AE 452 Aeronautical Engineering Design II

Energy Maneuvrability Methods

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Energy equations

- **Potential energy** of altitude can be **exchanged** for **kinetic energy** of speed or turn rate.
- The **<u>sum of aircraft energy</u>** (potential+kinetic) must be managed to attain success in a dogfight.

$$E = Wh + \frac{1}{2}\frac{W}{g}V_\infty^2$$

$$h_e = \frac{E}{W} = h + \frac{V_\infty^2}{2g},$$ specific energy or energy height

$$P_s = \frac{dh_e}{dt} = \frac{dh}{dt} + \frac{V_\infty}{g}\frac{dV_\infty}{dt},$$ specific excess power

Energy equations

$P = (T - D)V_\infty$, excess power

$$P_s = \frac{(T-D)V_\infty}{W} = \frac{dh}{dt} + \frac{V_\infty}{g}\frac{dV_\infty}{dt}$$

$$D = q_\infty S C_D = q_\infty S(C_{Do} + K C_L^2)$$

$$L = nW = q_\infty S C_L = nW \Rightarrow C_L = \frac{n(W/S)}{q_\infty}$$

Substituting yields:

$$P_s = V_\infty\left[\frac{T}{W} - \frac{q_\infty C_{Do}}{W/S} - n^2\frac{K}{q_\infty}\frac{W}{S}\right]$$

P_s plots

- P_s can be calculated for different **Mach numbers** and **load factors** at a given altitude provided that aerodynamic coefficients and installed thrust data are available.
- From P_s charts at various altitudes, several other charts can be plotted by cross-plotting.

P_s plots

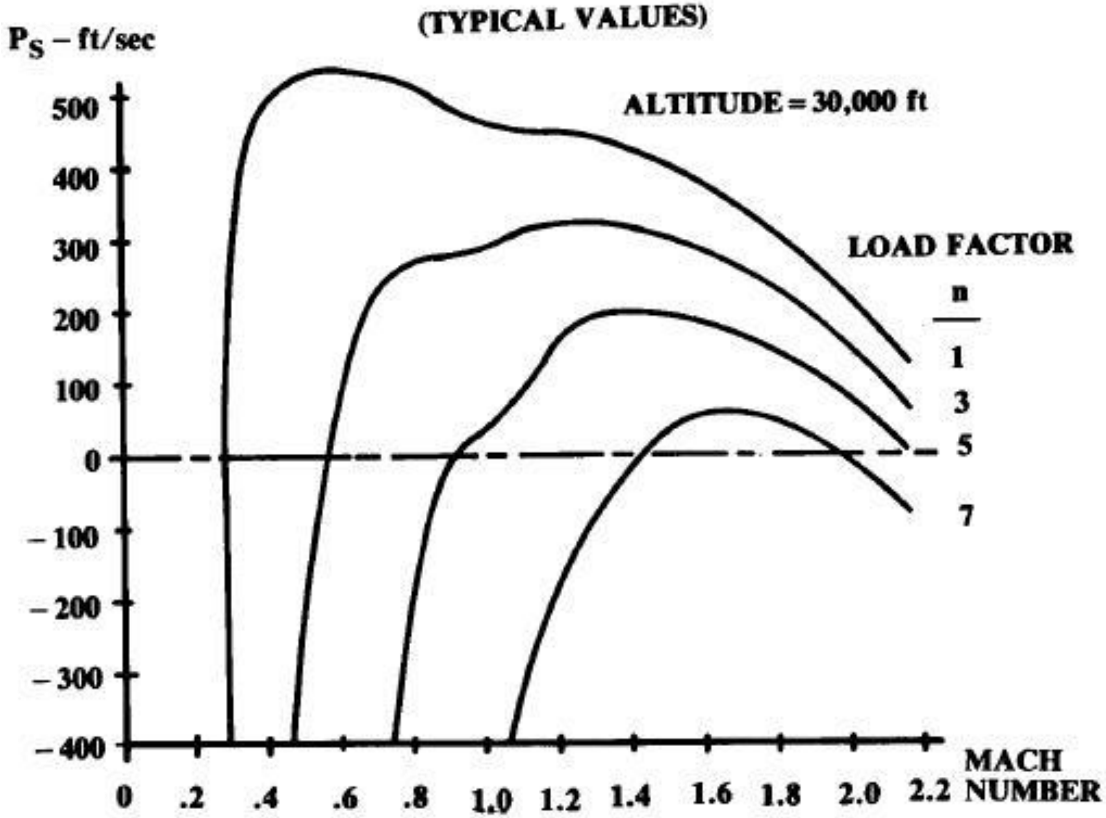

Fig. 17.8 P_S vs Mach number and load factor.

