Applying the Wheatstone Bridge Circuit

measurement with confidence

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The relationship between the applied strain ε and the relative change of the resistance of a strain gauge

A strain gauge converts a mechanical strain into a change of an electrical resistance.

"gauge factor":

characteristic of the strain gauge and has to been checked experimentally:

$$\frac{\Delta R}{R} = k \cdot \varepsilon$$

Test of the gauge factor

Thomas Kleckers, HBM

Value of the relative change of the resistance

$\Delta R/R_0 = k \cdot \varepsilon$

$k \sim 2 \quad R_{(typ.)} = 120\Omega$ or 350Ω

$\varepsilon \longrightarrow$ for example – (strain level for transducers)

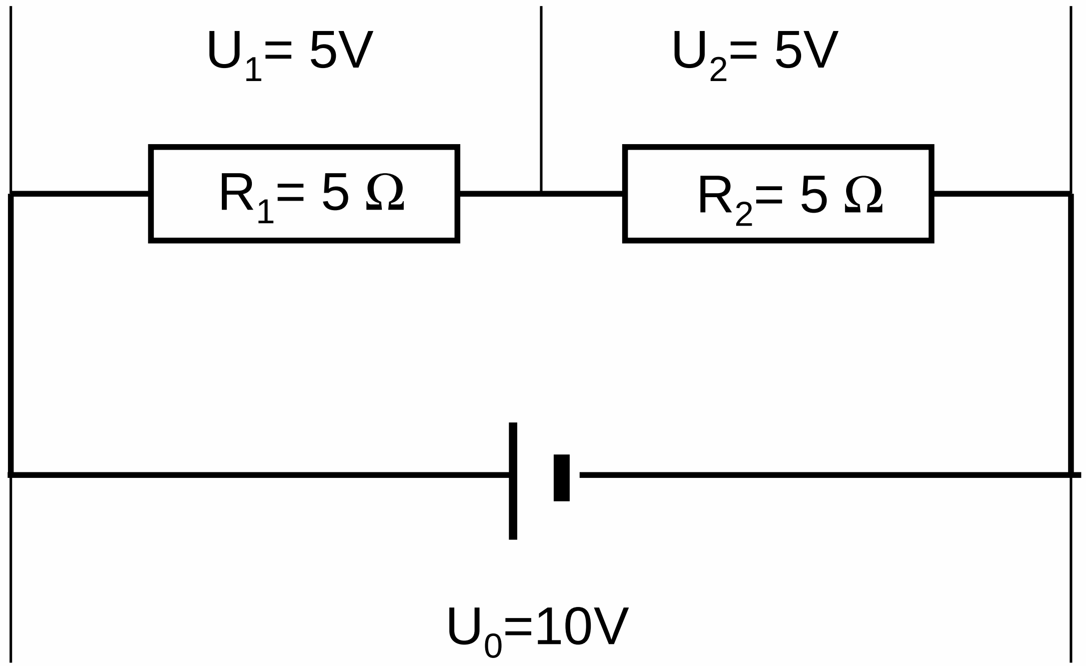
→ 1000 µm/m = 1mm/m (0,1%)

