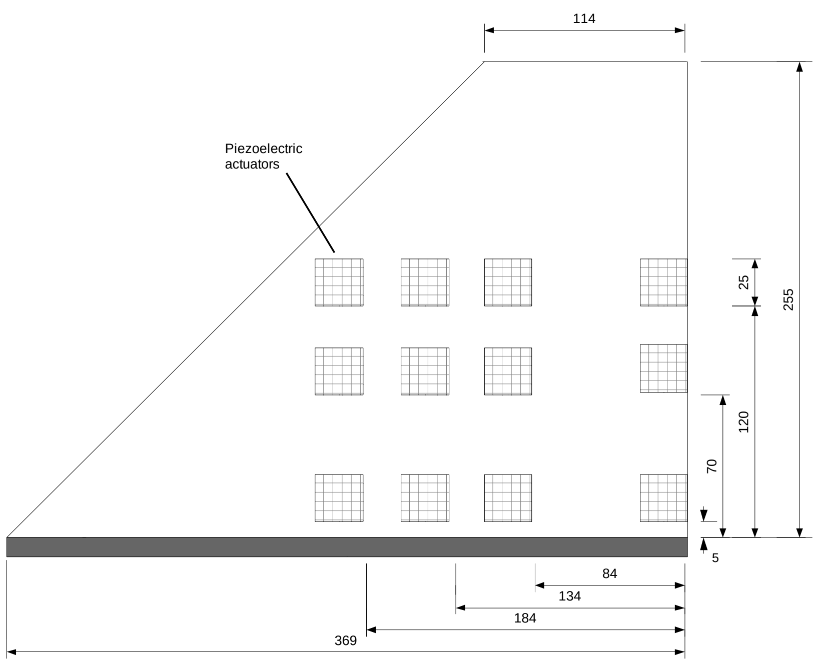

Figure 1. Smart fin (all dimensions in mm)

Table 1. Properties of the smart fin

Property	Aluminum 6061-T6	PZT BM500
Density [kg/m^3]	2710	7650
Thickness [mm]	0.737	0.5
Young's Modulus [GPa]	69.0	64.5
Piezoelectric charge constant [m/V]	-	-1.75×10^{-10}

Open-loop Flutter Estimation

In order to verify the state space approach, an open-loop flutter analysis of the smart fin was performed, and then the results were compared to those obtained by MSC®/NASTRAN/Aeroelasticity I.

MSC®/PATRAN was used to obtain the Finite Element Model of the structure. Figure 2 shows the finite element model of the smart fin. QUAD4 shell elements were utilized for the modeling of the smart fin. PCOMP cards in MSC®/NASTRAN were used to specify the properties of the composite lay-up for the actuator and aluminum substrate.

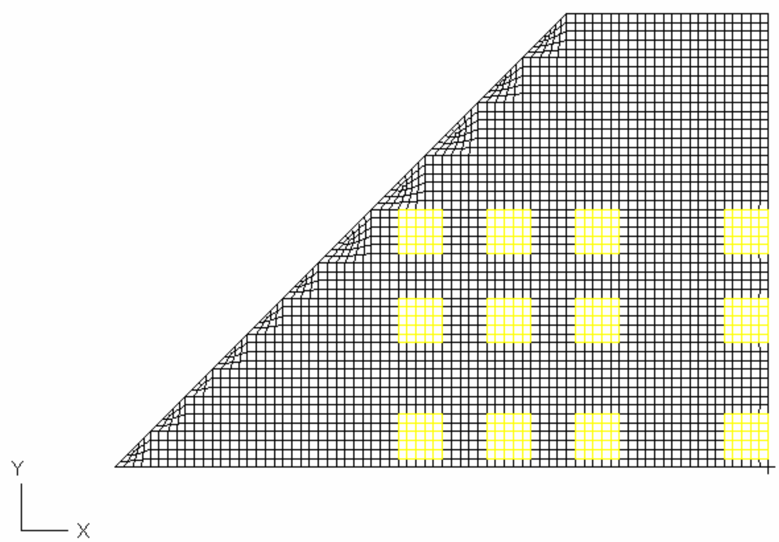

Figure 2. Finite element model of the smart fin

The generalized unsteady aerodynamic loads in modal domain were calculated for a chosen range of reduced frequencies (*k = 0.01, 0.03, 0.05, 0.1, 0.16, 0.2, 0.3, 0.35, 0.4, 0.6*) by using a linear two dimensional aerodynamic theory, Doublet-Lattice Method in MSC®/NASTRAN, and the loads were extracted using Direct Matrix Abstraction Programme (DMAP) from MSC®/NASTRAN. The flight conditions are an air density of $\rho = 1.186$ kg/m^3 and low subsonic velocity regime (Mach number $M=0.2$). In the analysis, the first four elastic modes were considered. These modes and natural frequencies are given in Table 2. The discrete air loads from MSC®/NASTRAN were approximated by Roger's approximation and least-squares method using MATLAB® code [6]. Figure 3 and 4 show approximated air loads for Q_{11} and Q_{22} with the tabulated aerodynamic MSC®/NASTRAN data. The chosen values of the aerodynamic lag term coefficients used in Roger's approximation are $\beta_1 = 0.03$, $\beta_2 = 0.1$, $\beta_3 = 0.2$, $\beta_4 = 0.25$ which are in the reduced frequency range of interest and provide a good curve fitting.

