

Aeronautical Engineering Design II

Sizing Matrix and Carpet Plots

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Empty weight estimation and refined sizing

- Empty weight of the airplane was calculated using statistical group weights method by a detailed estimation of the weight of each component of the aircraft.
- Empty weight thus calculated will usually not be equal to the empty weight previously calculated using approximate group weights method.
- It is possible to go back to the detailed weight equations and recalculate the empty weight using statistical group weights method \Rightarrow too prohibitive in terms of time required.



Empty weight estimation and refined sizing

- An approximate non-computerized method relies on statistical data:

$$\frac{W_e}{W_o} \searrow \text{ as } W_o \nearrow$$

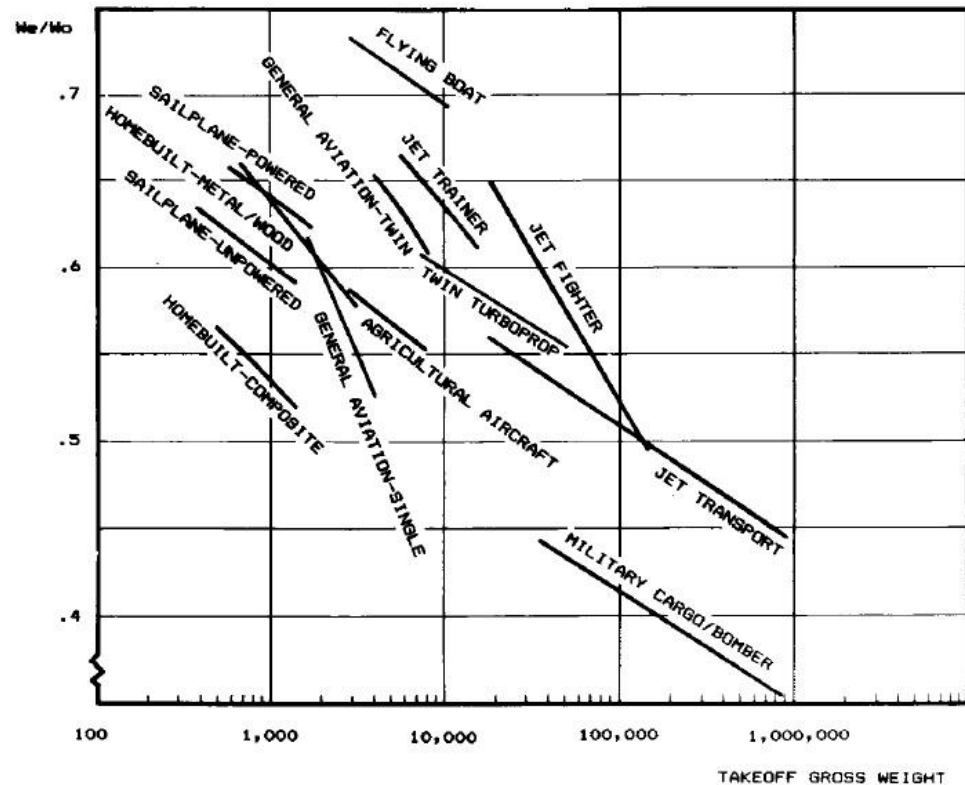


Fig. 3.1 Empty weight fraction trends.

Empty weight estimation and refined sizing

- Empty weight:

$$W_e = W_{e,as\ drawn} \left[\frac{W_o}{W_{o,as\ drawn}} \right]^{1+c}, \quad c \approx -0.1$$

- For c :
 - Either use a statistical value ($c \approx -0.1$) or:
 - Calculate for a specific airplane. Make an arbitrary change (e.g. 10%) in W_o and recalculate W_e with all effects considered including changes in wing and tail areas, increased fuselage size, heavier landing gear and larger engine. With the new value of W_o and W_e , solve for c in the above equation.

