Path Optimization of Dual Airfoils Flapping in a Biplane Configuration with RSM in a Parallel Computing Environment

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Summary. The path of dual airfoils in a biplane configuration undergoing a combined, non–sinusoidal pitching and plunging motion is optimized for maximum thrust and/or propulsive efficiency. The non–sinusoidal, periodic flapping motion is described using Non-Uniform Rational B-Splines (NURBS). The Response Surface Methodology (RSM) is employed for the optimization of NURBS parameters in a parallel computing environment. A gradient based optimization algorithm, steepest ascent method is started from the optimum point of response surfaces. Unsteady, low speed laminar flows are also computed in parallel using a Navier-Stokes solver based on domain decomposition. It is shown that the parallel optimization process with RSM suggests a quick and accurate initial guess for a gradient based optimization algorithm.

1 INTRODUCTION

Flow characteristics of flapping wings are currently investigated experimentally and numerically to shed some light on the lift, drag and propulsive power considerations for a MAV flight(1; 2). It should be noted that in order to maximize the thrust and/or the propulsive efficiency of a flapping airfoil, its kinematic parameters, such as the flapping path, the frequency and the amplitude of the flapping motion, need to be optimized.

In earlier studies, the present authors employed a gradient based optimization of sinusoidal and non–sinusoidal flapping motion parameters of flapping airfoils(28; 30; 31). These optimization studies with a limited number of optimization variables show that the thrust generation and efficiency of flapping airfoils may be increased significantly. However, the gradient based global optimization process becomes computationally expensive as the number of optimization variables increases in the non–sinusoidal flapping motion definition with NURBS.

Response surface methodology (RSM) is mainly employed for the construction of global approximations to a function based on its values computed at

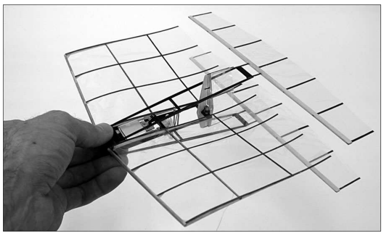

Fig. 1. Flapping-wing MAV model (Jones and Platzer)

various points(13). The method may also be employed for the optimization of a function when the objective function is expensive in terms of computational resources(13; 15; 14; 16).

In the present study, the thrust generation of dual airfoils flapping in a biplane configuration undergoing a combined non–sinusoidal pitching and plunging motion is optimized using RSM. First, a single airfoil undergoing a non-sinusoidally flapping motion is considered. RSM for 3 optimization variables is assessed and optimization data are compared to the gradient based optimization method in terms of the performance and the accuracy. Next, the non–sinusoidal flapping motion of dual airfoils with seven optimizationvariables is considered.

2 Response Surface Methodology, RSM

RSM is based on building approximate models for unknown functional relationships between input and output data. In this study, the function is the average thrust coefficient, C_t, which is based on the integration of the drag coefficient over a flapping period. It is a function of flapping parameters, V_i, in a given flight condition and can be written as

$$C_t = \eta(V_1, V_2, V_3, \dots) \tag{1}$$

The function $\eta(\mathbf{V})$ is in fact the solution of the Navier-Stokes equations. An approximate response surface, $g(\mathbf{V}) \cong \eta(\mathbf{V})$ may then be constructed over some $\mathbf{V}$ region (13). In this study, $g(\mathbf{V})$, is chosen to be a quadratic function of V_i's:

$$g(V_i) = a_{11}V_1^2 + 2a_{12}V_1V_2 + 2a_{13}V_1V_3 + \cdots + a_{22}V_2^2 + \dots \tag{2}$$

The constants, a_{ij}, are evaluated throught a least-square minimization of the error between the response surface and a certain number of the Navier-Stokes solutions based on a design of experiment (DoE). In this study, the Box-Behnken(17) DoE method is employed.

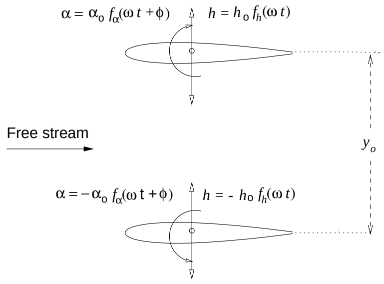

Fig. 2. Out-of-phase flapping motion of two airfoils in a biplane configuration

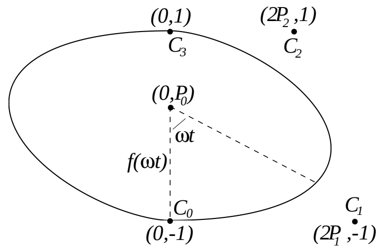

Fig. 3. Flapping path defined by a 3^{rd} degree NURBS

3 Periodic Path defined by NURBS

A smooth curve S based on a general n^{th} degree rational Bezier segment is defined as follows(33):