P_s plots

- $\dot{\Psi}$ (turn rate) vs. P_s at a given altitude.
- $\dot{\Psi}_{design} - \dot{\Psi}_{threat} > 2^o/s$ is significant.

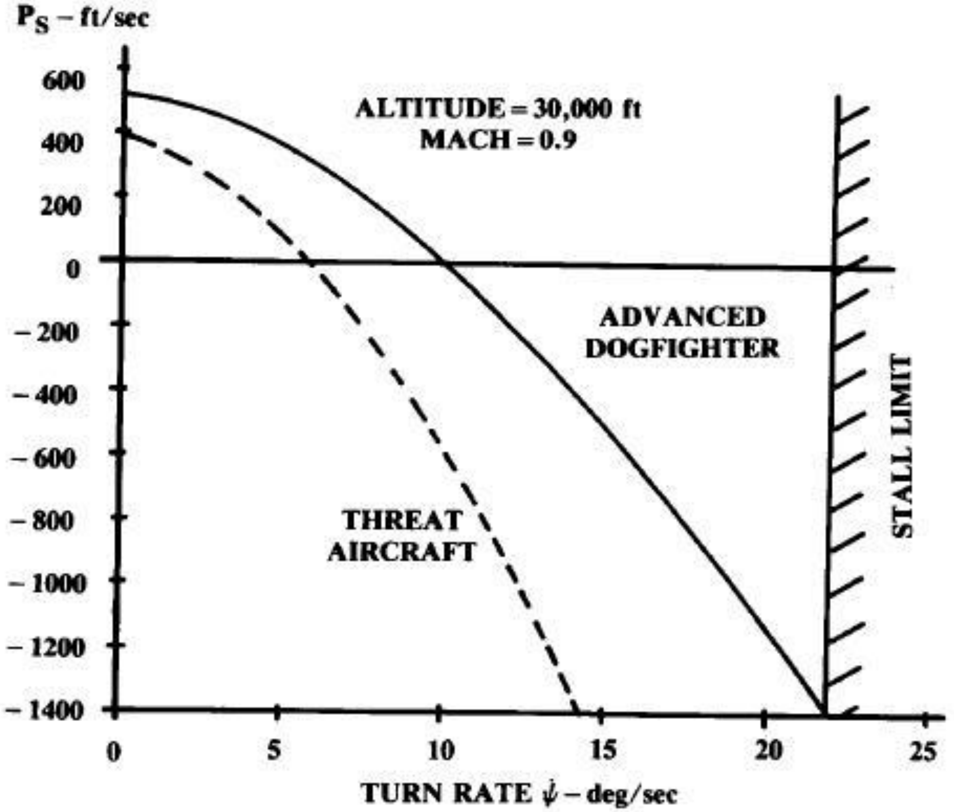

Fig. 17.9 Turn rate vs P_S.

P_s plots

- $P_s = 0$ contours as a function of Mach # vs. altitude, allows comparison between two fighters.
- To win a dogfight, a new design should have $P_s = 0$ contours that envelop those of threat airplanes.

