AE 172 Aircraft Performance
Aerodynamics of the Airplane

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Outline

- Source of aerodynamic forces
- Aerodynamic lift, drag and moment
- Aerodynamic coefficients
- Variation of lift, drag and moment coefficients with angle of attack, Reynolds number and Mach number
- Aerodynamic center
- NACA airfoil nomenclature
- Lift of a finite wing
- Drag
- Drag polar

Source of aerodynamic forces

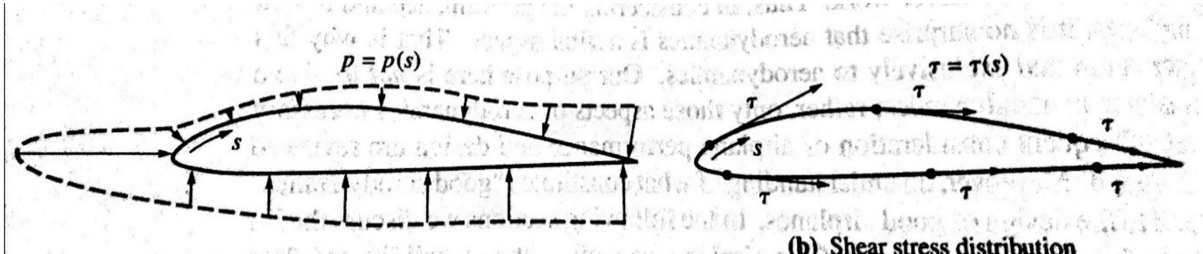
Aerodynamic forces on a body

=

Pressure Forces (distributed)

+

Shear Stresses (distributed)



(a) Pressure distribution (schematic only; distorted for clarity)

(b) Shear stress distribution

Source of aerodynamic forces

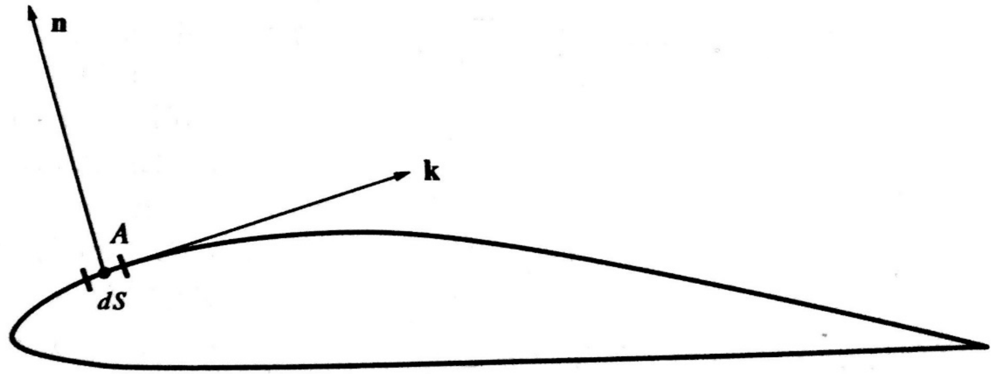


Figure 2.2 Sketch of the unit vectors.

Resultant aerodynamic force is the sum of two surface integrals:

$\vec{R} = -\iint_S p\vec{n}dS + \iint_S \tau\vec{k}dS$: forces due to pressure + forces due to shear stresses (friction)

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Aerodynamic lift, drag and moment

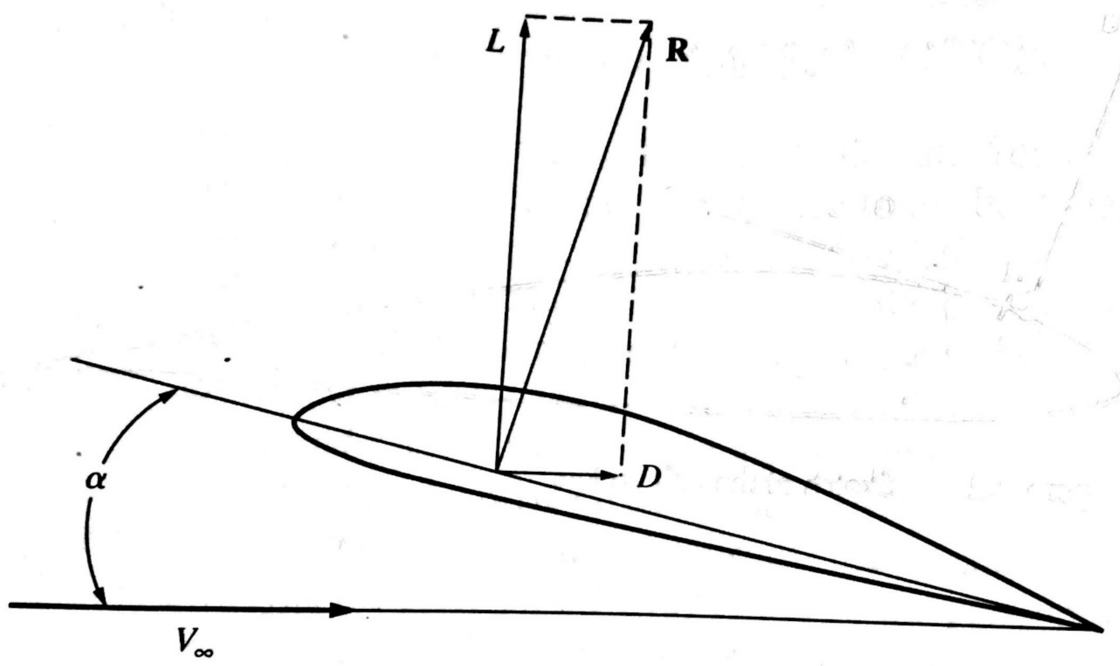

Figure 2.3 Lift, drag, and resultant aerodynamic force.

Aerodynamic lift, drag and moment

Chord line: line joining the leading and trailing edges.

V_∞: freestream velocity (relative wind).

α: angle of attack, angle between the **freestream velocity vector and chord line.**

L: lift force, component of the resultant aerodynamic force **perpendicular to the freestream velocity vector**.

D: drag force, component of the resultant aerodynamic force **parallel to the freestream velocity vector**.

Aerodynamic lift, drag and moment

Where should the resultant aerodynamic force be placed?

Anywhere, as long as the moment around the chosen point accompanies $\vec{R}$.

Aerodynamic lift, drag and moment

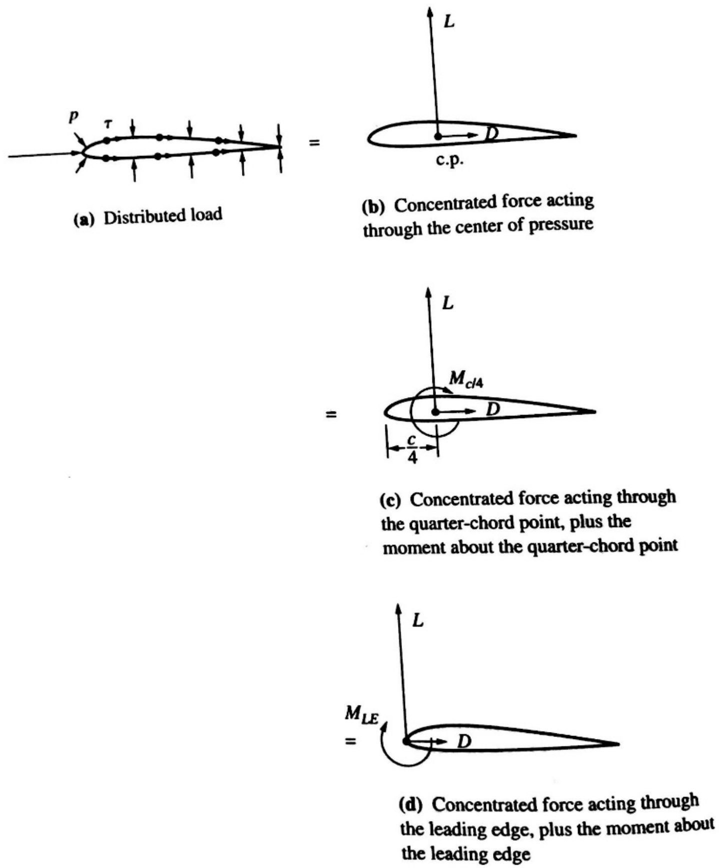

Figure 2.5 Three ways of representing the actual distributed load exerted by pressure and shear stress on the surface of the airfoil by a concentrated force at a point and the moment at that point.

Aerodynamic lift, drag and moment

Center of pressure: the point through which the concentrated resultant force acts. The point on the body about which the net aerodynamic moment is zero.

A moment is positive when it tends to rotate the body so as to increase its angle of attack (clockwise).

Aerodynamic coefficients

Aerodynamic forces and moments on a body depend on several parameters:

$L = L(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$

$D = D(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$

$M = M(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$: aerodynamic moment

a_∞: speed of sound, accounting for compressibility (high speed effects).

Aerodynamic coefficients

$C_L = \frac{L}{q_\infty S}$: lift coefficient,

$C_D = \frac{D}{q_\infty S}$: drag coefficient,

$C_M = \frac{M}{q_\infty S c}$: moment coefficient,

$q_\infty = {}^1/_2\, \rho_\infty V_\infty^2$: dynamic pressure, determines the magnitudes of the aerodynamic forces and moments.

S: planform area of the wing, base area of a missile (characteristic area).

c: chord length of a wing, diameter of a missile (characteristic length).

ρ_∞ : density of air.

μ_∞ : viscosity of air.

Aerodynamic coefficients

V_∞ : freesream velocity or airspeed.

$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty}$: Reynolds number.

$M = \frac{V_\infty}{a_\infty}$: Mach number.

Alternatively:

$$C_L = f_1(\alpha, Re, M)$$

$$C_D = f_2(\alpha, Re, M)$$

$$C_M = f_3(\alpha, Re, M)$$

Aerodynamic lift, drag and moment

- **Dynamic similarity:** If the Reynolds number, Mach number are the same for two flows, the aerodynamic forces and moments will be the same for two geometrically similar bodies at the same angle of attack.
- **Essence of wind tunnel testing:** scaled models of large vehicles can be tested in wind tunnels to reflect real flight conditions provided that the Re, M, α during wind tunnel tests and actual flight conditions are identical.