$$S(u) = (x(u), y(u)) = \frac{\sum_{i=0}^{n} W_i B_{i,n}(u) C_i}{\sum_{i=0}^{n} W_i B_{i,n}(u)} \qquad 0 \le u \le 1 \qquad (3)$$

where $B_{i,n}(u) \equiv \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$ are the classical n^{th} degree Bernstein polynomials, and $C_i = (x_{pi}, y_{pi})$, are called control points with weights, W_i. Note that $S(u = 0) = C_0$ and $S(u = 1) = C_n$. A closed curve which describes the upstroke and the downstroke of a flapping path is then defined by employing a NURBS composed of two 3^{rd} degree rational Bezier segments. The periodic flapping motion is finally defined by 3 parameters. The first parameter P_0 defines the center of the rotation vector on a closed curve. The remaining two points, P_1 and P_2 are used to define the x coordinates of the control points, which are $C_1 = (2P_1, -1)$ and $C_2 = (2P_2, 1)$ (Figure 3).

The x and y coordinates on the periodic NURBS curve may be obtained as a function of the parameter u:

$$x(u) = \frac{2P_1 u(1-u)^2 + 2P_2 u^2(1-u)}{2u^2 - 2u + 1} \qquad y(u) = \frac{2u - 1}{2u^2 - 2u + 1} \qquad (4)$$

A non-sinusoidal periodic function, f, is then defined by $y(u)$. For a given ωt position, the Equation 5 is solved for u. Once u is determined, $y(u) \equiv f(\omega t)$ is evaluated using Equation 4.

$$\tan(\omega t) = -\frac{x(u)}{y(u) - P_0} \qquad (5)$$

4 Numerical Method

Unsteady, viscous flowfields around flapping airfoils in a biplane configuration are computed by solving the Navier-Stokes equations on moving and

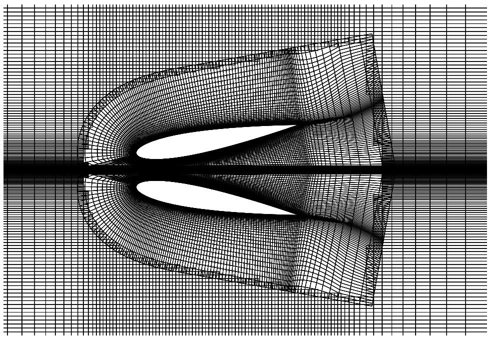

Fig. 4. Moving and deforming overset grid system

deforming overset grids. A domain decomposition based parallel computing algorithm is employed. `PVM` message passing library routines are used in the parallel solution algorithm. The computed unsteady flowfields are analyzed in terms of time histories of aerodynamic loads, and unsteady particle traces.

4.1 Flow Solver

The strong conservation-law form of the 2-D, thin-layer, Reynolds averaged Navier-Stokes equations is solved on each subgrid. The convective fluxes are evaluated using Osher's third–order accurate upwind–biased flux difference splitting scheme. The discretized equations are solved in parallel by an approximately factored, implicit algorithm. The overset grid system (Figure 4) is partitioned into subgrids. The holes in the background grid are excluded from the computations by an *i-blanking* algorithm. The conservative flow variables are interpolated at the intergrid boundaries formed by the overset grids(34).

4.2 Flapping Motion

The flapping motion of the upper airfoil in plunge, h, and in pitch, α, is defined by:

$$\begin{aligned} h(t) &= h_0\ f_h(\omega t) \\ \alpha(t) &= \alpha_0\ f_\alpha(\omega t + \phi) \end{aligned} \tag{6}$$

where h_o and α_o are the plunge and pitch amplitudes, f is a periodic function based on NURBS, ω is the angular frequency which is given in terms of the reduced frequency, $k = \frac{\omega c}{U_\infty}$. ϕ is the phase shift between plunge and pitching motions. The pitch axis is located at the mid-chord. The flapping motion of the lower airfoil is in counter-phase. The average distance between the dual airfoils is set to $y_0 = 1.4$.

The flapping motion of the airfoils are imposed by moving the airfoil grids over the background grid (Figure 4). The airfoil grids are deformed as they come close to the symmetry line between the airfoils.