Table 2. Natural frequencies of the smart fin in zero-flow condition

Mode number	**Frequency (Hz)**
1 (First bending)	13.7
2 (First torsion)	39.7
3 (Second bending)	63.0
4 (Second torsion)	128.1

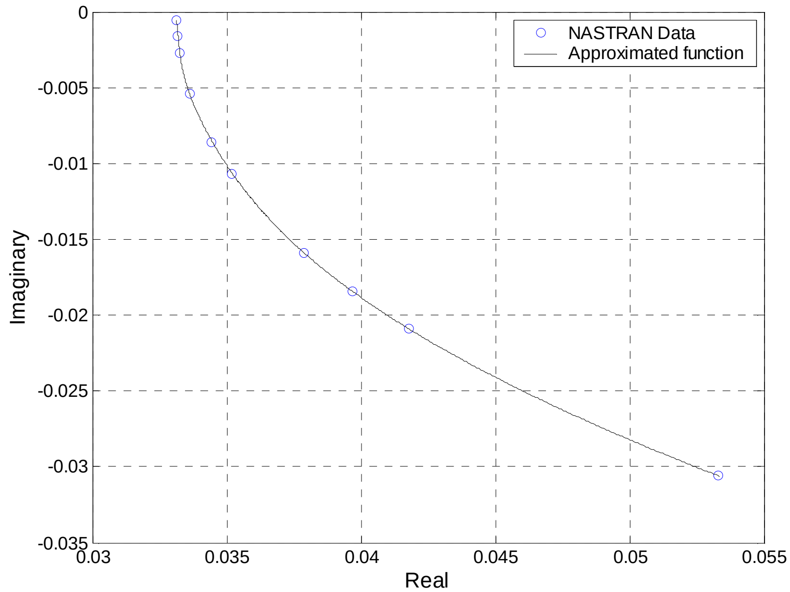

Figure 3. Approximation of *Q(1,1)* by Roger's approximation

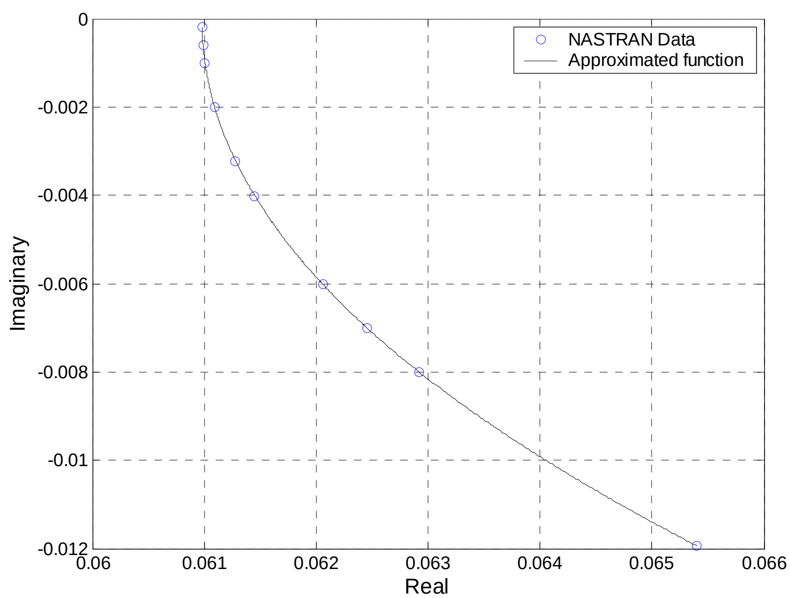

Figure 4. Approximation of *Q(2,2)* by Roger's approximation

The approximated air loads estimated in modal domain are used along with structural matrices to build a state space model from Equation 13 to conduct an open loop flutter analysis. By solving Equation 13 for several flight velocities (in the range of *20* and *120 m/sec*), the roots of the system are obtained. Figure 5 shows the velocity root-locus plot of the smart fin model. The plot traces the roots of the system as the airspeed changes. The horizontal axis is the real part while the vertical axis is the imaginary part of the roots. The imaginary axis represents the point of neutral stability. Flutter is represented on the root locus plot by a pole crossing this axis into the right half plane.

The flutter analysis results of state space approach have been compared with MSC®/NASTRAN/Aeroelasticity analysis in Table 3. The open-loop flutter estimation by state space approach appears to be reasonable.

