SECOND ORDER RUNGE-KUTTAA METHODS

Employ the Improved Euler (Heun’s) method to solve a first order ODE of a physical process, which has an analytical, but non-polynomial solution. In addition, derive a second order RK method (RK2) by choosing $p_1$ arbitrarily between 0 and 1, and solve the same problem again. You have to modify the Fortran code, euler.f for the numerical solutions.

In your report,

- describe the physical process, the mathematical model and the ODE.
- obtain Improved Euler solutions for various step sizes, and compare them to the analytical solution and the Euler solution on the same graph. Determine the maximum step size for a convergent solution.
- compare the relative error distributions for the various step sizes on the same graph.
- compare Improved Euler and RK2 solutions together for the same step sizes, and plot the relative error distributions for various step sizes.
- discuss all the results obtained in the previous steps.
- include the Fortran program developed, but not include any data files.

For a 50% bonus:

- Estimate the truncation error at a certain location for various constant step sizes and assess the order of accuracy of the numerical solution. Note that the leading truncation error term is in the form of $E = |A| \Delta x^n$. When the logarithm of the error estimation, $\log_{10} E$, is plotted with respect to the step size, $\Delta x$;

$$\log_{10} E = \log_{10} |A| + n \log_{10} \Delta x$$

the slope of the curve represents the order of accuracy of the numerical method employed.