E.g.:120Ω-strain gauge at 1mm/m = 0,24Ω change of the resistance

→ Measurements with an ohmmeter are impossible

Basics

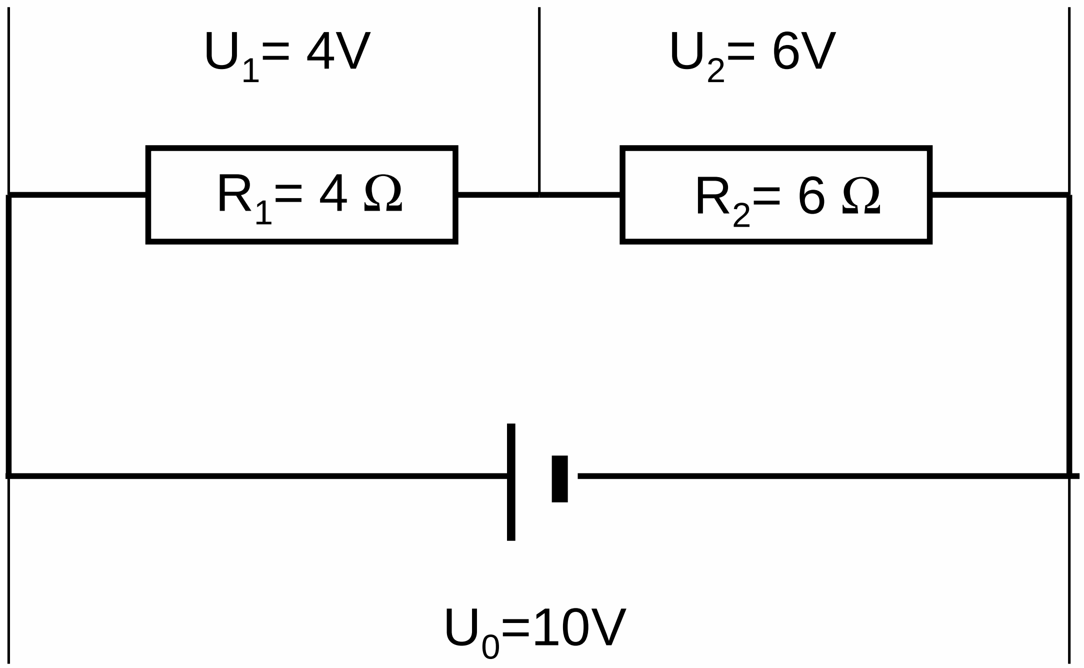
$$\frac{U_1}{U_0} = \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad U_1 = U_0 \frac{R_1}{R_1 + R_2}$$