Empty weight estimation and refined sizing

Methodology:

- With an initial estimate of W_o (as drawn) and W_e (as drawn), calculate the fuel weight W_f by using the refined sizing analysis.
- Using $W_o = W_{e,as\ drawn} + W_f + W_c + W_p$ obtain a new value of W_o .
- Using $W_e = W_{e,as\ drawn} \left[\frac{W_o}{W_{o,as\ drawn}} \right]^{1+c}$ obtain a new value of W_e .
- Iterate until convergence.

Sizing matrix method

- The sizing procedure described ensures that the as-drawn aircraft will meet the required mission range or other important performance criteria.
- The configuration geometry was initially selected to meet the requirements based on assumptions for lift, drag, thrust, etc.
- The aircraft sized using the method described will have different characteristics and may no longer meet all requirements or may exceed all of them \Rightarrow the aircraft is overdesigned and is not the lightest possible design.

Sizing matrix method

- Thrust-to-weight ratio (T/W) and wing loading (W/S) are varied from the baseline values (typically $\pm 20\%$).
- Each combination of T/W and W/S produce a different airplane with different aerodynamics, propulsion and weights. Size each of the airplanes separately to determine the takeoff weight of each to perform the design mission.
- Analyze each combination separately for performance.
- If the T/W and W/S variations are wide enough, at least one aircraft will meet the requirements.



Sizing matrix method

- What combination of T/W and W/S will meet all the requirements at a minimum weight.

	$W/S = 50$	$W/S = 60$	$W/S = 70$
$T/W = 1.1$	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">1</div> <p> $W_0 = 56,000$ lb $P_s = 700$ fps (M0.9, 30k ft, 5g's) $S_{TO} = 340$ ft $a = 46$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">2</div> <p> $W_0 = 49,000$ lb $P_s = 330$ fps $S_{TO} = 430$ ft $a = 42$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">3</div> <p> $W_0 = 46,000$ lb $P_s = 30$ fps $S_{TO} = 660$ ft $a = 39$ s </p> </div>
$T/W = 1.0$	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">4</div> <p> $W_0 = 48,500$ lb $P_s = 430$ fps $S_{TO} = 450$ ft $a = 50.5$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">5</div> <p> RESIZED BASELINE $W_0 = 43,700$ lb $P_s = 30$ fps $S_{TO} = 595$ ft $a = 47$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">6</div> <p> $W_0 = 42,000$ lb $P_s = -190$ fps $S_{TO} = 800$ ft $a = 45$ s </p> </div>
$T/W = 0.9$	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">7</div> <p> $W_0 = 44,000$ lb $P_s = 140$ fps $S_{TO} = 670$ ft $a = 56$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">8</div> <p> $W_0 = 39,000$ lb $P_s = -230$ fps $S_{TO} = 810$ ft $a = 53$ s </p> </div>	<div style="border: 1px solid black; padding: 5px;"> <div style="text-align: right; font-size: small;">9</div> <p> $W_0 = 36,000$ lb $P_s = -320$ fps $S_{TO} = 1070$ ft $a = 51$ s </p> </div>

Require: $P_s \geq 0$ at (M0.9, 30k ft, 5g's)
 $S_{TO} \leq 500$ ft
 $a \leq 50$ s from M0.9 to M1.5

Fig. 19.1 Sizing matrix.

Sizing matrix plot

- Start optimization of T/W and W/S by cross-plotting the sizing matrix data in terms of chosen performance characteristics and takeoff weights.
- From the takeoff weight graphs, wing loadings corresponding to regularly spaced arbitrary gross weights are determined.

