Stability of Three-Dimensional Boundary-Layers

AE 549 Linear Stability Theory and Laminar-Turbulent Transition

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Three-dimensional stability

- A fundamental difference between the stability of 3-D and 2-D boundary-layers is that a 3-D boundary-layers is subject to crossflow instability.
- To understand the effect of three-dimensionality of the mean flow on stability, it is necessary to have a family of boundary-layers, where the magnitude of the crossflow can be varied in a systematic manner.
- The two-parameter yawed-edge (swept leading edge) flows are suitable for this purpose resulting in Falkner-Skan-Cooke family of profiles.
Flow geometry for three-dimensional b.l. flow
3-D flow with pressure gradient

- The two parameters are:
  - Falkner-Skan or Hartree parameter $\beta_H$,
  - Flow angle $\theta$, which is the ratio of the spanwise freestream velocity to the chordwise freestream velocity, $\tan \theta = W_{sl}^*/U_{cl}^*$.

- The inviscid velocity in the plane of the wedge and normal to the leading edge (in $x_c$-direction, i.e. chordwise velocity) is defined as:
  $$U_{ce}^* = C^* x_c^* m,$$

  where the wedge angle is $\beta\pi/2$ and $\beta = 2m/m + 1$, with $m$ being the dimensionless pressure gradient defined as:
  $$m = \frac{x_c^*}{U_{ce}^*} \frac{dU_{ce}^*}{dx_c^*}$$

- The velocity parallel to the leading edge (in $z_s$ direction, i.e. spanwise velocity) is
  $$W_{se}^* = \text{constant}. $$
3-D flow with pressure gradient

- Flow in the chordwise direction, \( x_c \) is defined by the Falkner-Skan equation, which is independent of the spanwise flow, and \( f' = U_c \):
  \[
  2f'''' + ff' + \beta_H(1 - f'2) = 0.
  \]
- Falkner-Skan length scale is defined as:
  \[
  L^* = \left[ \frac{v^* x_c^*}{(m + 1)U_{ce}^*} \right]^{1/2}.
  \]
- Once \( f \) is solved, flow in the spanwise direction, \( z_s \) is defined from the following equation:
  \[
  g'' + fg' = 0,
  \]
  where \( g = W_s^*/W_{se}^* \).
- Boundary conditions:
  \[
  f'(0) = g(0) = 0, \quad \text{(no slip)}
  \]
  \[
  f'(y) \to 1, g(y) \to 1 \quad \text{as} \quad y \to \infty. \quad \text{(freestream)}
  \]
3-D flow with pressure gradient

- Dimensionless streamwise (x-direction) and crossflow (z-direction) velocity components:

\[ U(y) = f'(y) \cos^2 \theta + g(y) \sin^2 \theta, \]
\[ W(y) = [-f'(y) + g(y)] \cos \theta \sin \theta. \]

- Falkner-Skan parameter \( \beta \) fixes both \( f'(y) \) and \( g(y) \).

- It can be seen from the above equation that all crossflow profiles \( W(y) \) have the same shape for a given pressure gradient, i.e. \( \beta \) value.

- Magnitude of the crossflow velocity will change with flow direction \( \theta \).

- However, streamwise profiles \( U(y) \) will change shape as \( \theta \) varies.
Velocity profiles for $\beta_H = -0.05$ and $\theta = 45^\circ$
Velocity profiles for $\beta_H = 0$ and $\theta = 45^\circ$
Velocity profiles for $\beta_H = 0.1$ and $\theta = 45^\circ$
Composite profile for $\beta_H = -0.1$ and $\theta = 45^\circ$
Neutral stability curves for $\beta_H = -0.1$
Neutral stability curves for $\beta_H = 0.1$
Effect of flow angle on $Re_{cr}$ for $\beta_H = -0.1, 0, 0.1$
Conclusions

• For $\beta < 0$, increasing flow angle renders the flow more stable.
• For $\beta > 0$, increasing flow angle renders the flow more unstable.
• Flow angle has no effect on stability when $\beta = 0$.
• When the flow angle is around $0^\circ$, the critical Reynolds number of the three-dimensional flow is very close to that of the two-dimensional flow.
  
  When $\theta = 0^\circ$, $U(y) = f'$ and $W(y) = 0$.
• When the flow angle is $\theta = 90^\circ$, the critical Reynolds number of the three-dimensional flow is very close to that of the Blasius flow ($\beta = 0^\circ$).
  
  When $\theta = 90^\circ$, $U(y) = g$, which is very close to the Blasius profile and $W(y) = 0$. 