Spannungsabfall = voltage drop

potential divider

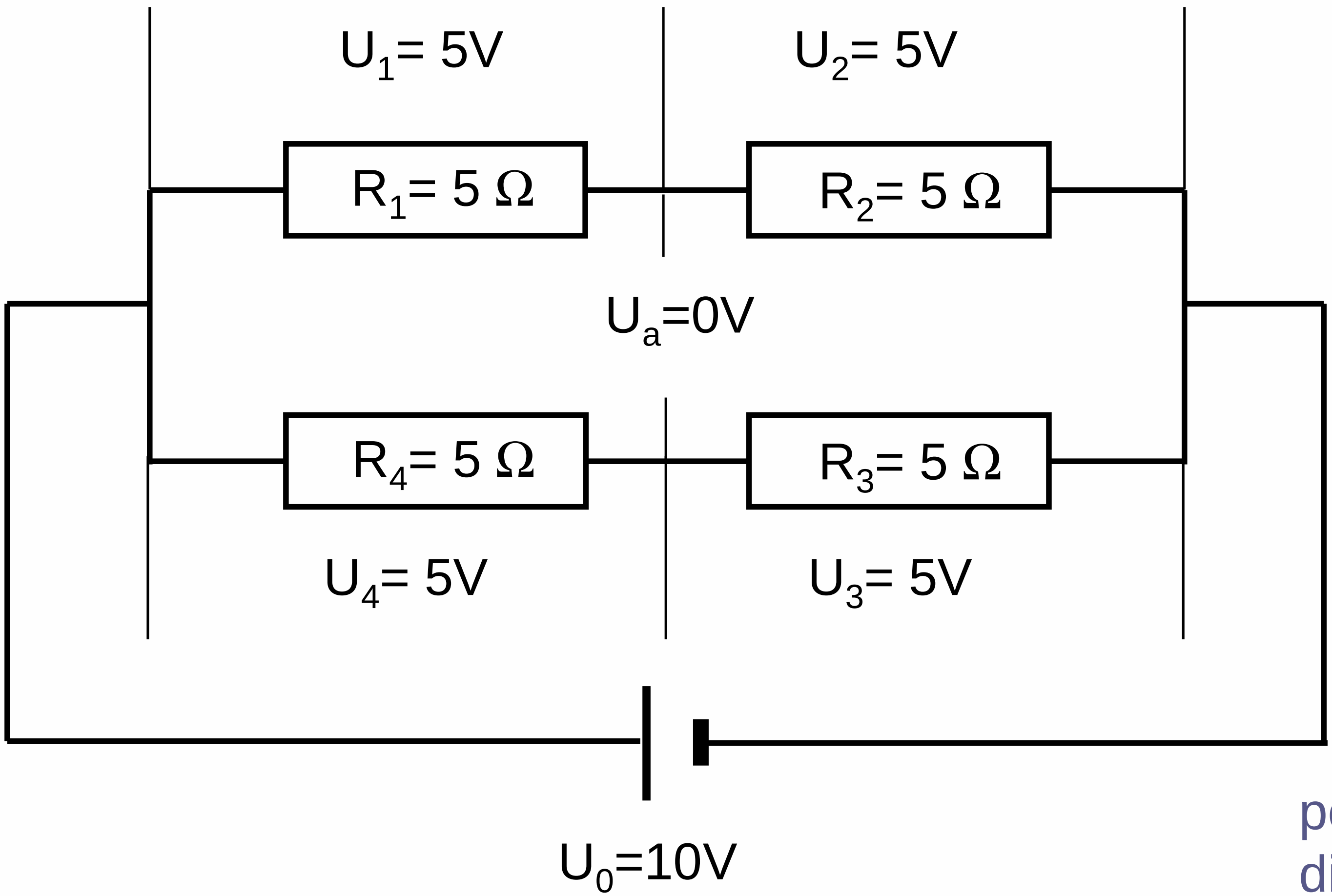
Basics



$$\frac{U_1}{U_0} = \frac{R_1}{R_1 + R_2} \Rightarrow U_1 = U_0 \frac{R_1}{R_1 + R_2}$$