Variation of coefficients

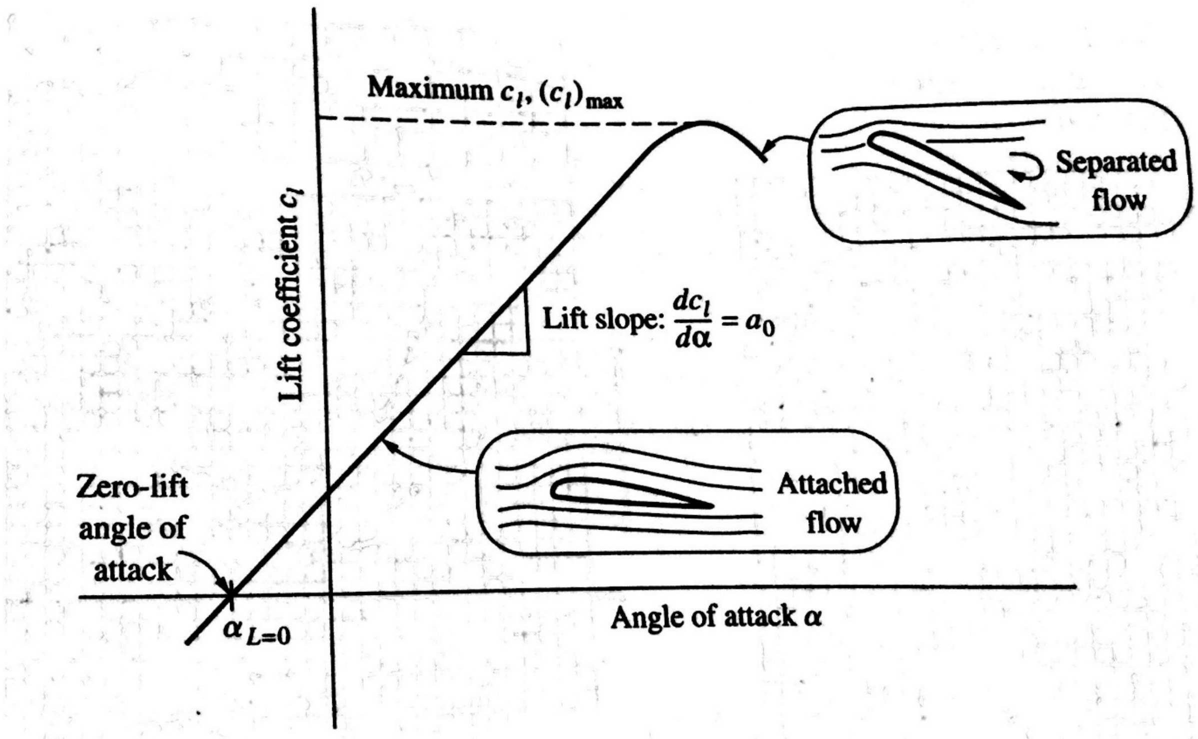

Figure 2.7 Sketch of a generic lift curve.

$a_o = 2\pi/radian = 0.11/deg$ for most airfoils.

Variation of coefficients

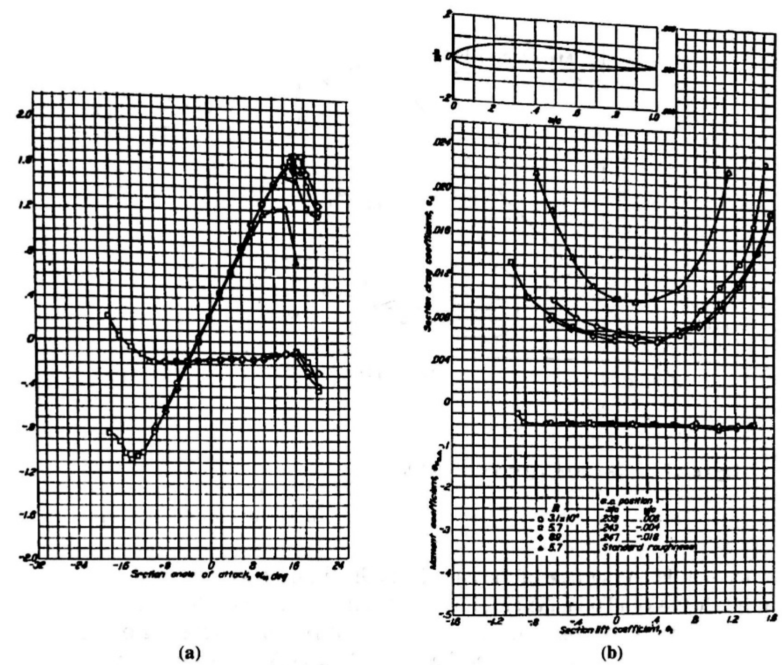

Figure 2.6 Data for the NACA 2412 airfoil. **(a)** Lift coefficient and moment coefficient about the quarter-chord versus angle of attack. **(b)** Drag coefficient and moment coefficient about the aerodynamic center as a function of the lift coefficient. (From Abbott and von Doenhoff, Ref. 19.)

- Linear part of the lift curve is unaffected by Re.
- $c_{l,max}$ and α_s decrease with Re since flow separation is a viscous phenomenon.
- $\alpha_{L=0}$ is a negative angle.

Variation of coefficients

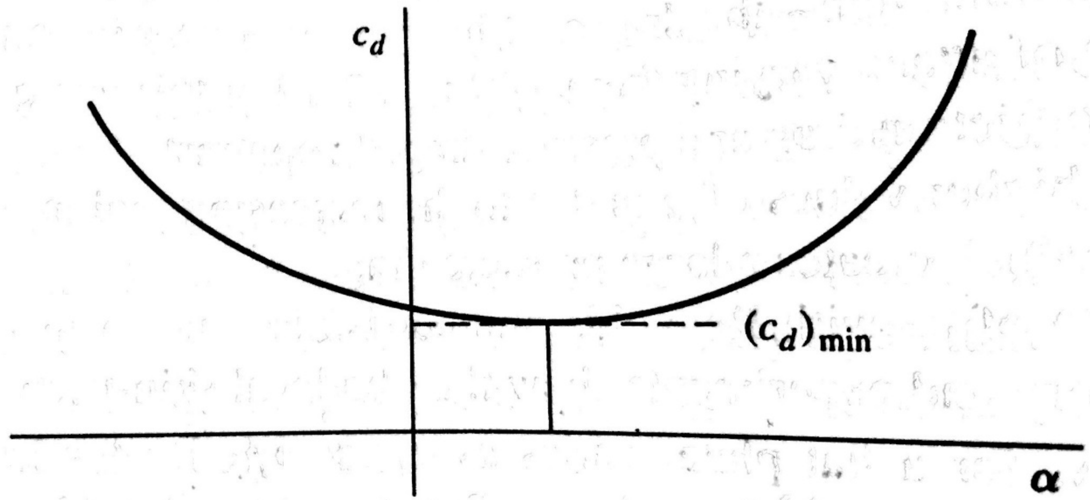

Figure 2.9 Sketch of a generic drag curve.

- $c_{d,min}$ occurs at positive c_l for positively cambered airfoils.

Variation of coefficients

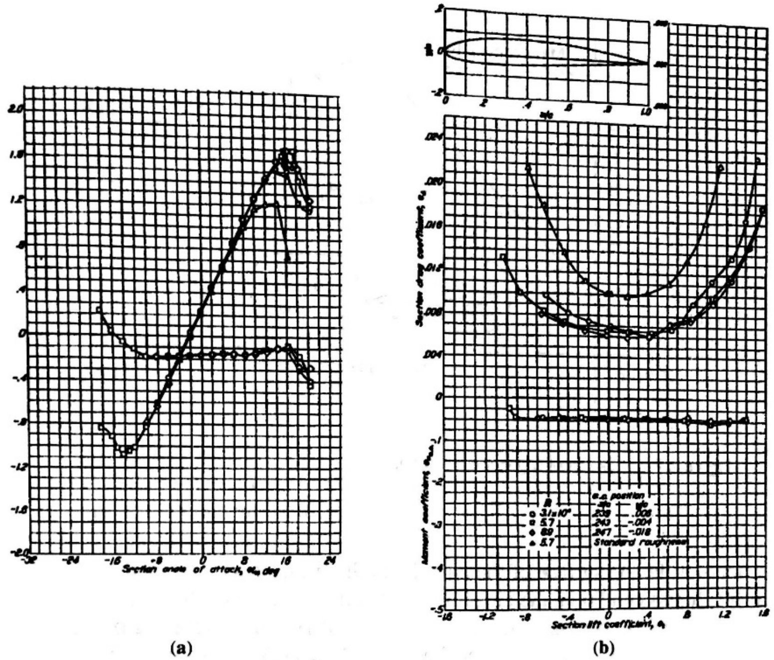

Figure 2.6 Data for the NACA 2412 airfoil. **(a)** Lift coefficient and moment coefficient about the quarter-chord versus angle of attack. **(b)** Drag coefficient and moment coefficient about the aerodynamic center as a function of the lift coefficient. (From Abbott and von Doenhoff, Ref. 19.)

- c_d decreases with Re since skin friction is inversely proportional to Re.
- c_d varies steeply with α for low and high α since flow separation is important for those α values.

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Variation of coefficients

$C_f \sim \frac{1}{\sqrt{Re}}$: laminar flow,

$C_f \sim \frac{1}{Re^{0.2}}$: turbulent flow.