Sizing matrix plot

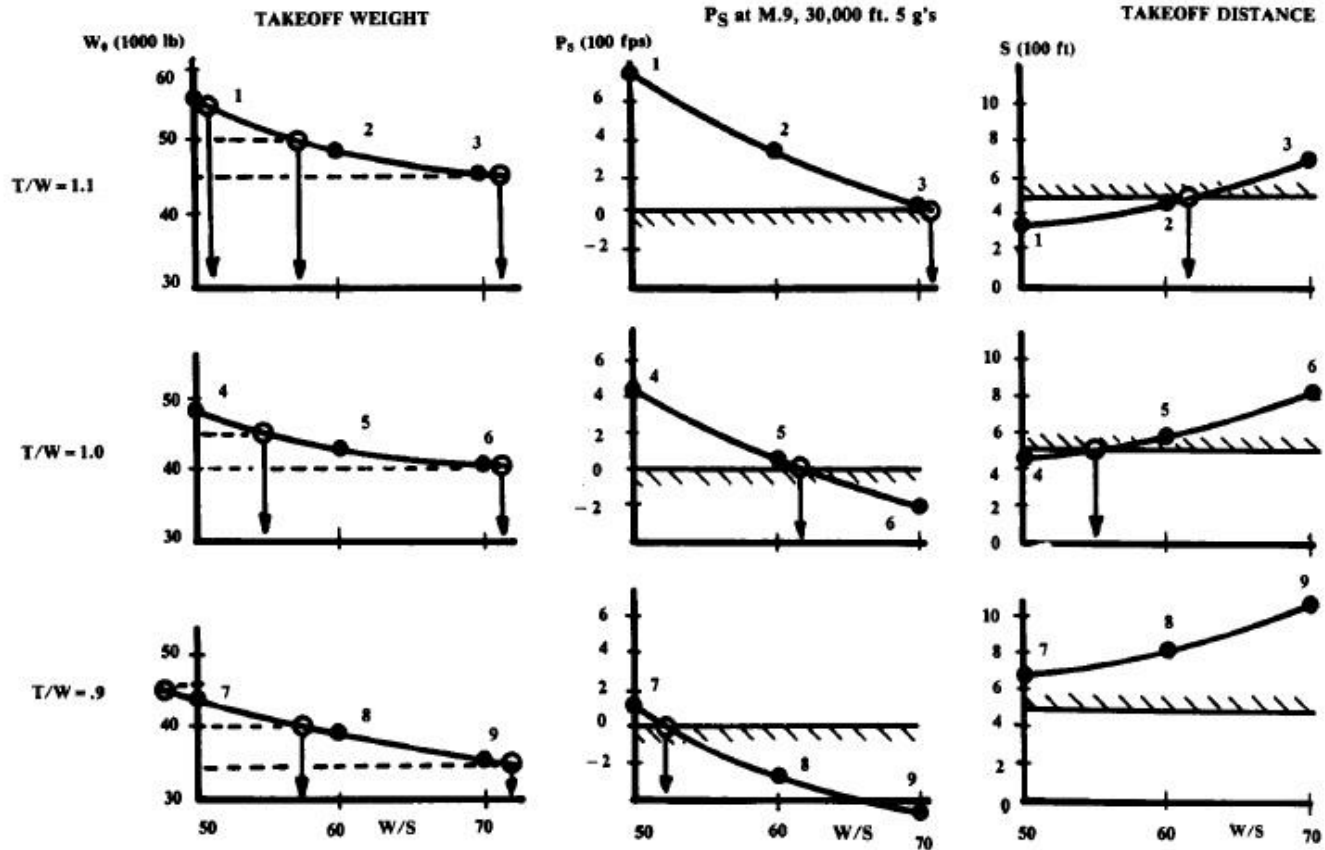


Fig. 19.2 Sizing matrix cross plots.

Sizing matrix plot

- T/W and W/S values for the arbitrary gross weights are transferred to a $W/S - T/W$ graph.
- Smooth curves are drawn connecting various points with identical gross weight to produce lines of constant takeoff gross weight.
- From these curves, it is possible to determine the sized takeoff weight for any combination of T/W and W/S .