P_s plots

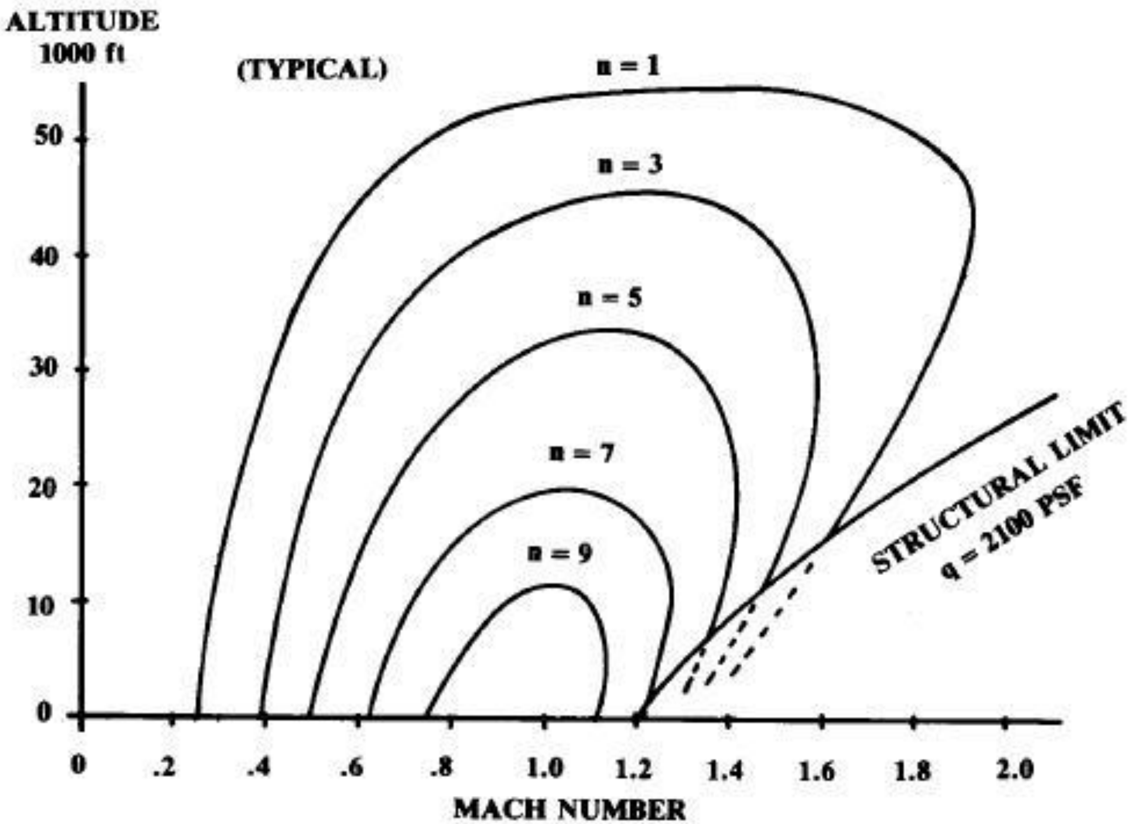

Fig. 17.10 $P_S = 0$ contours.

P_s plots

- $P_s = constant$ contours at a given load factor as a function of Mach # vs. altitude.
- A separate chart is prepared for each load factor n.
- A chart for $\boldsymbol{n = 1}$ is especially important because it yields the **rate of climb** and the **aircraft ceiling**, as well as determining the **optimal climb trajectory**.