C_f: skin friction coefficient

$\tau = C_f q_\infty$

Variation of coefficients

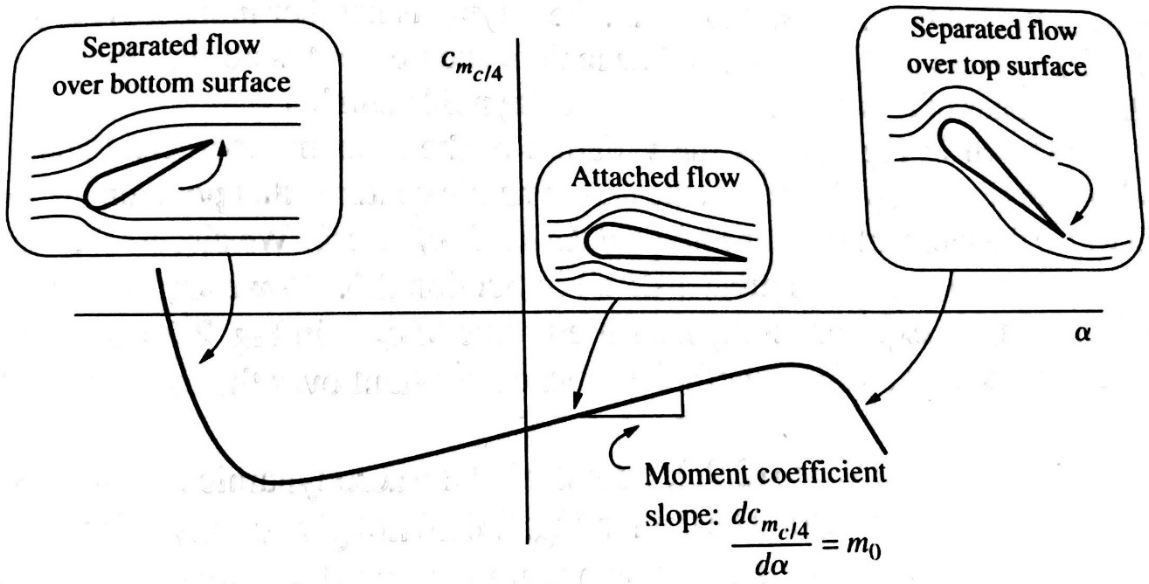

Figure 2.8 Sketch of a generic moment curve.

- Moment coefficient is almost unaffected by Re.

Variation of coefficients

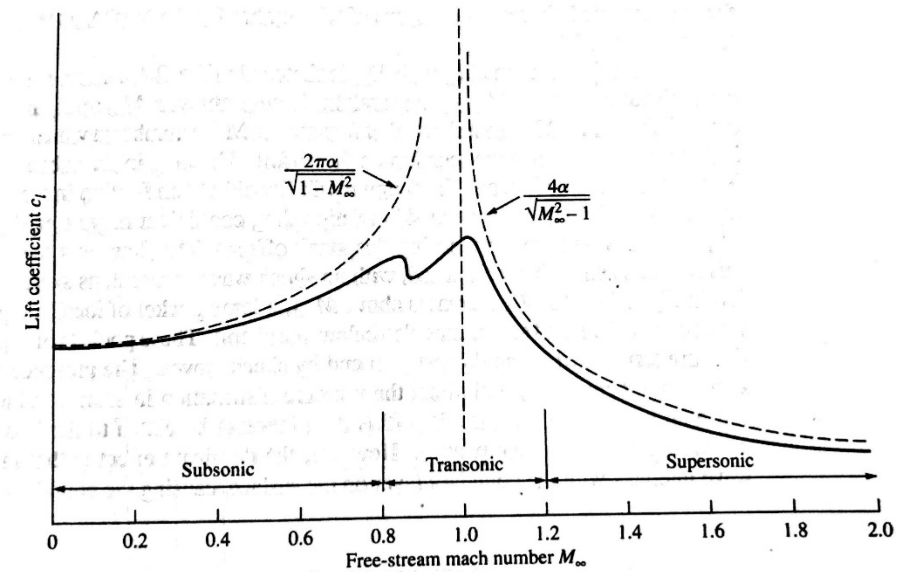

Figure 2.10 Sketch of a generic lift coefficient variation with Mach number.

- For subsonic speeds, the pressure difference between the upper and lower surfaces of an airfoil increases as Mach number increases.
- Oscillatory behavior near $M = 1.0$ is due to shock wave-boundary-layer interaction.

Variation of coefficients

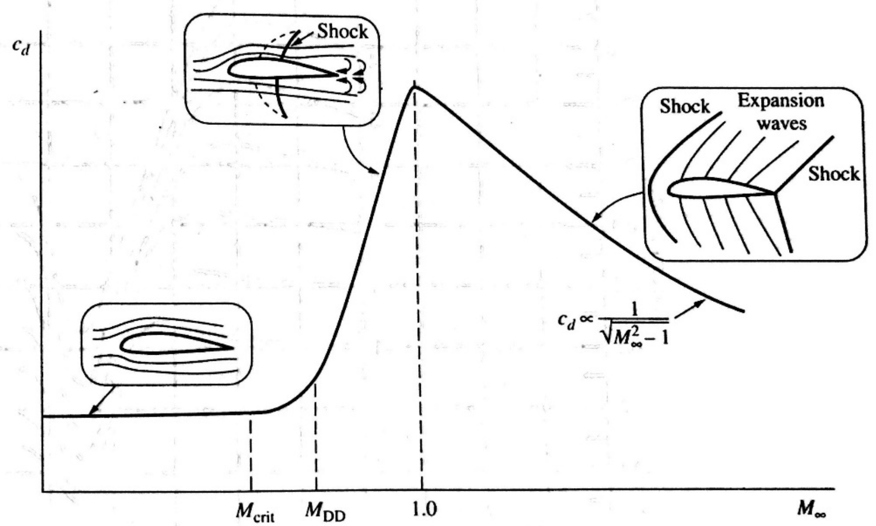

Figure 2.11 Sketch of a generic drag coefficient variation with Mach number.

- Increase of c_d with Mach number near $M = 1$ is due to flow separation due to shock wave-boundary-layer interaction.
- For $M > 1$, there is no shock wave on the airfoil surface, hence c_d decreases with Mach number.

Variation of coefficients

- Variation of c_m with Mach number is qualitatively similar to variation of c_l because moment of an airfoil is due to pressure distribution just like lift.
- Variations of C_L, C_D and C_M (3-D) with Re, M, α is generally similar to the variations of c_l, c_d and c_m (2-D).

Aerodynamic center

- **Aerodynamic center:** is the point on the body about which the aerodynamic moment is independent of angle of attack.

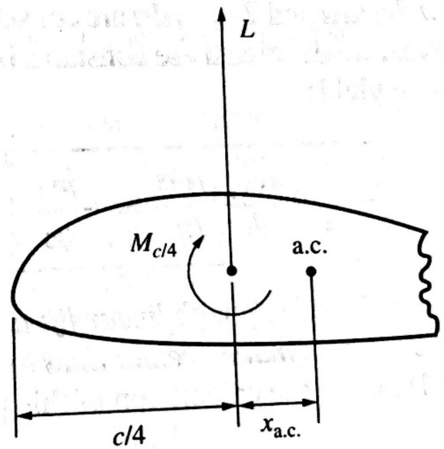

Figure 2.14 Schematic for finding the location of the aerodynamic center.

- For most airfoil shapes, aerodynamic center is very close to $c/4$. According to thin airfoil theory, it is exactly at $c/4$.

NACA airfoil nomenclature

- **Airfoil families:** NACA, RAE, Göttingen, Eppler, etc.
- Most airfoil data was obtained during 1920-1960 in wind tunnel tests.
- Today, there are computer programs producing optimized airfoil shapes.

NACA airfoil nomenclature

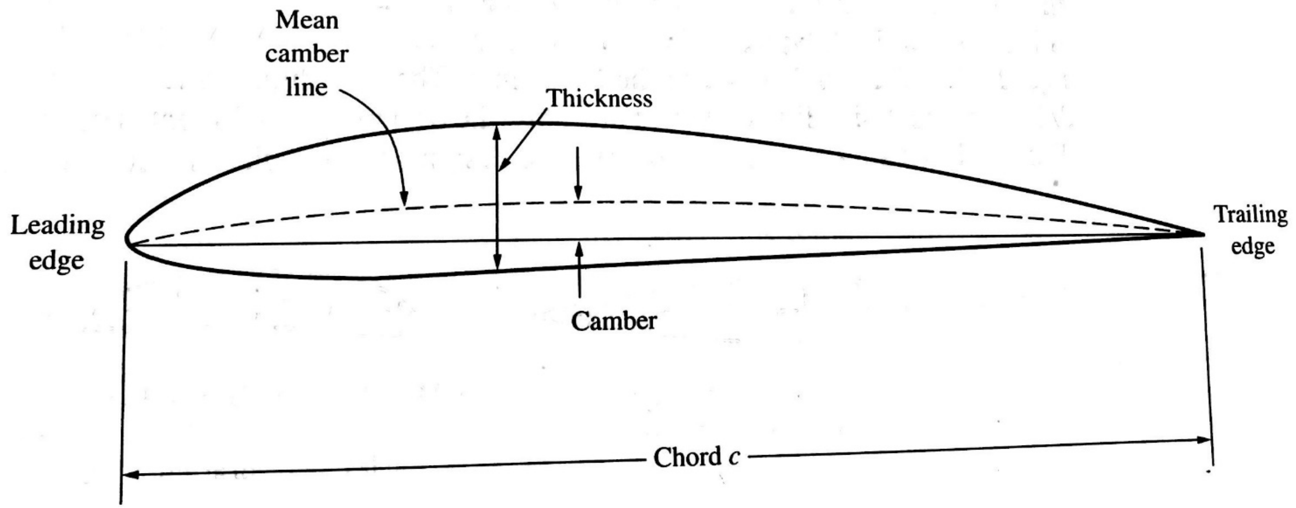

Figure 2.15 Airfoil nomenclature.

NACA airfoil nomenclature

- **Mean camber line:** locus of points halfway between the upper and lower surfaces, measured perpendicular to the mean camber line itself (or the chord line with little error).
- **Chord line:** straight line joining the leading and the trailing edges.
- **Leading edge:** most forward point of the mean camber line.
- **Trailing edge:** most backward point of the mean camber line.

NACA airfoil nomenclature

- **Camber:** maximum distance between the camber line and the chord line measured perpendicular to the chord line.
- Camber, slope of the camber line and the thickness distribution control the lift and moment characteristics of an airfoil.

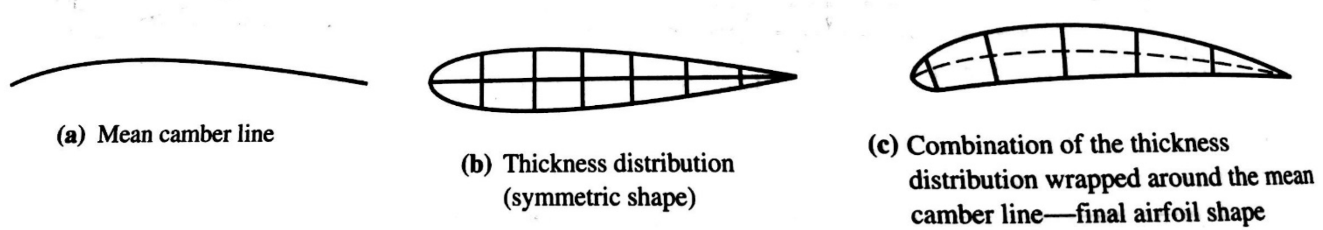

Figure 2.16 Buildup of an airfoil profile.