Sizing matrix plot

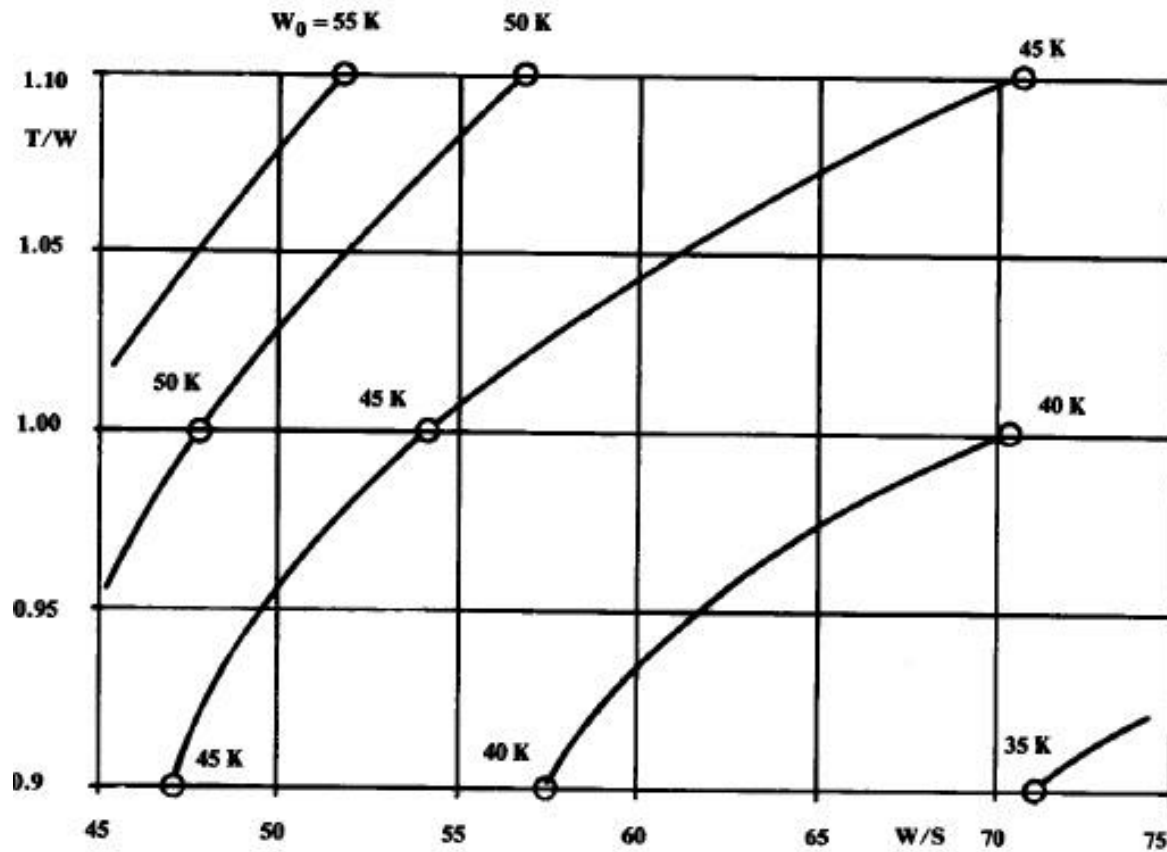


Fig. 19.3 Sizing matrix plot (continued).

Sizing matrix plot

- W/S values that exactly meet various performance requirements are obtained from the performance plots for different T/W values.
- The combinations of T/W and W/S that exactly meet a performance requirement are transferred to the $W/S - T/W$ and connected by smooth curves.
- The desired solution is the lightest aircraft that meets all the requirements.
- It is usually a combination where two constraint lines meet.