P_s plots

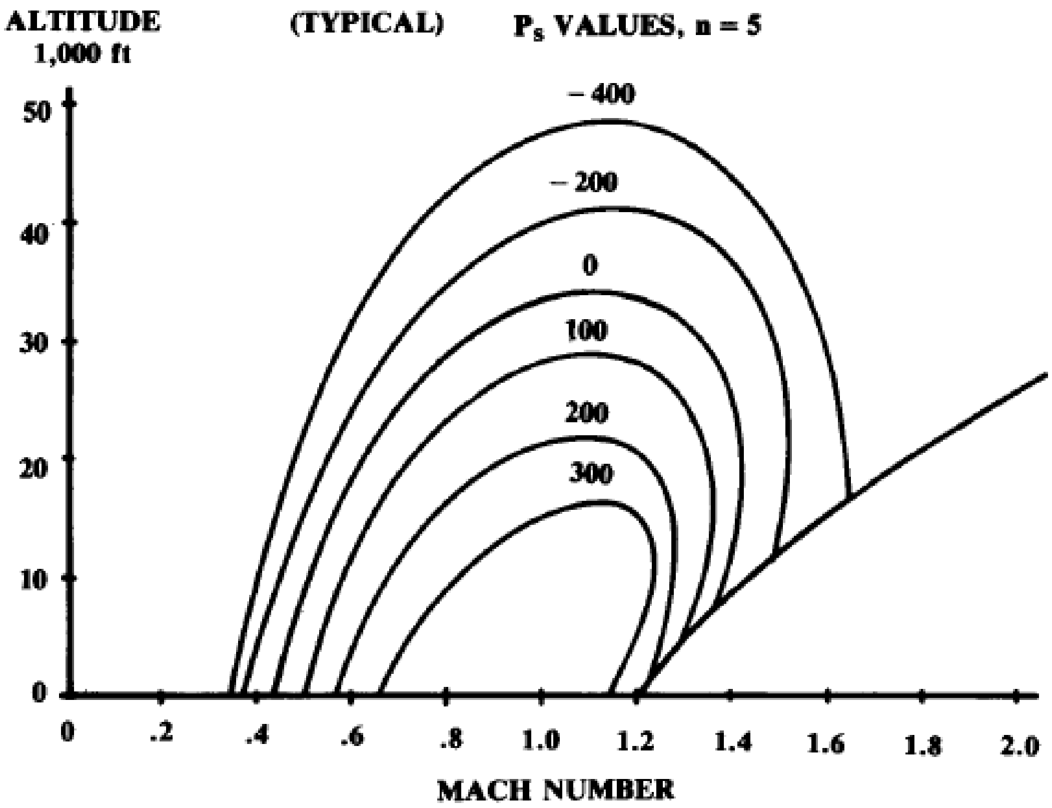

Fig. 17.11 P_S contours, constant load factor.

Contours of energy height, Mach # vs. altitude

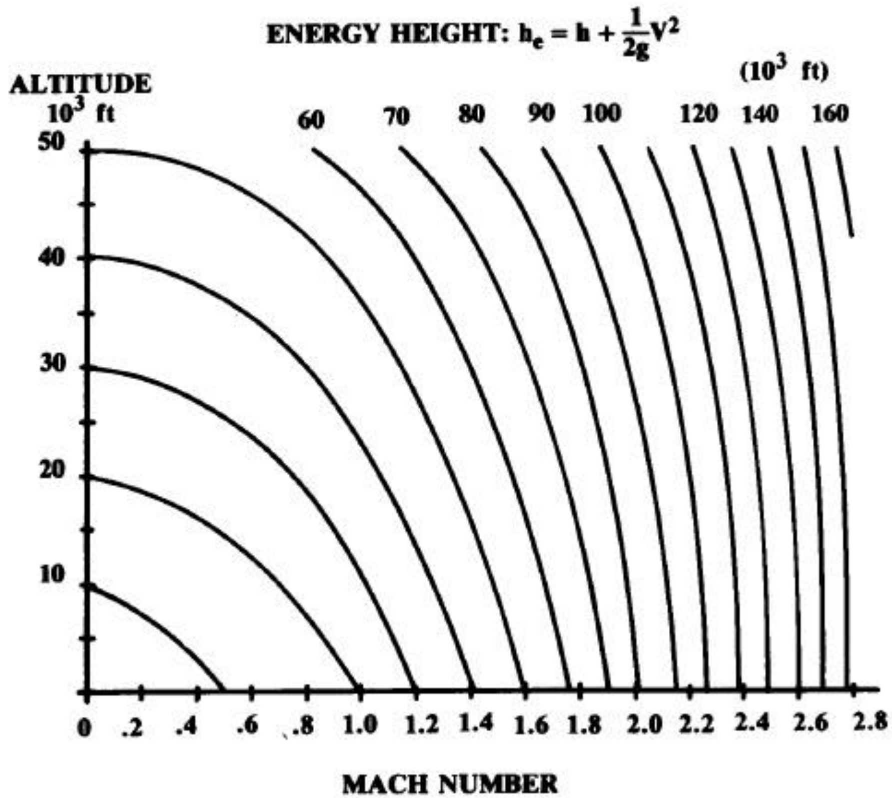

Fig. 17.12 Lines of constant energy height.