NACA airfoil nomenclature

- NACA 4 digit airfoils:

NACA 4412

- 12 → maximum thickness in percent of the chord (12 %)
- 4 (second digit) → location of maximum camber in tenths of chord (40 %)
- 4 (first digit) → maximum camber in percent of chord (4 %)

NACA airfoil nomenclature

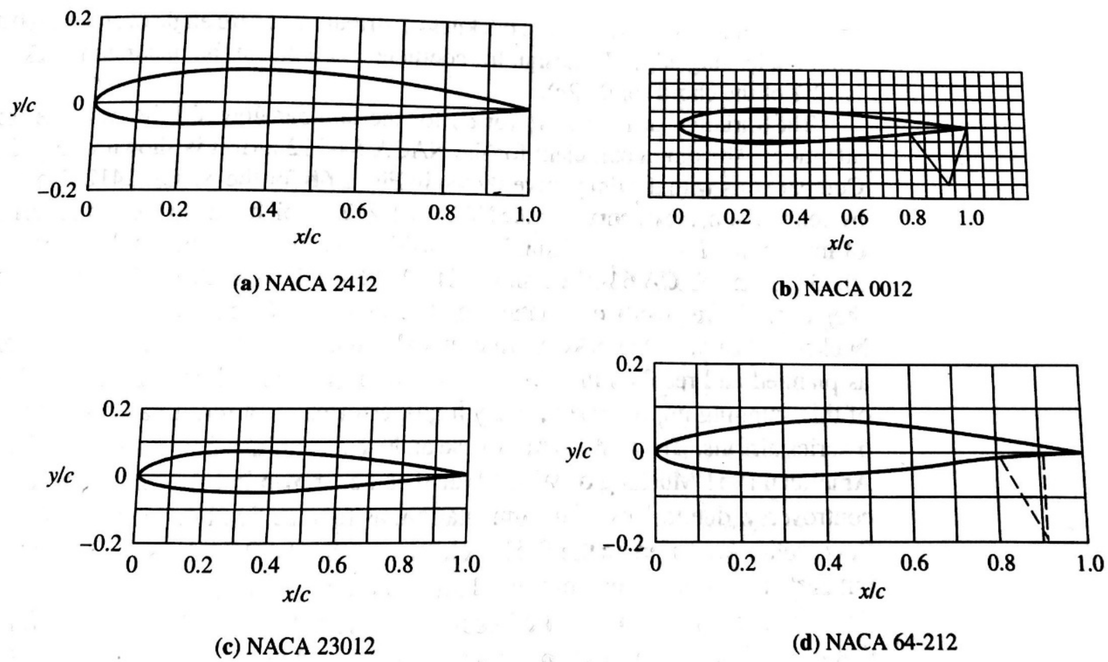

Figure 2.17 Various standard NACA airfoil shapes, all with 12% thickness.

NACA airfoil nomenclature

- NACA 5 digit airfoils:

NACA 23012

- 12 → maximum thickness in percent of the chord (12 %)
- 3 → *0.5 = location of maximum camber in percent of chord (15 %)
- 2 → *1.5 = design lift coefficient in tenths (0.3)

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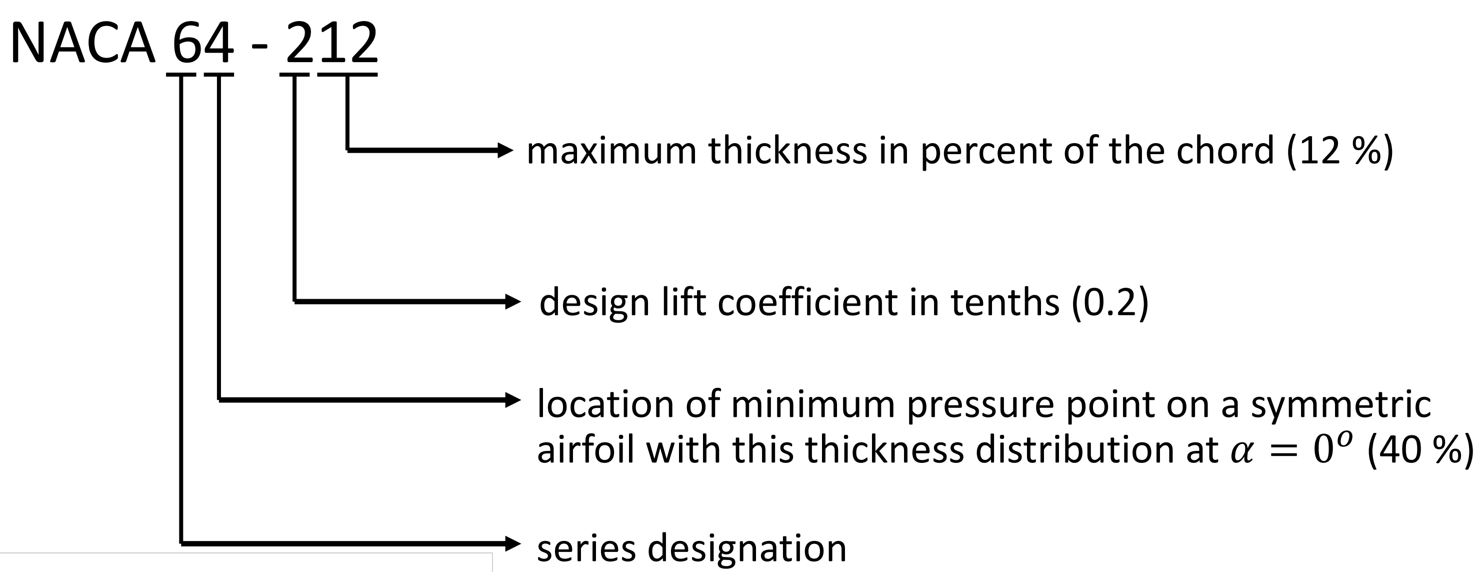
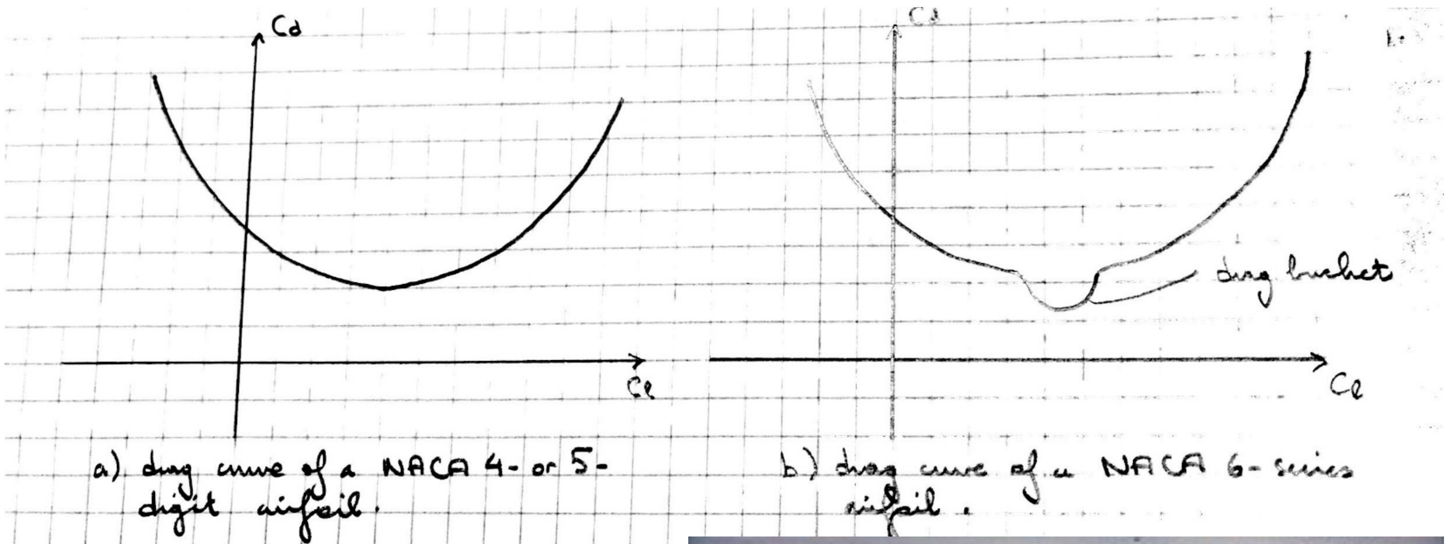
NACA airfoil nomenclature

- NACA 6 series airfoils:

These were designed for laminar flow to reduce skin friction drag.



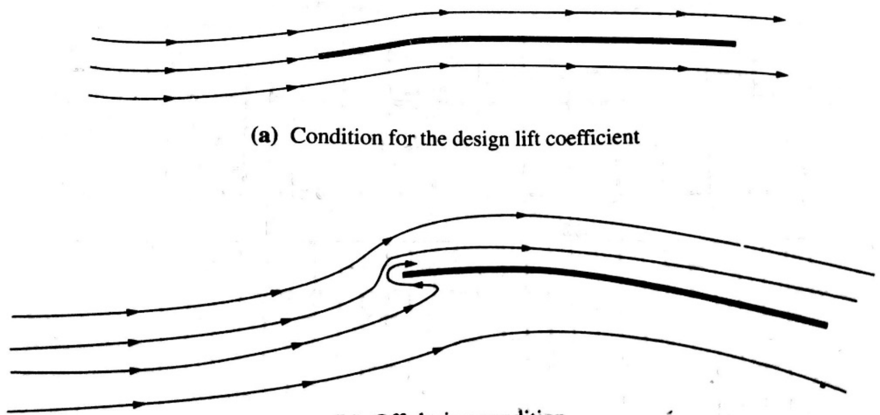
NACA airfoil nomenclature



P-51 mustang was the first airplane to use a laminar flow airfoil (NACA 6 series).