Sizing matrix plot

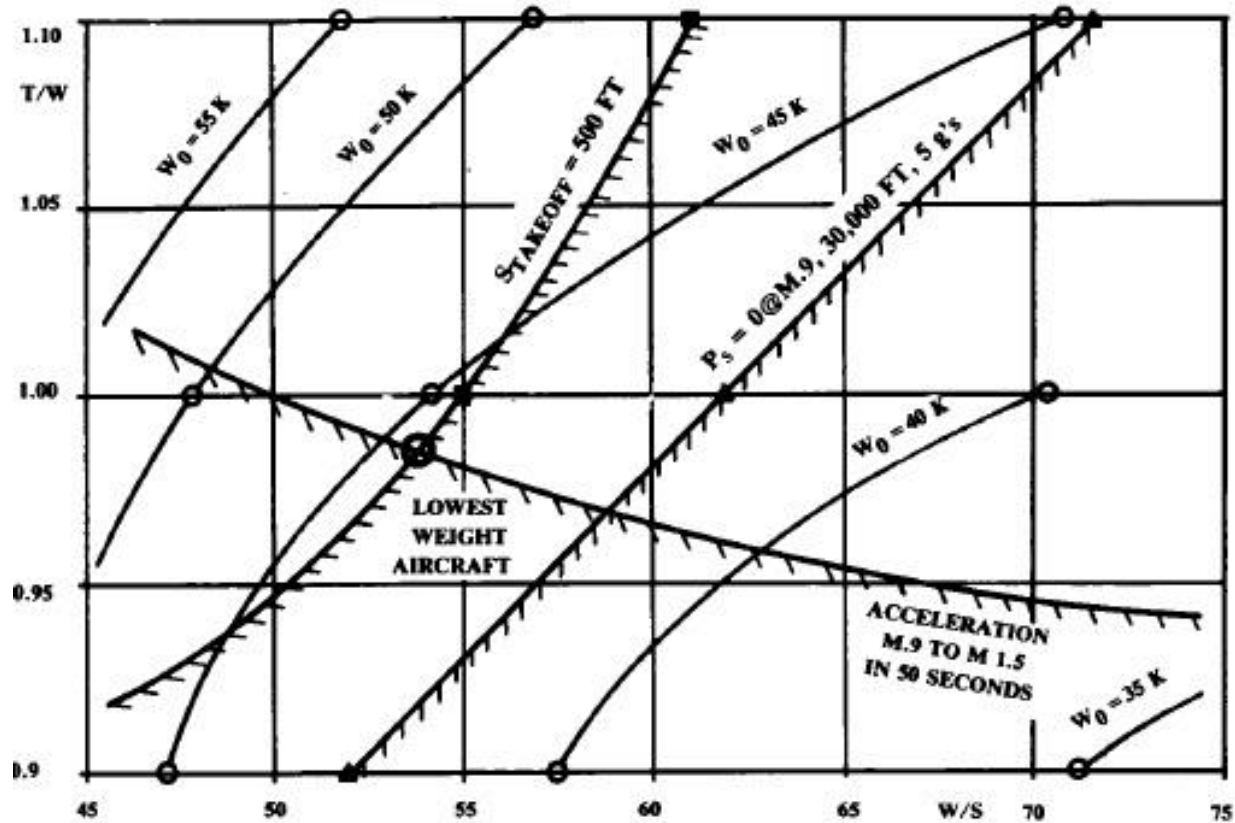


Fig. 19.4 Sizing matrix plot (concluded).