Minimum time to climb trajectory

- Time to change energy height:

$$\frac{dh_e}{dt} = P_s \Rightarrow dt = \frac{dh_e}{P_s} \Rightarrow t_{1\rightarrow 2} = \int_{h_{e1}}^{h_{e2}} \frac{dh_e}{P_s}$$

$\Rightarrow$time to change energy height is minimized if P_s is maximized at each energy height.

Minimum time to climb trajectory

Method to calculate minimum time to climb trajectory:

1. Superpose $P_s = constant$ curves in Mach # vs. altitude format at $n = 1$ and contours of energy height in Mach # vs. altitude format.
2. The locus of points where a $\boldsymbol{P_s}$ **curve** is exactly **tangent** to an **energy height curve** is the trajectory for minimum time to climb.

Minimum time to climb trajectory

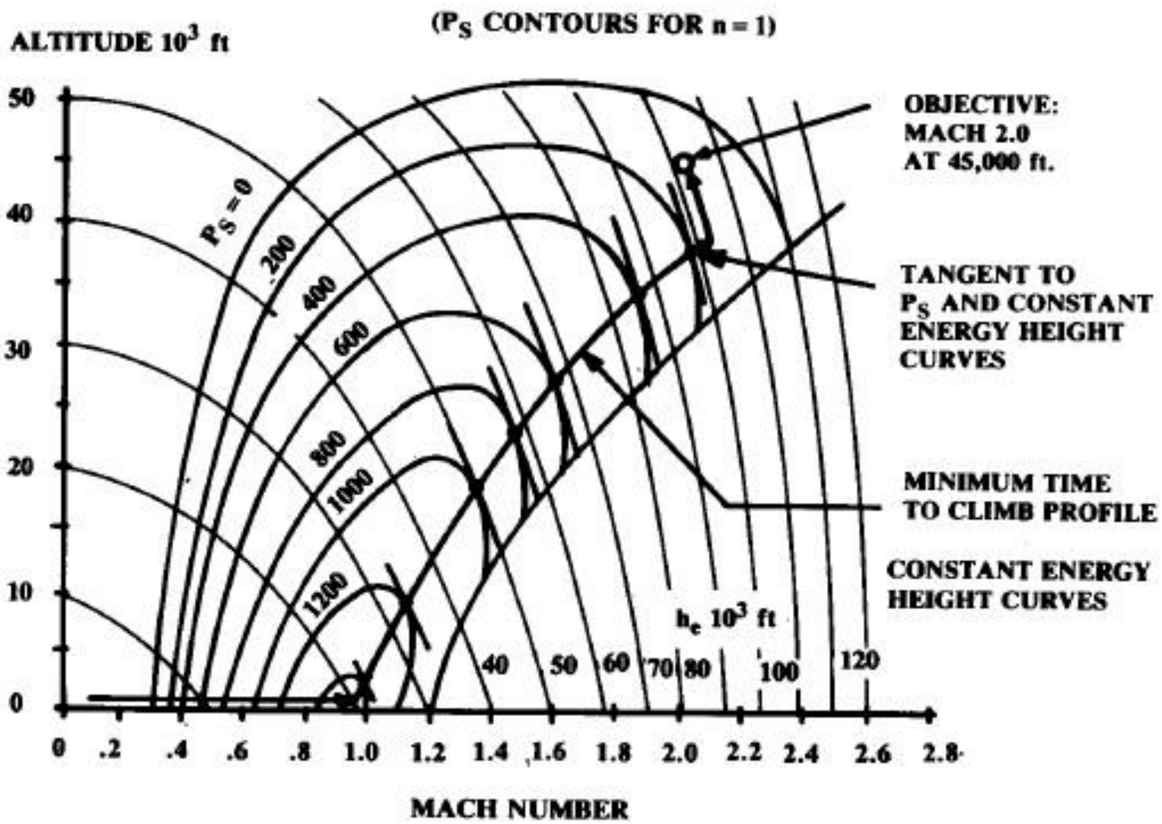

Fig. 17.13 Minimum time-to-climb trajectory, high-thrust fighter.