NACA airfoil nomenclature

- Design lift coefficient:



(a) Condition for the design lift coefficient

(b) Off-design condition

Figure 2.19 Sketch illustrating the definition of the theoretical design lift coefficient.

There is only one angle of attack at which the local flow direction at the leading edge is tangent to the mean camber line. The theoretical lift coefficient for the camber line at this angle of attack is the **design or ideal lift coefficient**.

Lift for a finite wing

- Airfoil: wing with infinite span. Real wings have **finite** span.

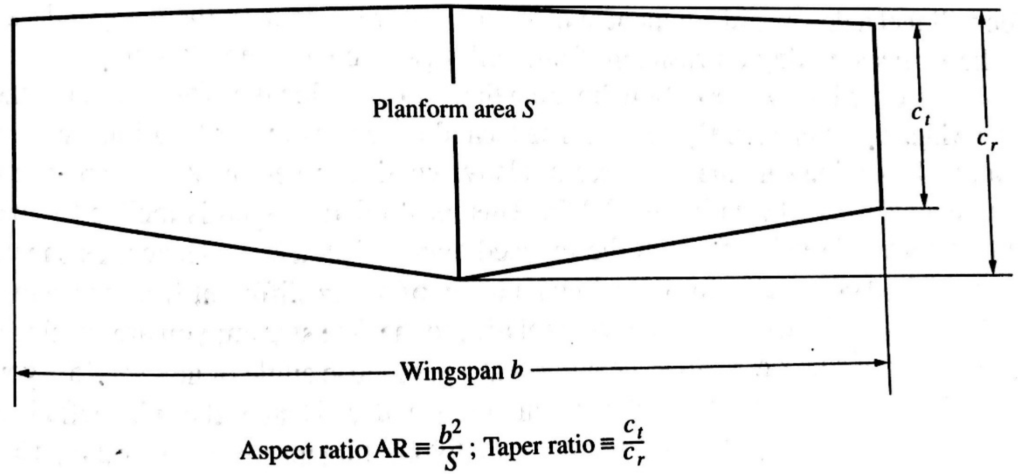


Figure 2.20 Finite-wing geometry.

- Aspect ratio, $AR = \dfrac{b^2}{S}$

- Taper ratio, $\lambda = \dfrac{c_t}{c_r}$

Lift for a finite wing

- c_l for a NACA 2412 airfoil at $\alpha = 4^o$ is 0.65.
- C_L for a finite wing made of NACA 2412 airfoil at $\alpha = 4^o$ is <u>not</u> 0.65!

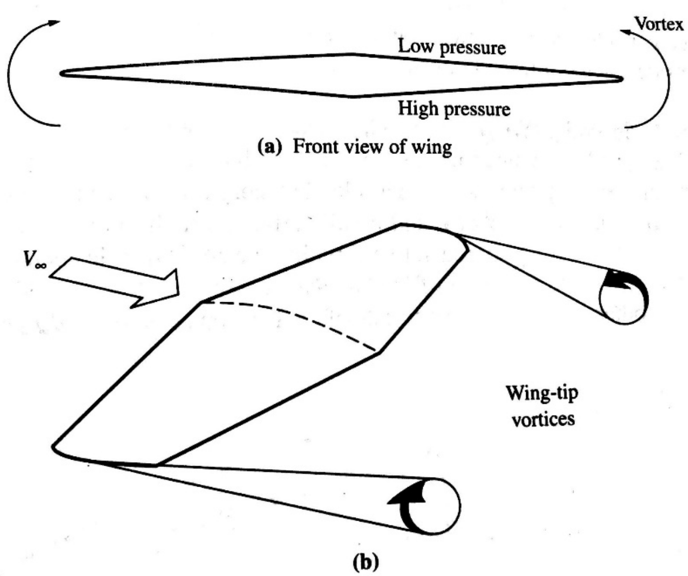


Figure 2.21 Wing-tip vortices.

Lift for a finite wing

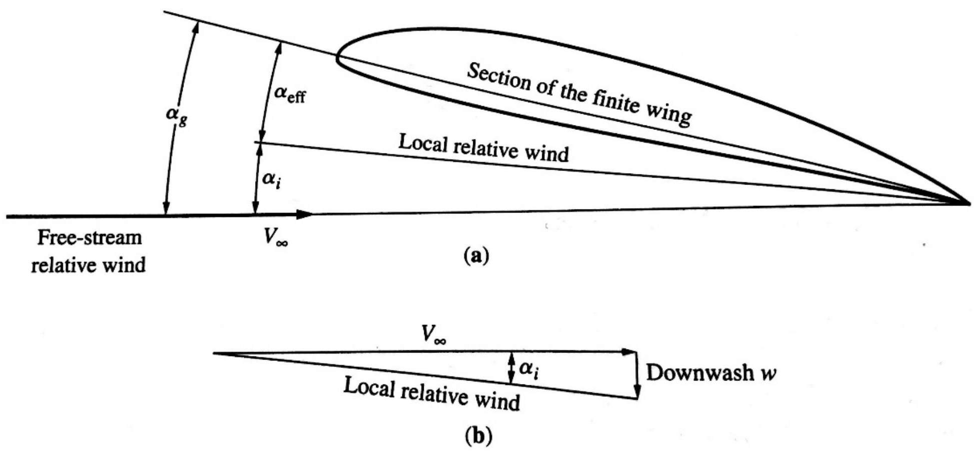


Figure 2.22 Illustration of induced and effective angles of attack, and downwash w.

α_i: induced angle of attack

Lift for a finite wing

Main types of wing planforms:

i. High aspect ratio straight wings

ii. Low aspect ratio straight wings

iii. Swept wings

iv. Delta wings

Lift for a finite wing

High aspect ratio straight wings:

According to Prandtl's lifting line theory:

$$a = \frac{a_o}{1+a_o/(\pi e AR)}$$; for incompressible (low-speed) flow ($M \leq 0.3$), valid for $AR \geq 4$.

e: Oswald span efficiency factor ($\approx 0.9 - 0.95$)

Lift for a finite wing

Corrections due to compressibility are done using the Prandtl-Glauert rule ($0 \le M \le 0.7$):

$$a_{o,comp} = \frac{a_o}{\sqrt{1-M^2}}$$

Combining this correction and Prandtl's formula:

$a_{comp} = \dfrac{a_o}{\sqrt{1-M^2}+a_o/(\pi e AR)}$; lift curve slope of a high aspect ratio, straight wing in compressible flow.

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Lift for a finite wing

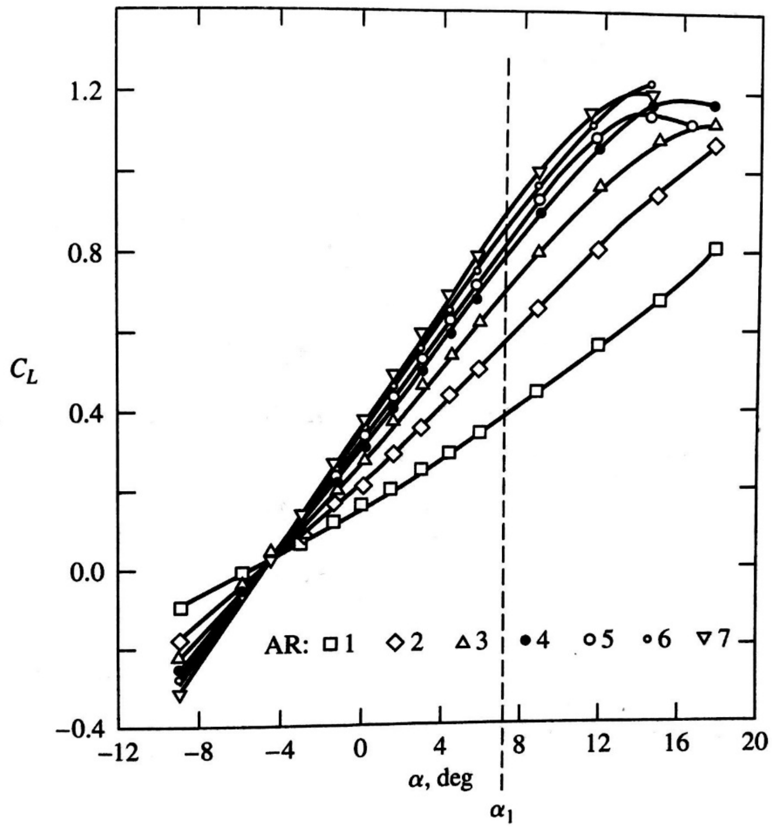

Figure 2.23 Effect of aspect ratio on the lift curve.

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Lift for a finite wing

For supersonic flow over a high aspect ratio straight wing, the lift slope can be approximated from supersonic linear theory:

$$a_{comp} = \frac{4}{\sqrt{M^2-1}}$$

Lift for a finite wing

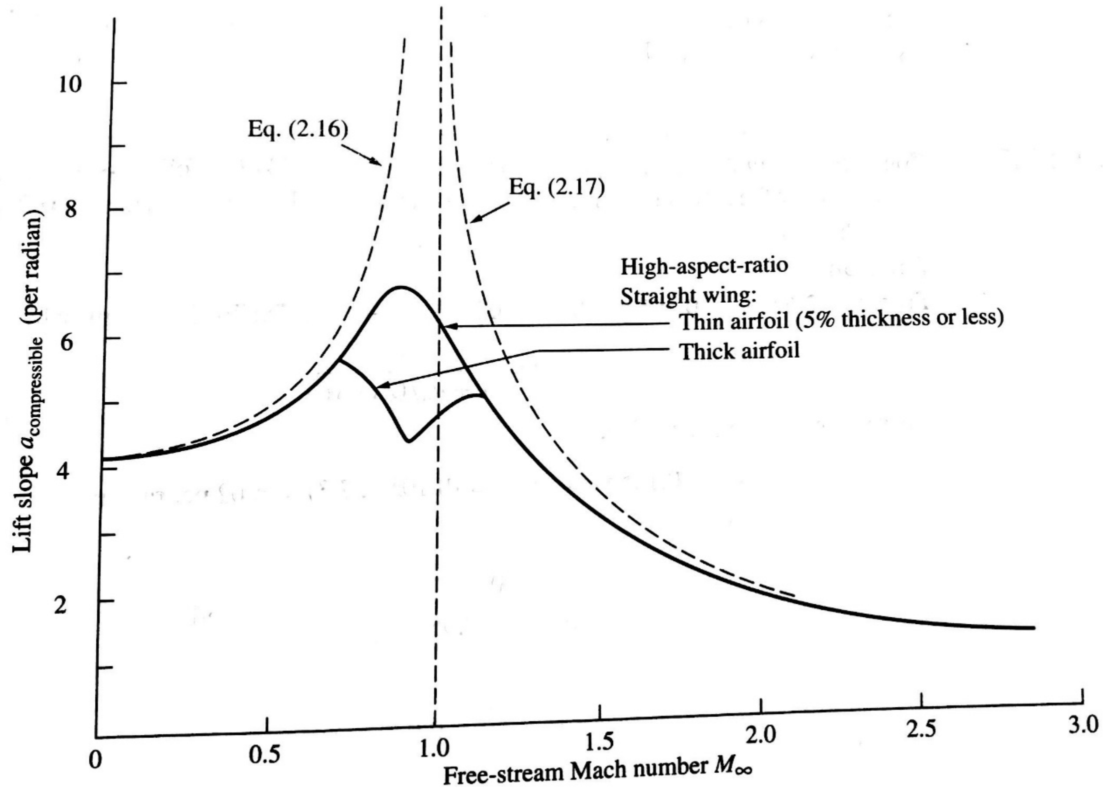


Figure 2.24 Effect of Mach number on the lift slope.