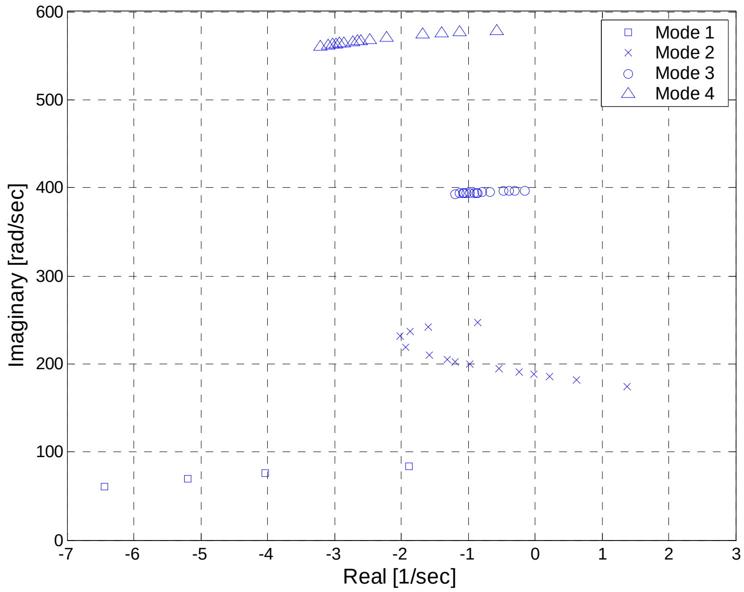

Figure 5. Root-locus of the state-space model of the smart fin as a function of the flight velocity

Table 3. Comparison of open-loop flutter characteristics

	Flutter Velocity [m/sec]	Flutter Frequency [Hz]
MSC®/NASTRAN/Aeroelasticity I	109.74	29.94
State Space Approach	110.09	29.91
% Deviation from NASTRAN	0.32	0.10

CONCLUSION

In this paper a method for the state-space representation of the unsteady aerodynamic loads on a smart structure was presented. The aerodynamic loads acting on the structure at a set of reduced frequencies were obtained from a finite element program, MSC®/NASTRAN. The loads at these discrete points were approximated in the complex frequency domain as rational functions, then a state-space representation of the aeroelastic model was constructed for the smart structure. The state-space model was used to predict the flutter characteristics of the structure. These results were compared with the results obtained from the MSC/NASTRAN/Aeroelasticity solutions and very good agreement was observed. In future studies this state-space model will be used in representing the uncertainties and designing robust controllers for the active flutter suppression of the smart structure.

References

[1] Han, J.H., Tani, J. and Qiu, J., *Active Flutter Suppression of a Lifting Surface Using Piezoelectric Actuation and Modern Control Theory,* Journal of Sound and Vibration 291, p: 706-722, 2006.

[2] Chen, Y., Wickramasinghe, V.and Zimcik, D., *Active Control of a Hybrid Actuation System for Aircraft Vertical Fin Buffet Load Alleviation,* Aeronautical Journal of the Royal Aeronautical Society, p: 315-326, 2006.

[3] Raja, S., Sreedeep, R., Pashilkar, A.A. and Kamesh, J.V., *Active Flutter Suppression of a Composite Plate with PZT Multilayered Benders using LQG Control,* International Conference on Smart Materials Structures and Systems, p: 34-42, July 2005.

[4] Sheta, E.F., Moses, R.W. and Huttsell, L.J., *Active Smart Material Control System for Buffet Alleviation,* Journal of Sound and Vibration, p: 854-868, November 2005.

[5] Waisman, H., Abramovich, H., *Open-loop Flutter Analysis of a Composite UAV model using the Active Stiffening Effect,* Finite Elements in Analysis and Design 40, p: 1283-1295, 2004.

[6] MATLAB, *User's Manual Version 6.5*, The MathWorks, Inc., 2003.

[7] MSC/NASTRAN, *Quick Reference Guide*, Version 70.5, The MacNeal-Schwendler Corporation, 1998.

[8] Heeg, J., *Analytical and Experimental Investigation of Flutter Suppression by Piezoelectric Actuation,* NASA TP 3241, February 1993.

[9] Meirovitch L., *Elements of Vibration Analysis*, the McGraw-Hill Company, 1986.

[10] Karpel, M., *Design for Active Flutter Suppression and Gust Alleviation Using State-Space Aeroelastic Modeling,* Journal of Aircraft, Vol. 19, p: 221-227, March 1982.

[11] Karpel, M., *Design for Active and Passive Flutter Suppression and Gust Alleviation,* NASA Contractor Report 3482, November 1981.

[12] Edwards, J.H., *Applications of Laplace Transform Methods to Airfoil Motion and Stability Calculations,* AIAA Paper, 1979.

[13] Abel, I., *An Analytical Technique for Predicting the Characteristics of a Flexible Wing Equipped with an Active Flutter Suppression System and Comparison with Wind Tunnel Data,* NASA TP 1367, 1979.

[14] Vepa, R., *Finite State Modeling of Aeroelastic Systems,* NASA Contractor Report 2779, 1977.

[15] Roger, K.L., *Airplane Math Modeling and Active Aeroelastic Control Design*, AGARD-CP-228, p: 4.1–4.11, 1977

[16] Albano, E. and Rodden, W. P., *A Doublet-Lattice Method for Calculating Lifting Disturbances on Oscillating Surfaces in Subsonic Flows,* AIAA Journal, Vol. 7, p: 279-285, February 1969.