Minimum time to climb trajectory

- For a current technology fighter, where excess power is great, minimum time to climb is obtained by accelarating to transonic speeds at constant altitude and pitching up into a steep climb at **approximately constant dynamic pressure**.

Minimum time to climb trajectory

- For older generation jets where excess power is less and is almost zero at transonic speeds, P_s contours form bubbles.
- The minimum time to climb trajectory requires jumping from one bubble to another having the same numerical P_s value.
- This requires a dive through $M = 1$.

$$t_{1\rightarrow 2} = \frac{\Delta h_e}{P_{s,average}}$$

Minimum time to climb trajectory

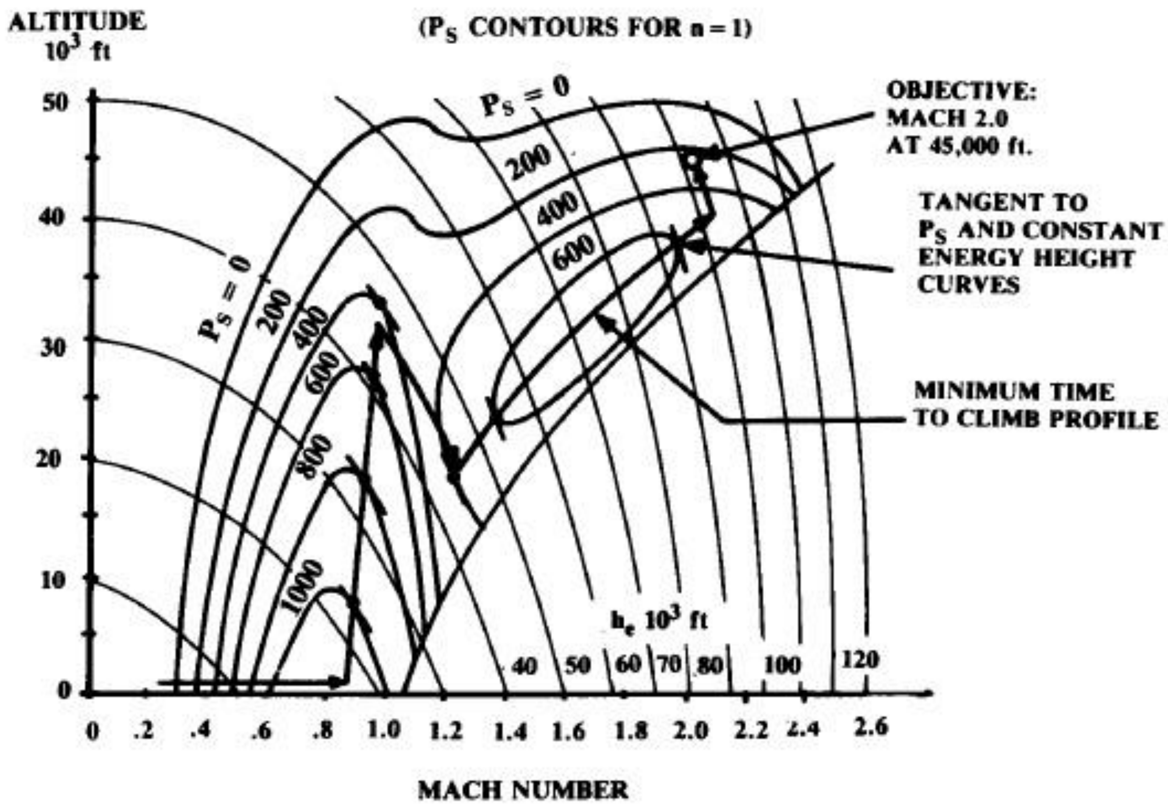

Fig. 17.14 Minimum time-to-climb, low thrust fighter (circa 1960).