4.3 Optimization based on the Steepest Ascent Method

Optimization based on the Steepest Ascent is also performed for validation. The gradient vector of the objective function is given by

$$\nabla O(\mathbf{V}) = \frac{\partial O}{\partial V_1}\mathbf{v_1} + \frac{\partial O}{\partial V_2}\mathbf{v_2} + \cdots$$

where V_i's are the optimization variables. The components of the gradient vector are evaluated numerically by computing an unsteady flow solution for a perturbation of the optimization variables one at a time.

4.4 Parallel Processing

The parallel solution algorithm is based on the domain decomposition. The moving and deforming overset grid system is decomposed into its subgrids first, and the solution on each subgrid is computed in parallel. Intergrid and overlapping boundary conditions are exchanged among subgrid solutions at each time step of the unsteady solution. The unsteady flow solutions needed for the RSM and the gradient vector components, are also carried out in parallel. `PVM` (version 3.4.5) library routines are used for inter–process communication. Computations are performed in a cluster of Linux based computers with dual Xeon and Pentium-D processors.

5 Results and Discussion

The unsteady, laminar flow solutions over the flapping airfoils are obtained by a Navier-Stokes solver in a parallel computing environment. The flowfields are computed at a low Mach number of 0.1 and a Reynolds number of 10000.

5.1 Validation Study

The optimization of a single airfoil flapping on a non-sinusoidal path is studied first. The flapping motion is a combination of non-sinusoidal pitching and sinusoidal plunging. The values for the reduced flapping frequency, $k \equiv \frac{\omega c}{U_\infty}$, and the plunge amplitude, h_0, are fixed at $k = 1.0$ and $h_0 = 0.5$. The optimization variables for this case are the pitch amplitude, α_0, the phase shift between plunging and pitching, ϕ, and the NURBS parameter, $P_{0\alpha}$ (Table 1).

Table 4 gives the design points used for constructing the response surface. Design points are chosen based on Box-Behnken matrix of runs(17). The P_1 and P_2 values are constrained within the range 0.2 to 5.0, and P_0 in the range -0.9 to 0.9, in order to define a proper flapping motion which does not impose excessively large accelerations.

Table 1. Fixed parameters and optimization variables in validation study

Case	k	h_0	$P_{1\alpha}$	$P_{2\alpha}$	P_{0h}	P_{1h}	P_{2h}	$P_{0\alpha}$	α_0	ϕ
	1.0	0.5	1.0	1.0	0.0	1.0	1.0	V	V	V

Table 2. Initial conditions for steepest ascent method

Case	k	h_0	$P_{1\alpha}$	$P_{2\alpha}$	P_{0h}	P_{1h}	P_{2h}	$P_{0\alpha}$	α_0	ϕ	C_t
	1.0	0.5	1.0	1.0	0.0	1.0	1.0	**0.0**	**20°**	**90°**	0.09

Table 3. Optimization results for the validation study

Case	$\alpha_o(^o)$	$\phi(^o)$	$P_{0\alpha}$	C_t
RSM	9.3	90.6	0.03	**0.17**
Steepest Ascent	9.2	90.7	−0.01	**0.15**

Table 4. RSM design points for the validation study

DoE	$\alpha_o(^o)$	$\phi(^o)$	$P_{0\alpha}$
1	5.0	30.0	0.0
2	5.0	150.0	0.0
3	35.0	30.0	0.0
4	35.0	150.0	0.0
5	5.0	90.0	−0.9
6	5.0	90.0	0.9
7	35.0	90.0	−0.9
8	35.0	90.0	0.9
9	20.0	30.0	−0.9
10	20.0	30.0	0.9
11	20.0	150.0	−0.9
12	20.0	150.0	0.9
13	20.0	90.0	0.0

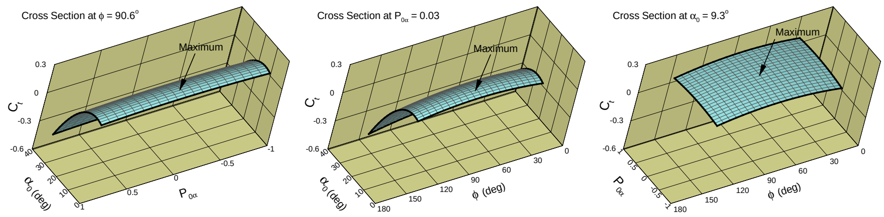

Fig. 5. Response surfaces for the validation case

The parallel computations for 13 unsteady flow solutions take about 2 hours of wall clock time using 40 processors. The cross sections of the constructed response surface are shown in Figure 5. The same optimization case is also studied using the steepest ascent method. The initial conditions required for this method are given in Table 2. The parallel computations, which required 58 unsteady flwo solutions, take about 20 hours of wall clock time using 15 processors.