Carpet plots

- Carpet plot is based on superimposing the takeoff weight plots like in

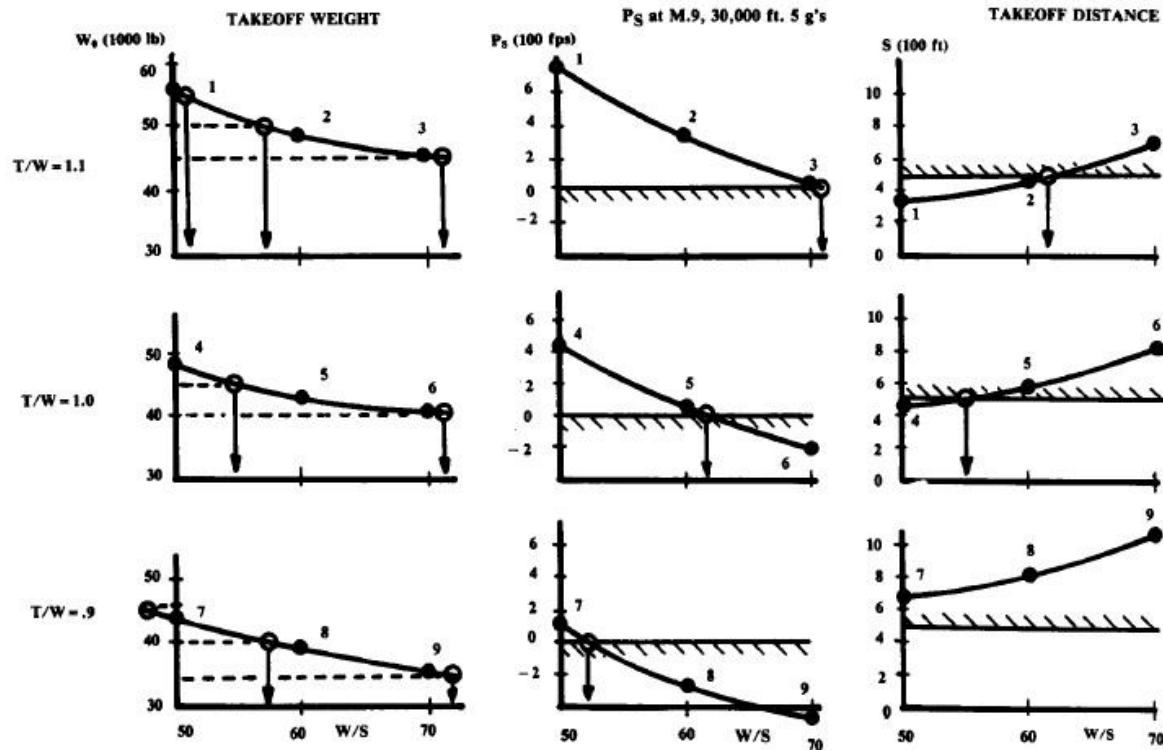


Fig. 19.2 Sizing matrix cross plots.

Carpet plots

- Plot W_o as a function of W/S for say $T/W = 1.1$,
- Plot W_o as a function of W/S for say $T/W = 1.0$. To avoid clutter, shift the horizontal axis to left by some arbitrary distance.
- Plot W_o as a function of W/S for say $T/W = 0.9$, again shifting the horizontal axis.