Minimum fuel to climb trajectory

- Fuel specific energy

= change in specific energy/change in fuel weight

$$f_s = \frac{dh_e}{dW_f} = \frac{dh_e/dt}{dW_f/dt} = \frac{P_s}{CT} \text{ using } C = \frac{W_f/t}{T}$$

$$W_{f1\rightarrow 2} = \int_{h_{e1}}^{h_{e2}} \frac{dh_e}{f_s}$$

Minimum fuel to climb trajectory

Method to calculate minimum fuel to climb trajectory:

1. Calculate and plot f_s values as a function of Mach # at each altitude.
2. Minimum fuel to climb trajectory passes through those points for which f_s contours are tangent to h_e contours, i.e. when f_s is minimized for each energy height.

$$W_{f1\rightarrow 2} = \frac{\Delta h_e}{f_{s,average}}$$

Minimum fuel to climb trajectory

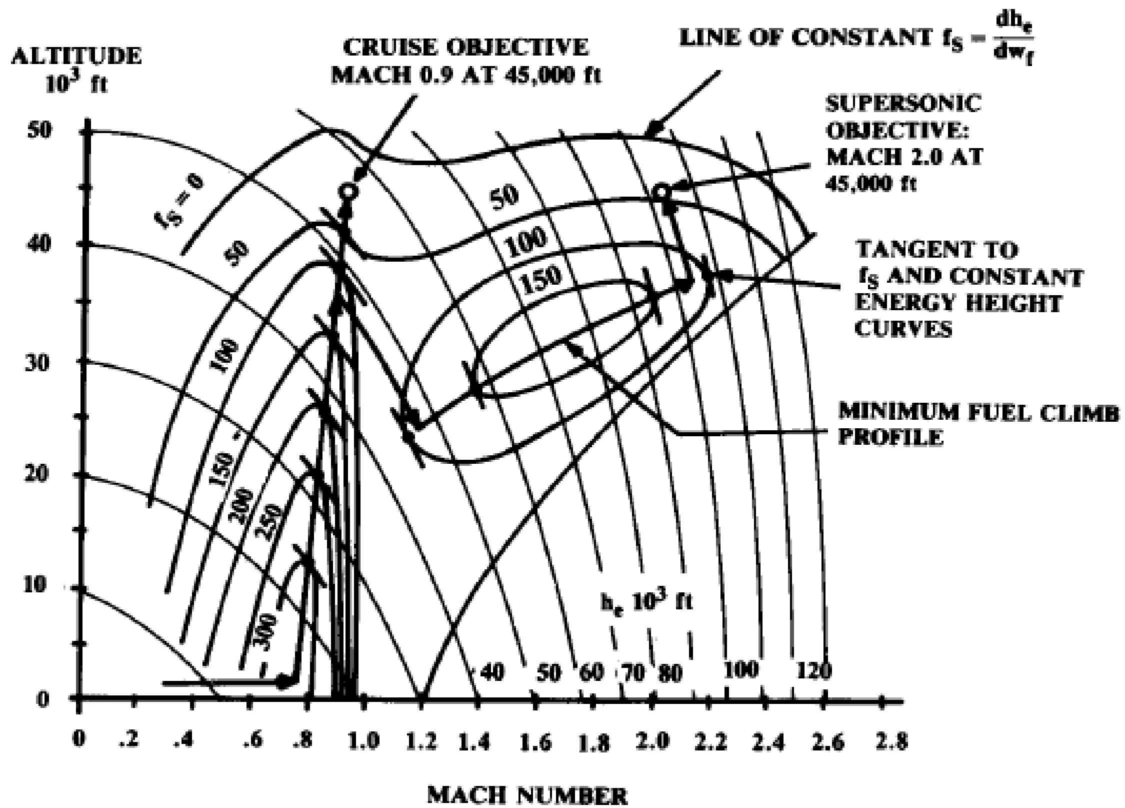

Fig. 17.15 Minimum fuel to climb.

Energy method for mission segment weight fractions

- For mission segments involving **an increase in energy height** (climb, accelaration or both):

$$\frac{W_i}{W_{i-1}} = exp\left[-\frac{C\Delta h_e}{V_\infty(1-D/T)}\right] = exp\left[-\frac{C\Delta h_e}{V_\infty\left(1-\frac{1}{(T/W)(L/D)}\right)}\right]$$

- This formulation **cannot be used** for mission segments **involving decreases** in energy height (descents, decelarations, etc.).

