1. Introduction

The purpose of this experiment is to calibrate the supersonic wind tunnel Mach number and get familiarized with different pressure measurements and relate these pressure readings to the Mach number of the tunnel. The pressures that are measured are the total pressure of the settling chamber $P_{01}$, the test section static pressure $P_{w,test}$ at the wall, the pitot tube pressure $P_{02}$ behind a detached bow shock, and wall static pressures measured at 9 different locations ($P_{w1}, P_{w2}, \ldots, P_{w9}$).

2. Setup

The supersonic wind tunnel GA-10 is a blowdown facility, where stored high pressure dry air is discharged through a variable cross-section test chamber to yield supersonic flow with different Mach numbers. Shock waves can be observed on different models having blunt or sharp leading edges.

Test chamber nozzle performance is measured by nine static pressure ports. Model pressure (pitot tube) and test chamber pressure (wall) are measured through two static ports. The pressure readings are displayed on the control panel of the wind tunnel.

3. Experimental procedure

a. Startup

1. By using the handle next to the test chamber adjust the tunnel test section for $M=1.5$.
2. Turn the compressor knob to position A in order to let the compressor fill the tanks with air. Observe the pressure and temperature gauges on the control panel so that these parameters do not exceed the limits.
3. When the pressure level in the tanks has reached the sufficient level for the experiments to be conducted ($\approx 6-7$ bar), turn the compressor knob to off position. As the pressure will drop quickly, start the experiment immediately.

b. Experiment

1. Run the tunnel at the corresponding run-pressure $P_{01}=1.35$ bar.
2. As soon as the pressure has been stabilized, press the remote pressure lock button to freeze the pressure readings.
3. Stop the run and record the corresponding 9 wall pressure readings $P_{w#,x}$ for the tap locations given in Table 1 and pitot-tube pressure $P_{02}$, and test section static pressure $P_{w,test}$.

4. Calculations

a. Calibration of Tunnel Mach Number:

Assuming that the tunnel test section is arranged to operate at a given Mach number, then it is essential that the actual Mach number in the working section is verified.

There are three possible ways in which this can be accomplished. If a pitot-tube is fixed to the sting, then this pressure $P_{02}$, with the test section wall static pressure $P_{w,test}$ and the settling chamber pressure $P_{01}$ gives three different pressure measurements for
a given Mach number and by using the ratios given below a Mach number can be determined.

a) Ratio between settling chamber pressure $P_{01}$ (stagnation or reservoir pressure) and wall static pressure $P_{w,\text{test}}$: $P_{01}/P_{w,\text{test}}$

$$\frac{P_{01}}{P_{w}} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}$$

b) $P_{01}/P_{02}$, Ratio between settling chamber pressure $P_{01}$ and Pitot tube pressure $P_{02}$.

When a pitot-tube is placed in a supersonic flow a normal shock wave is produced in front of the tube and the pressure measured by the pitot-tube is the total or the stagnation pressure $P_{02}$ after the normal shock wave.

A standard formula relating the stagnation pressure ratios upstream and downstream of a normal shock wave to the upstream Mach number $M_1$ can be used:

$$\frac{P_{02}}{P_{01}} = \left[\frac{\gamma+1}{(\gamma - 1)M_1^2 + 2} - \frac{\gamma+1}{2\gamma M_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}}$$

c) Ratio between pitot tube pressure $P_{02}$ and wall static pressure $P_{w,\text{test}}$: $P_{02}/P_{w,\text{test}}$. By combining the equation (1) and (2) given in (a) and (b) above:

$$\frac{P_{02}}{P_{w}} = \left[\frac{\gamma+1}{2M_1^2} - \frac{\gamma+1}{2\gamma M_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}}$$

This equation is known as the Rayleigh-Pitot tube formula and is usually given as tables. Therefore by measuring these three different pressures $P_{01}$, $P_{02}$, $P_{w,\text{test}}$ three alternative ways to determine the actual Mach number in the test section can be proposed.

Using the ratios $P_{01}/P_{w,\text{test}}$, $P_{02}/P_{01}$, and $P_{02}/P_{w,\text{test}}$ and the equations corresponding to these ratios calculate the Mach number of the test section, Plot actual Mach number vs. measured Mach number calculated by the three formulas above, Discuss the results obtained.

b. Determination of Nozzle Performance

i. Calculate the corresponding local Mach numbers for each position, $M(x)$ using the relationship:

$$\frac{P_{01}}{P_{w}} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$P_{01}$: settling chamber (total head) pressure (bar),

$P_{w,\#}$: wall static pressure (bar) at the given location, ($\# = 1, 2, ..., 9$)

$M$: Mach number.

$\gamma$: $C_p/C_v$ ($C_p$, $C_v$ specific heat coef.)

These are the measured Mach numbers.
ii. Using the relationship for the area ratios $A(x)/A^*$ calculate the local theoretical Mach numbers $M_T(x)$:

$$
\frac{A}{A^*} = \frac{1}{M_T} \left( 2 \left( 1 + \frac{\gamma - 1}{2} \frac{M_T^2}{\gamma + 1} \right) \right)^{\frac{\gamma+1}{2(\gamma-1)}}
$$

$A$ : area of that section

$A^*$: area of the throat section

iii. With these values of local theoretical Mach numbers, determine the local theoretical wall static pressure values $P_{w,T}(x)$ by Eq. (1).

iv. Make a table showing the measured and theoretical wall pressure values $P_{w,T}(x)$, also measured and theoretical local Mach number values for each $x$ station.

v. Plot the variation of measured and theoretical local wall pressure values, $P_{w,T}(x)$ and $P_{w,T}(x)$ vs. $x$.

vi. Plot the variation of measured and theoretical local Mach number values, $M(x)$ and $M_T(x)$ vs. $x$.

vii. Compare theoretical results with measured ones.

viii. Comment about results.

<table>
<thead>
<tr>
<th>Point no</th>
<th>Distance from section inlet (mm)</th>
<th>Cross sectional area ($m^2\cdot10^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1027.5</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
<td>667.5</td>
</tr>
<tr>
<td>3</td>
<td>52</td>
<td>647.5</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
<td>655.0</td>
</tr>
<tr>
<td>5</td>
<td>82</td>
<td>680.0</td>
</tr>
<tr>
<td>6</td>
<td>102</td>
<td>705.0</td>
</tr>
<tr>
<td>7</td>
<td>122</td>
<td>725.0</td>
</tr>
<tr>
<td>8</td>
<td>142</td>
<td>742.5</td>
</tr>
<tr>
<td>9</td>
<td>162</td>
<td>755.0</td>
</tr>
<tr>
<td>THROAT</td>
<td>-</td>
<td>647.5</td>
</tr>
</tbody>
</table>

Table 1.
Figure 1. Sketch of Supersonic Wind Tunnel

Figure 2. General Layout of the GA-10 Supersonic Wind Tunnel with Schlieren System