Lift for a finite wing

Low aspect ratio straight wings:

According to Helmbold's formulation:

$$a = \frac{a_o}{\sqrt{1+[a_o/(\pi e AR)]^2} + a_o/(\pi AR)}$$; for incompressible (low-speed) flow ($M \leq 0.3$), valid for $AR < 4$.

Correction due to compressibility:

$$a_{comp} = \frac{a_o}{\sqrt{1-M^2+[a_o/\pi AR]^2} + a_o/(\pi AR)}$$; lift curve slope of a low aspect ratio, straight wing in compressible flow ($0 \leq M \leq 0.7$).

Lift for a finite wing

For supersonic flow over a low-aspect ratio, straight wing, formulation of Hoerner and Borst can be used:

$$a_{comp} = \frac{4}{\sqrt{M^2-1}}\left(1 - \frac{1}{2AR\sqrt{M^2-1}}\right)$$

Lift for a finite wing

Swept wings:

- The main function of a swept wing is to reduce wave drag (drag due to shock waves or compressibility) at transonic and supersonic speeds.
- The pressure distribution over an airfoil section oriented perpendicular to the leading edge is mainly governed by the chordwise component of freestream velocity.

Lift for a finite wing

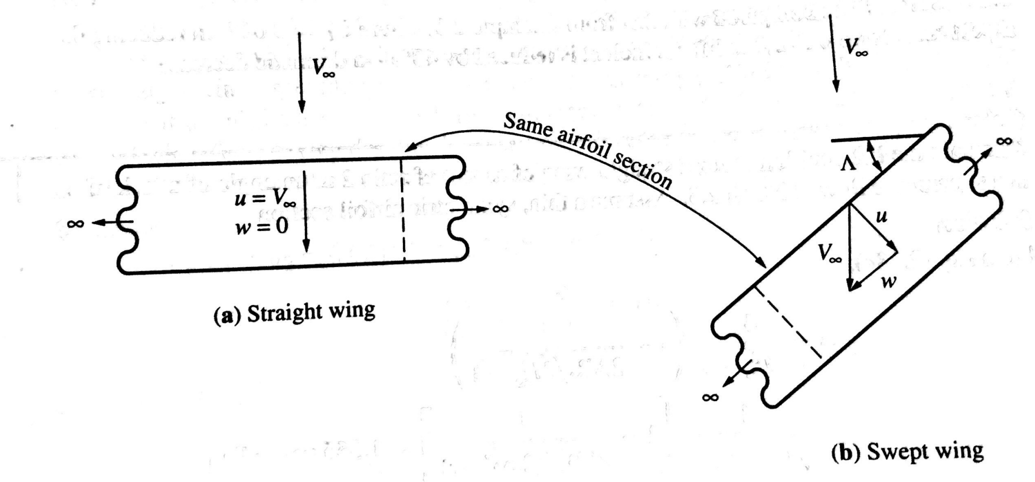


Figure 2.28 Effect of sweeping a wing.

$u_{swept} < u_{straight} \Rightarrow M_{n,swept} < M_{n,straight} \Rightarrow$ the effective Mach number over a swept wing will be less than the effective Mach number over a straight wing $\Rightarrow$ shock waves will be weaker over a swept wing, less wave drag.

Lift for a finite wing

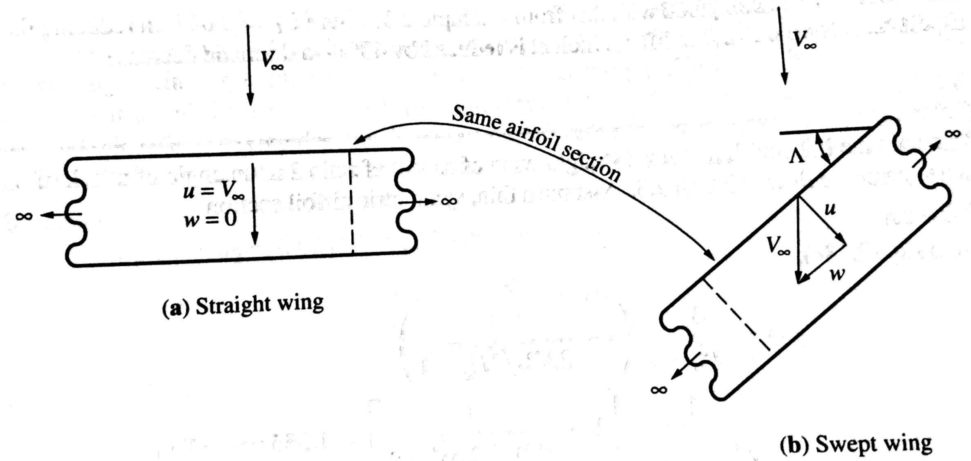


Figure 2.28 Effect of sweeping a wing.

$u_{straight} > u_{swept} \Rightarrow$ the pressure difference between the top and the bottom surfaces of a swept wing will be less than the pressure difference between the top and bottom surfaces of a straight wing $\Rightarrow$ lift on the swept wing will be less than the straight wing.

Lift for a finite wing

For tapered wings, sweep angle is referenced w.r.t. half chord line. As such, the lift curve slope becomes independent of the taper ratio.

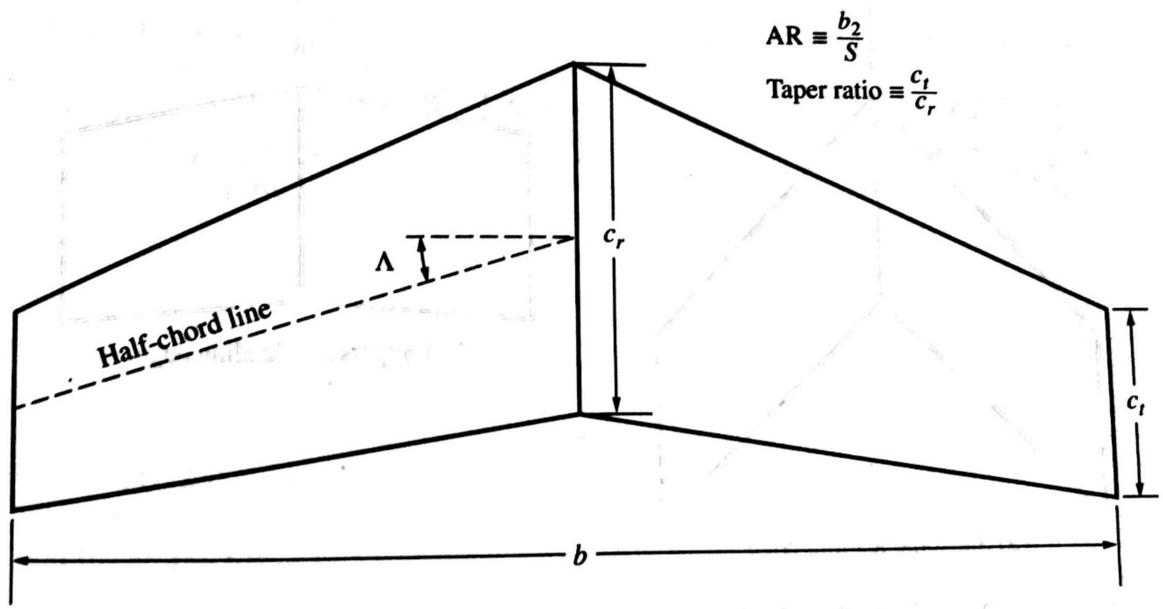


Figure 2.29 Swept-wing geometry.

Lift for a finite wing

Lift curve slope for an infinite swept wing according to Kuchemann:

$$a_{swept} = a_o \cos\Lambda .$$

Using this in Helmbold's equation:

$$a = \frac{a_o \cos\Lambda}{\sqrt{1+[(a_o \cos\Lambda)/(\pi AR)]^2}+(a_o \cos\Lambda)/(\pi AR)}$$; lift curve slope for swept wing in incompressible flow ($M \leq 0.3$).

Subsonic compressibility effects is added by noticing that $M_n = M\cos\Lambda$:

$$a = \frac{a_o \cos\Lambda}{\sqrt{1-M^2 cos^2\Lambda+[(a_o \cos\Lambda)/(\pi AR)]^2}+(a_o \cos\Lambda)/(\pi AR)}$$; lift curve slope for a swept wing in compressible flow ($0 \leq M \leq 0.7$).