Carpet plots

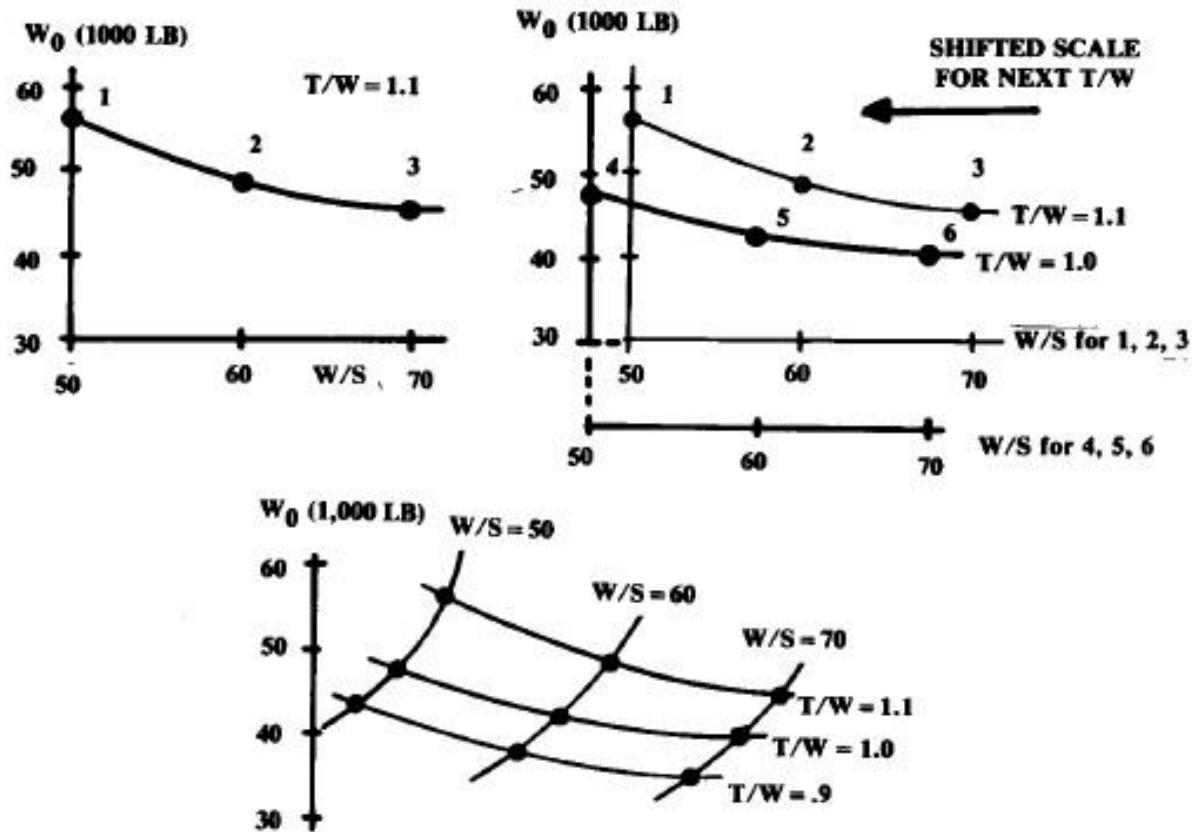


Fig. 19.5 Carpet plot format. (same results!)

Carpet plots

- Mark and connect with smooth constraint lines, the wing loadings that exactly meet requirements, say takeoff distance, P_S and acceleration time.
- The optimal airplane is the one corresponding to the lowest point on the curve that meets all the constraints.
- It is possible to create carpet plots in which the figure of merit is the cost rather than the weight.
- For most airplane types minimization of weight will also minimize the cost.