Table 3 gives the optimization results based on the RSM and the steepest ascent method. It is observed that while the optimum solution is about the same in both cases, the number of unsteady flow solutions is significantly smaller in the RSM than in the steepest ascend method.

5.2 Optimization for Dual Airfoils

Dual airfoils flapping in a biplane configuration is studied next with 8 optimization variables, namely, the NURBS parameters defining the plunging path, P_{0h}, P_{1h}, P_{2h}, the NURBS parameters defining the pitching path, $P_{0\alpha}$,

Table 5. RSM design points and the computed thrust values

Desing No.	P_{0h}	P_{1h}	P_{2h}	$P_{0\alpha}$	$P_{1\alpha}$	$P_{2\alpha}$	$\alpha_0(^o)$	$\phi(^o)$	C_t
1	0.0	0.5	0.5	0.0	1.0	1.0	9.0	90.0	0.42
2	0.0	0.5	2.0	0.0	1.0	1.0	9.0	90.0	0.49
...	...	...	...	...	...	...	...	...	...
112	0.0	1.0	1.0	0.0	1.0	1.0	12.0	105.0	0.46
113	0.0	1.0	1.0	0.0	1.0	1.0	9.0	90.0	0.42

Table 6. Optimization result based on RSM and the corresponding Navier-Stokes solution

	P_{0h}	P_{1h}	P_{2h}	$P_{0\alpha}$	$P_{1\alpha}$	$P_{2\alpha}$	$\alpha(^o)$	$\phi(^o)$	C_t
RSM	−0.30	2.00	2.00	−0.30	1.13	2.00	10.1	90.1	**0.89**
Navier-Stokes	−0.30	2.00	2.00	−0.30	1.13	2.00	10.1	90.1	**0.85**

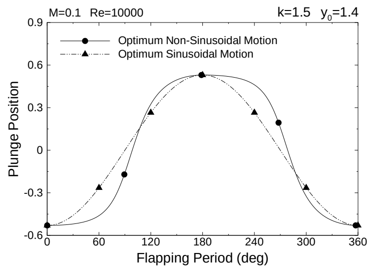

Fig. 6. Variation of optimum plunge position

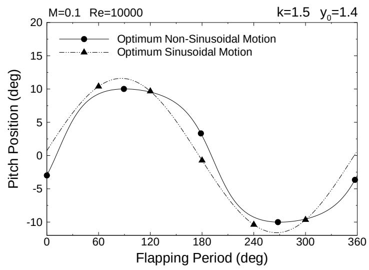

Fig. 7. Variation of optimum pitch position

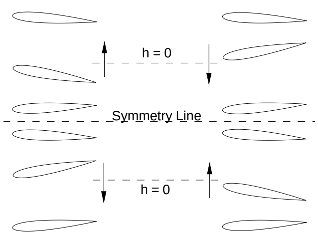

Fig. 8. Optimum non-sinusoidal flapping motion

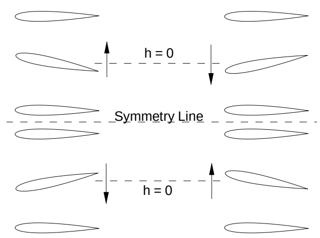

Fig. 9. Optimum sinusoidal flapping motion

$P_{1\alpha}$, $P_{2\alpha}$, the pitching amplitude, α_0 and the phase shift between plunging and pitching, ϕ. The values for the reduced flapping frequency, $k \equiv \frac{\omega c}{U_\infty}$, and the plunge amplitude, h_0, are fixed at $k = 1.5$ and $h_0 = 0.53$.

These values are from an earlier study which optimized the sinusoidal flapping motion of dual airfoils in biplane(32). At this study, the maximum thrust is computed to be $C_t = 0.45$.

Design of experiment due to Box-Behnken(17) is summarized in Table 5. A total of 113 unsteady flow solutions are computed, take about 10 hours of wall clock time using 64 processors. The response surface has 45 parameters (Eqn. 2). The optimization results are given in Table 6. The flow solution performed at the optimum conditions produces a thrust value of $C_t = 0.85$, which is about 4% off from the RSM prediction.

The optimum sinusoidal and non-sinusoidal flapping motions are given in Figures 6 6, Figure 8 and 9.

6 Conclusion

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