Lift for a finite wing

Delta Wings:

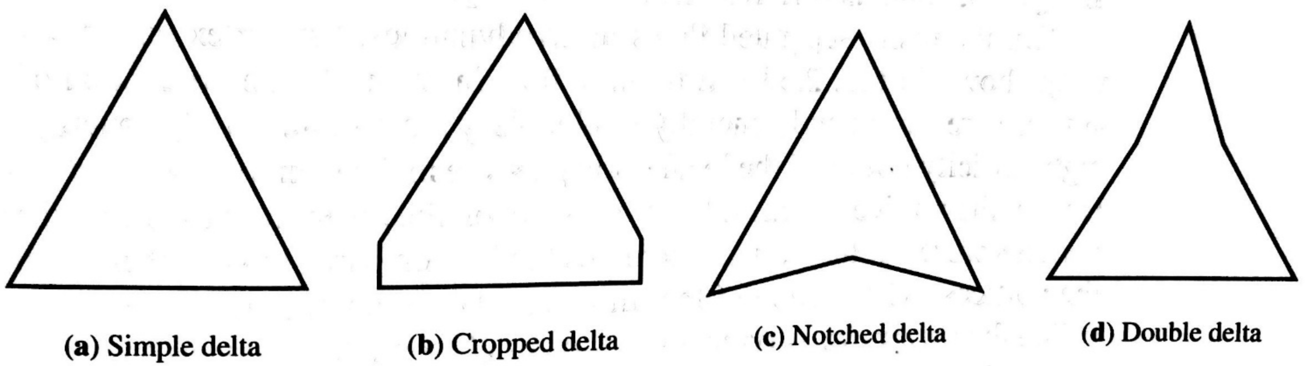


Figure 2.32 Four versions of a delta-wing planform. (After Loftin, Ref. 13.)

Lift for a finite wing

The flowfield over a low aspect ratio delta wing at low speeds is very different from that of a straight wing or a high aspect ratio wing.

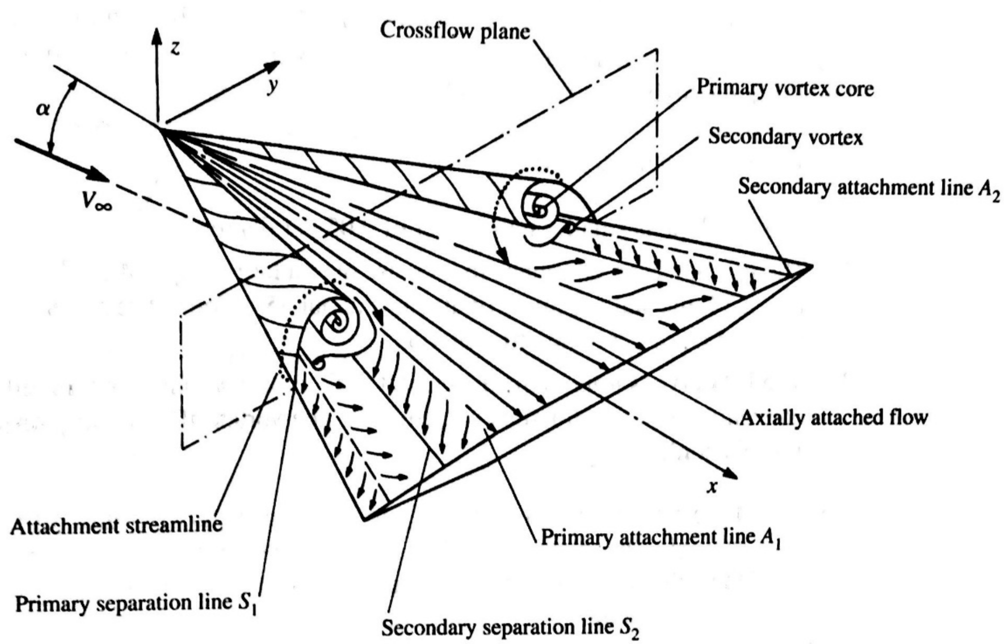
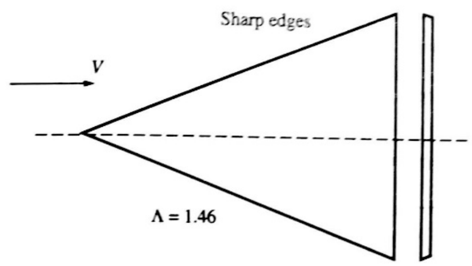


Figure 2.33 Schematic of the subsonic flow over the top of a delta wing at angle of attack. (Courtesy of John Stollery, Cranfield Institute of Technology, England.)

Lift for a finite wing



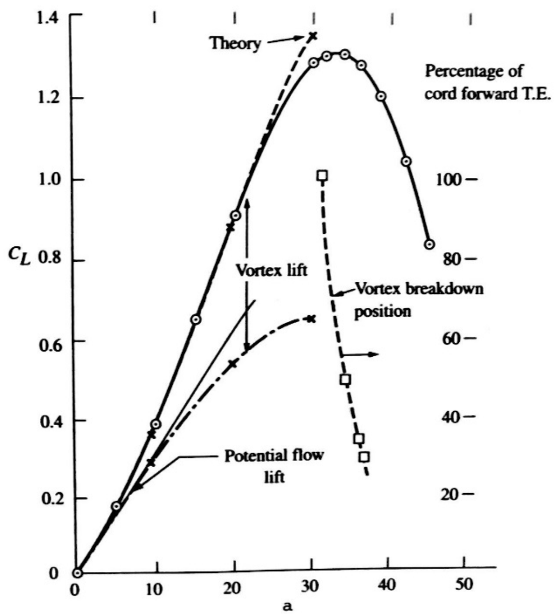


Figure 2.34 Lift coefficient curve for a delta wing in low-speed subsonic flow. (After Hoerner and Borst, Ref. 18.)

Lift for a finite wing

Characteristics of lift of low aspect ratio delta wings:

- The lift curve slope is small ($\approx 0.05/\mathrm{deg}$).
- Lift continues to increase over a large range of angle of attack.

$$\text{Stall angle} \approx 35^o, C_{L,max} = 1.35.$$

- Lift curve is non-linear because of the vortex effect.
- Lift for a delta wing in low speed flow (Smith):

$$\frac{C_N}{(s/l)^2} = 2\pi\left(\frac{\alpha}{s/l}\right) + 4.9\left(\frac{\alpha}{s/l}\right)^{1.7}$$

s: semi span of the delta wing

l: root chord of the delta wing

Examples of wing types

i. High aspect ratio straight wings: almost all general aviation aircraft, all aircraft before jet age (WWII fighters, transport a/c).

ii. Low aspect ratio straight wings: supersonic (jet) fighters.

iii. Swept wings: (transonic) jet transport a/c, (supersonic) jet fighters.

iv. Delta wings: (supersonic) jets fighters, (supersonic) transport a/c.

Examples of wing types

i. High aspect ratio straight wings

Examples of wing types

ii. Low aspect ratio straight wings

Examples of wing types

iii. Swept wings

Examples of wing types

iv. Delta wings

Drag

Lift must be produced with high efficiency $\Rightarrow$ L/D must be high.

There are two types of drag:

i. Pressure drag: is due to the net effect of pressure distribution in the drag direction.

ii. Friction drag: is due to the net effect of shear stress distribution acting in the drag direction.

Although the lift is mainly produced by the wing only, drag is produced by every component of the airplane that is in contact with the air flowing around it $\Rightarrow$ prediction of drag is much more difficult compared to the prediction of lift, there are no analytical formulae or general methods.

Physical nature of drag and its prediction is influenced by the Mach number.

Drag

Drag of an airfoil:

- Drag coefficient of an airfoil: profile drag coefficient
- Profile drag = skin friction drag + pressure drag due to flow seperation
- For an inviscid flow with to separation, (pressure) drag is zero on an airfoil $\Rightarrow$ **d'Alembert's paradox.**
- Pressure drag due to flow separation: form drag

$$c_d = c_{d,f} + c_{d,p}$$

$c_{d,p}$ → form drag coefficient

$c_{d,f}$ → skin friction drag coefficient

c_d → profile drag coefficient

Drag

Drag of finite wings:

- Subsonic drag on a finite wing = profile drag + induced drag (vortex drag or drag due to lift)
- Induced drag is due to wingtip vortices and is essentially a pressure drag.

$C_{Di} = \frac{D_i}{q_\infty S} = \frac{C_L^2}{\pi e AR}$; drag due to lift (lift doesn't come for free!)

$e = \frac{1}{1+\delta}$; Oswald span efficiency factor, dependent on the aspect ratio and taper ratio.

$e = 1$ for an elliptical wing

$\delta \approx 0.05$ for most combinations

Drag

Oswald span efficiency factor

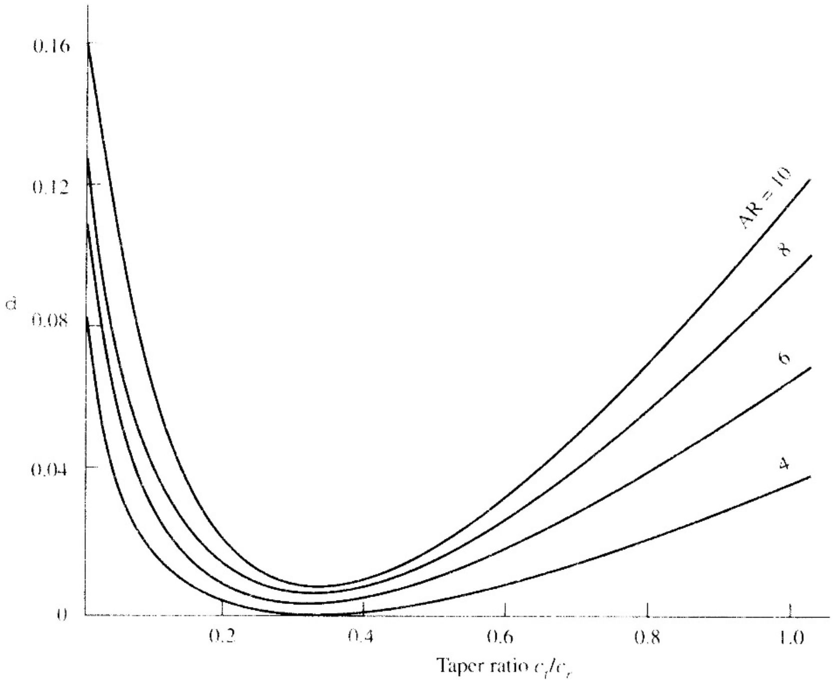


Figure 2.39 Induced drag factor as a function of taper ratio for wings of different aspect ratios.

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Drag

- Trying to design a wing with elliptical planform is not very feasible due to difficulty in manufacturing.
- Since the role of AR is much stronger, a sufficiently high AR wing is more feasible.
- However, the wing structural weight also increases with AR in order to support a long and narrow wing.