Carpet plots

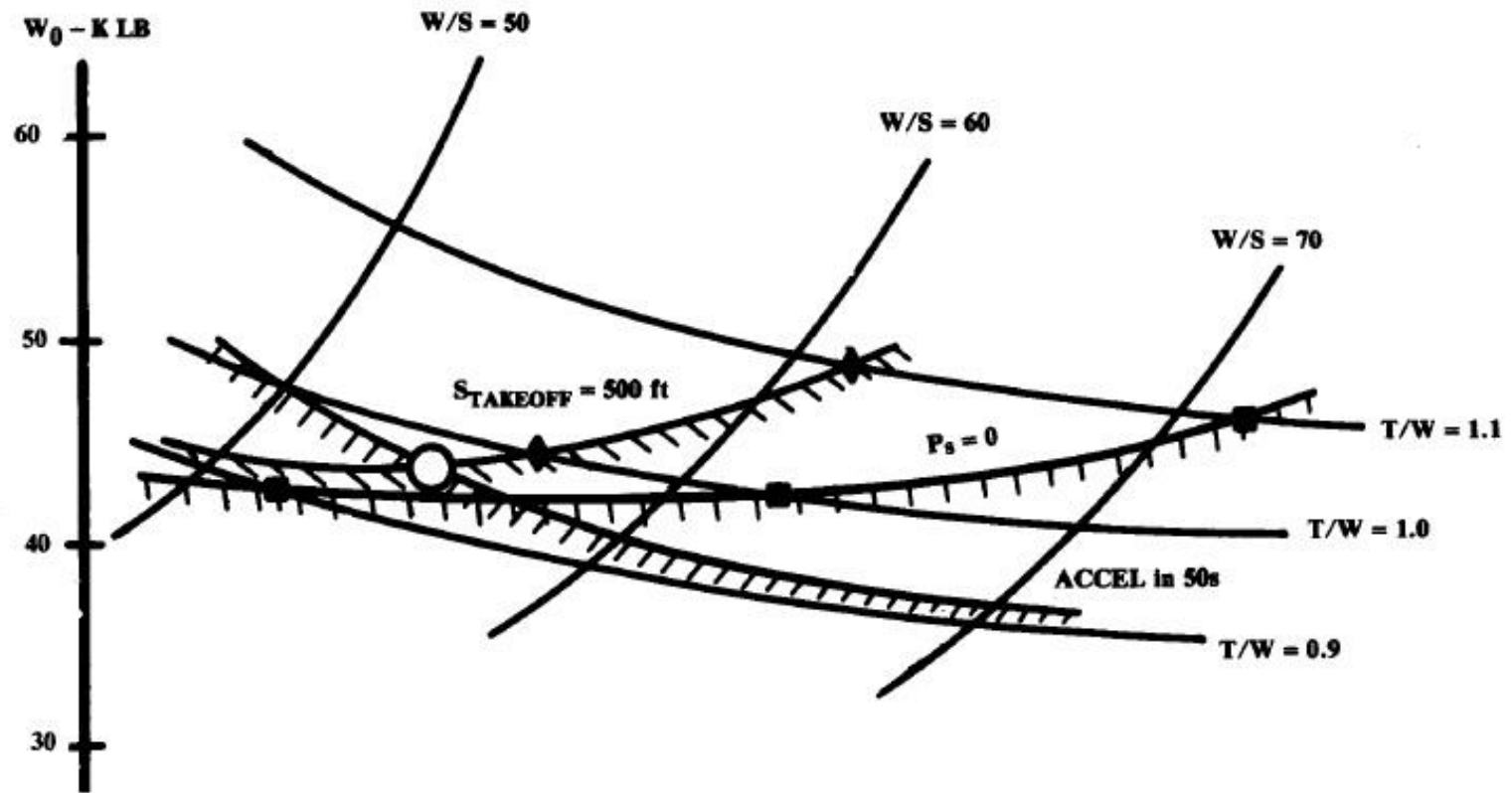


Fig. 19.6 Completed carpet plot.

Trade studies

- Trade studies allow the true optimum airplane emerge.
- The most basic trade study is the T/W and W/S carpet plot.
- Trade studies commonly undertaken in aircraft design:
 - Design trade: reduce the weight and cost of the airplane to meet a given set of mission and performance requirements.
 - Requirements trade: determine the sensitivity of the airplane to changes in the design requirements.
 - Growth sensitivity trade: determine how much the weight of the airplane will change if various parameters such as drag or specific fuel consumption should increase.

Trade studies

- Design trade studies must be calculated using a complete T/W and W/S carpet plot for each data point.
- For example, for an optimization of aspect ratio, sweep, T/W and W/S $3 \times 3 \times 3 \times 3 = 81$ parametric variations of the airplane are needed.

Sizing matrix data approximations

- Variation in T/W : variation in thrust and fuel flow \Rightarrow wetted area and wave drag increase due to change in nacelle size.
- Variation in W/S : variation in wetted area and wave drag \Rightarrow variation in K because the fuselage covers more or less of the wing span.

Sizing matrix data approximations

Approximations:

- $C_{D0} \sim S_{wet}$ due to wing area and nacelle size variations.
- $S_{wet,wing} \sim S$.
- $S_{wet,nacelle} \sim T$.
- $C_{D,wave}$ must be recalculated.
- $S_{wing,cross\ section} \sim S_{wing}$.
- Variation in K may be neglected but if the wing area changes, the airplane will fly at a different C_L , effecting K.

Sizing matrix data approximations

Approximations:

- $W_{wing} \sim S_{wing}^{0.7}$ (same holds for tail surfaces).
- $W_{engine} \sim T^{1.1}$.
- $T_{installed} \sim T_{uninstalled}$.