Operating envelope

- Operating envelope (flight envelope): maps the **combinations of altitude and velocity** that the aircraft has been designed to withstand.
- Level flight operating envelope is determined from the $P_s = 0$ and stall limit curves. $P_s = 0$ limit may be shown as maximum thrust and military thrust curves.
- Since P_s varies with aircraft weight, a constant weight must be assumed, i.e. takeoff weight, cruise weight or combat weight.

Operating envelope

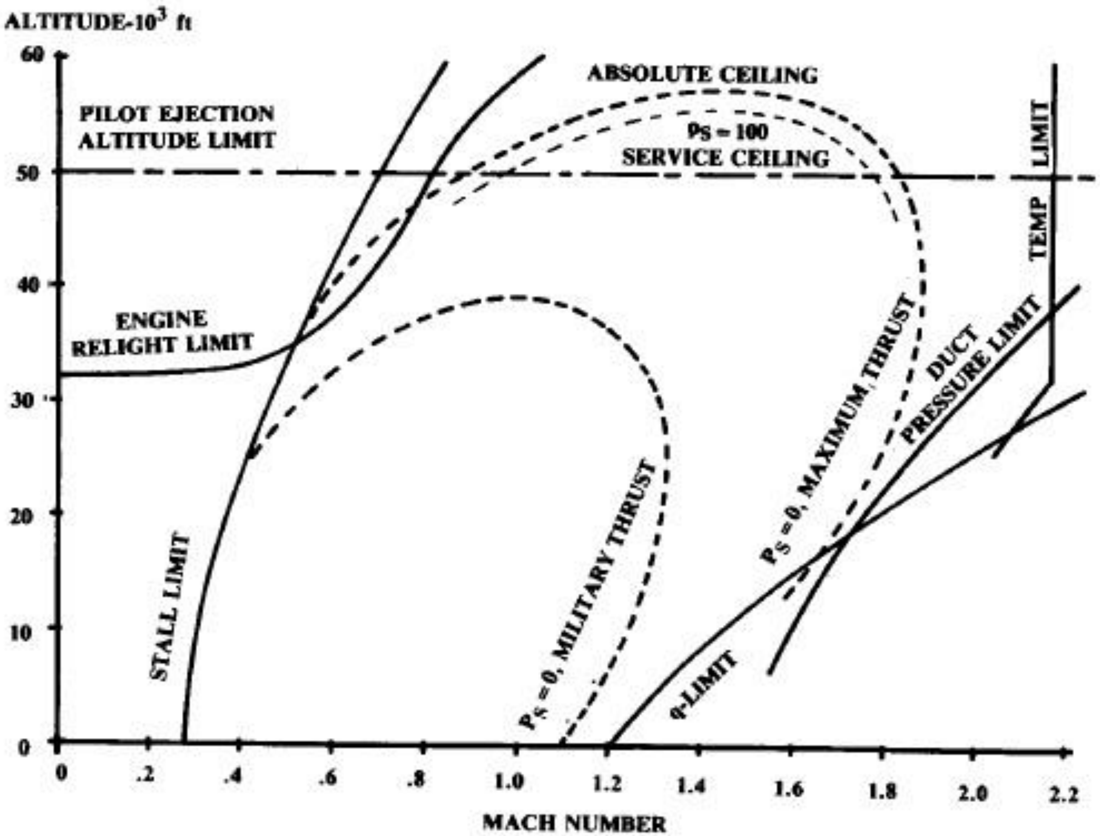

Fig. 17.16 Operating envelope.

Operating envelope

- **Absolute ceiling:** highest altitude where $P_s = 0$.
- **Service ceiling:** some rate of climb is required (and corresponding P_s.
  - FAR: 100 ft/min for propeller, 500 ft/min for jets.
  - Military: 100 ft/min.
- **Combat ceiling:** R/C=500 ft/min
- For jet fighters service ceiling is ≈ 50 000 ft since ejection above that altitude will not usually result in survival of the pilot.

Operating envelope

- Maximum q limit is defined in the design requirements and used by the structural designers for stress analysis.
- Typically, fighter aircraft are designed for $q = 1800 - 2100\, lb/ft^2$.
- This corresponds to **transonic speeds at sea level**.