Drag

Drag of fuselages:

- Drag of a fuselage = skin friction drag + pressure drag due to flow separation
- Skin friction drag is a function of the wetted surface area, S_{wet}.
- Drag of a fuselage + wing ≠ drag of fuselage alone + drag of wing alone

The difference is due to **interference drag**.

Drag of fuselages are estimated by making correlations with experimental data since no analytical methods or general methods exists.

Drag

Summary:

- **Skin friction drag:** drag due to **shear stress distribution** over the body.
- **Form drag (**pressure drag due to flow seperation): the drag due to pressure imbalance in the direction of drag caused by seperated flow.
- **Profile drag:** sum of pressure drag and skin friction drag for airfoils.
- **Interference drag:** an additional pressure drag caused by the interaction of flow fields around each component of the airplane.
- **Parasite drag:** profile drag for the entire airplane. Includes interference drag.

Drag

Summary:

- **Induced drag:** pressure drag due to wingtip vortices.
- **Zero-lift drag:** parasite drag when the airplane is at zero lift angle of attack. Usually a term used for the complete airplane.
- **Drag due to lift:** drag measured above the zero-lift drag. A term used for the complete airplane.

The drag polar

Zero lift drag: $D_o = C_{fe} q_\infty S_{wet}$

Zero lift drag coefficient: $C_{Do} = \frac{D_o}{q_\infty S}$

Combining the two equations:

$$C_{Do} = \frac{q_\infty S_{wet} C_{fe}}{q_\infty S} = \frac{S_{wet}}{S} C_{fe}$$

$\frac{S_{wet}}{S}$ is between 2-8 for most airplanes.

The drag polar

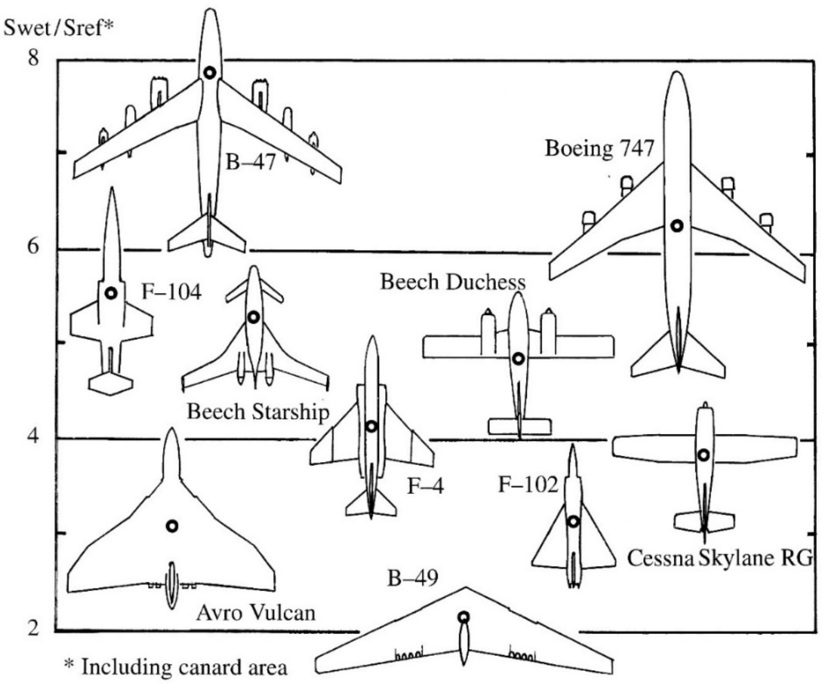

Fig. 3.5 Wetted area ratios.

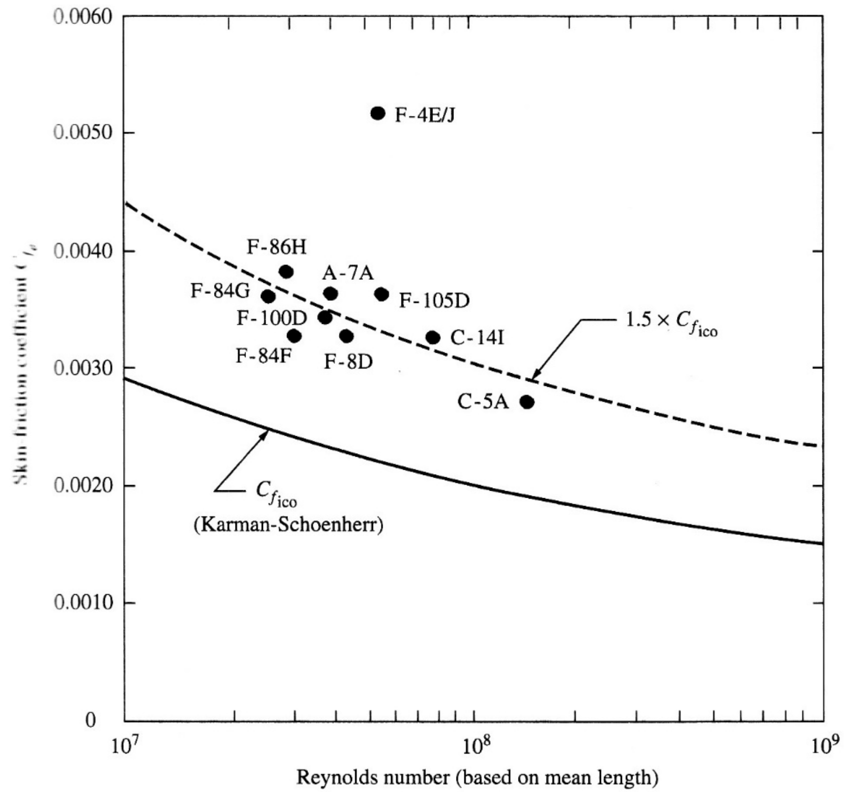

Figure 2.55 Equivalent skin-friction drag for a variety of airplanes. (After Jobe, Ref. 27.)

The drag polar

Total drag = parasite drag + induced drag

$$C_D = C_{Do} + KC_L^2 = C_{Do} + \frac{1}{\pi eAR} C_L^2 \text{ for subsonic flow.}$$

Above formulation assumes that the drag is minimum at $C_L = 0$. This may not always be the case.

An alternative drag polar expression:

$$C_D = C_{D,min} + K\left(C_L - C_{L,\mathrm{min}\,drag}\right)^2$$

For airplanes having wings with low or moderate camber $C_{Do} \approx C_{D,min}$ and $C_{L,\mathrm{min}\,drag} \approx 0$.

The drag polar

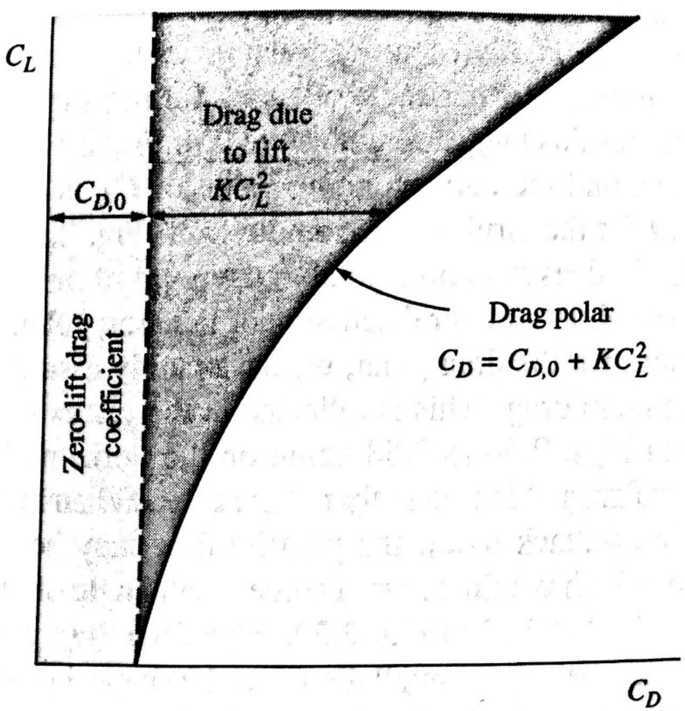

Figure 2.56 Schematic of the components of the drag polar.

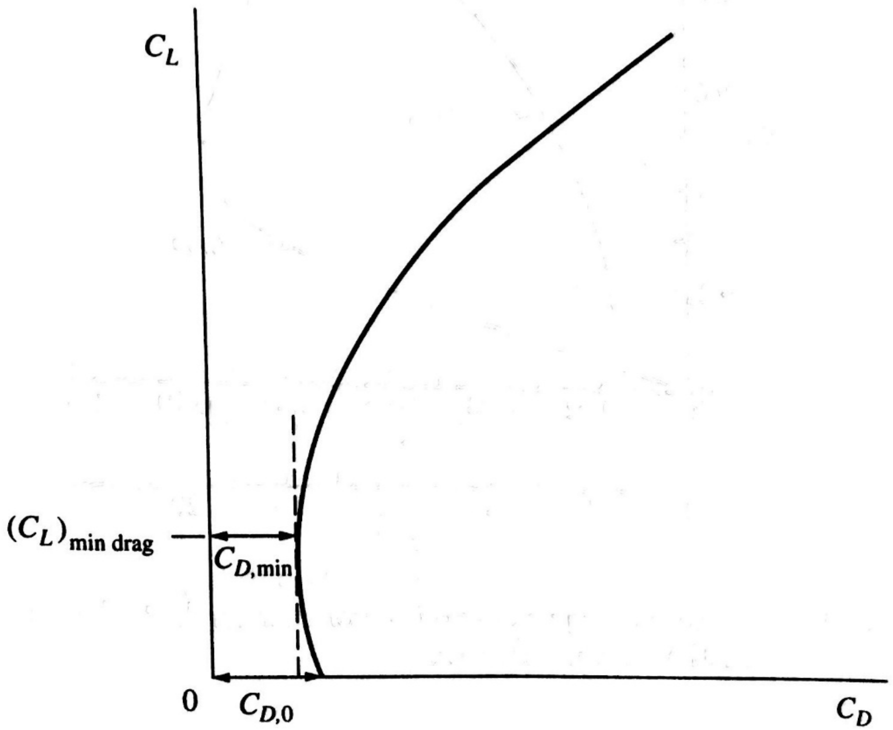

Figure 2.59 Illustration of minimum drag and drag at zero lift.