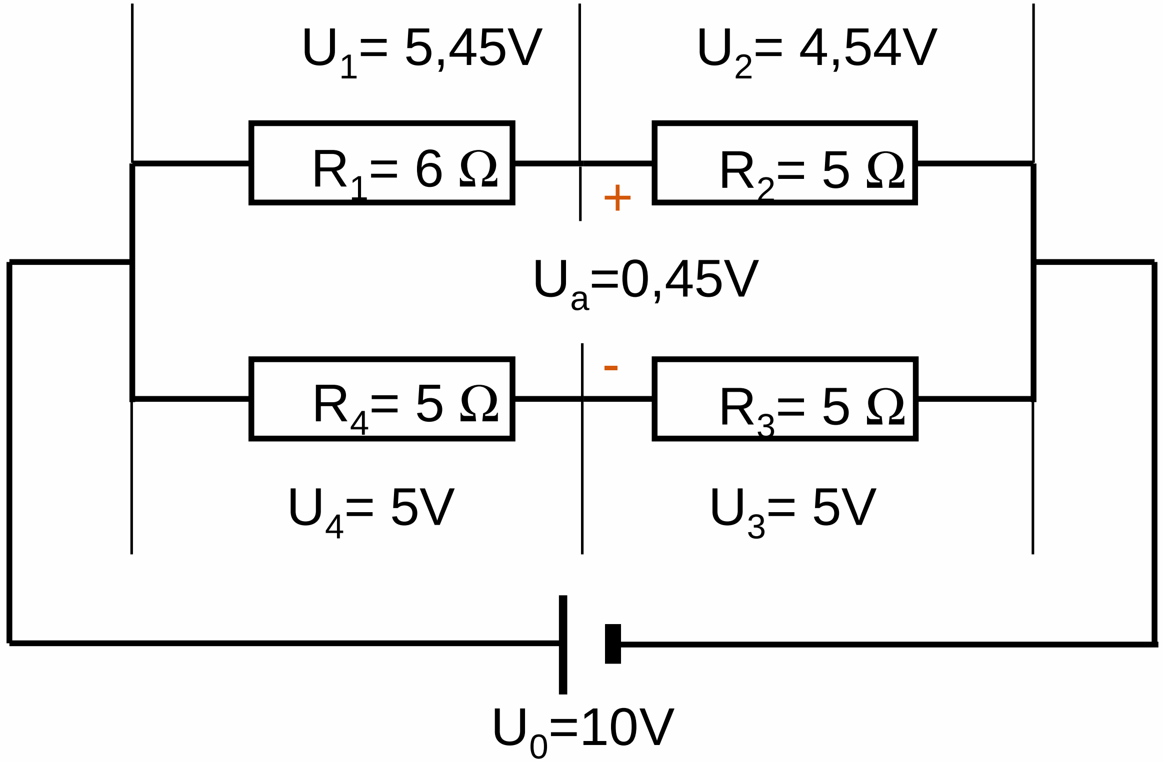
potential divider

Basics



potential divider

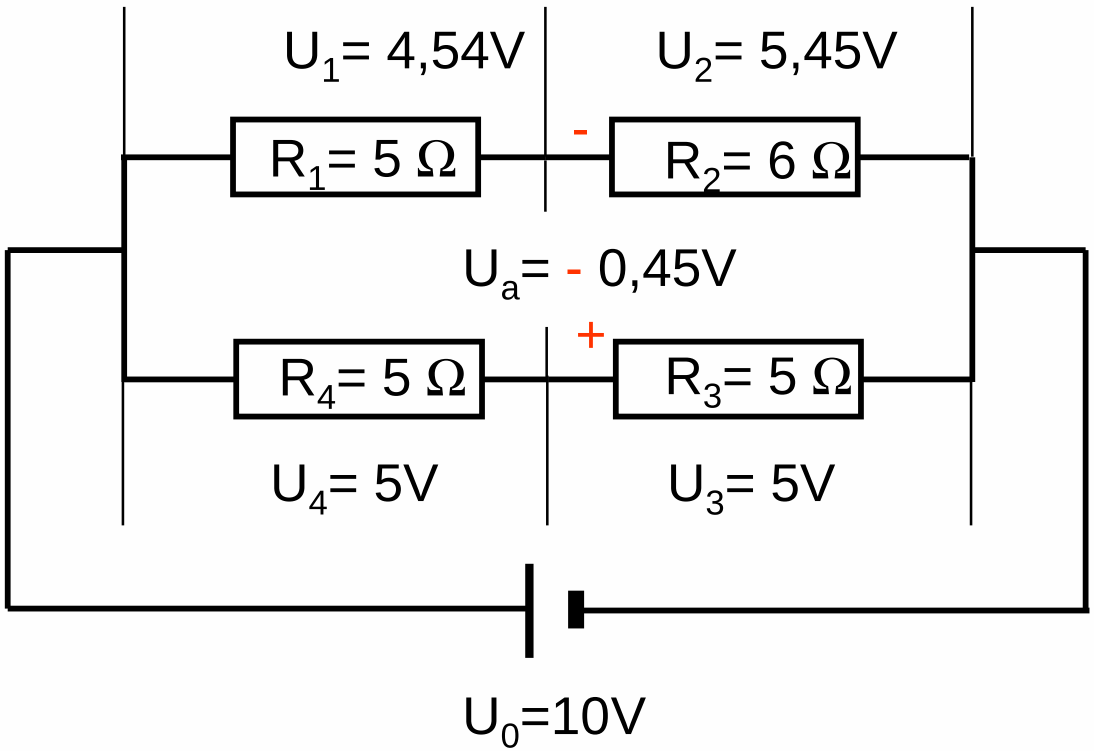
Basics



increasing of R_1 → output voltage U_a will be positive

decreasing of R_1 → output voltage U_a will be negative

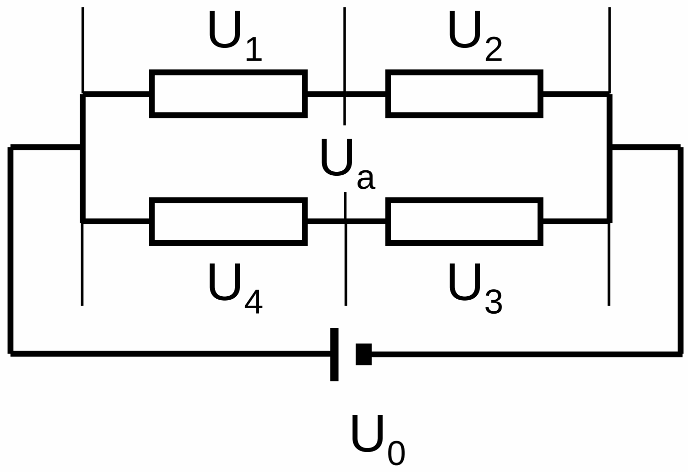
Basics



increasing of R_2 → output voltage U_a will be negative

decreasing of R_2 → output voltage U_a will be positive

Basics



$$U_1 = U_0 \frac{R_1}{R_1 + R_2}$$

$$U_4 = U_0 \frac{R_4}{R_3 + R_4}$$

$$U_a = U_0 \left(\frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

$$U_a = U_0 \left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \right)$$

Basics

$$U_a = U_0\left(\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2+\Delta R_2} - \frac{R_4+\Delta R_4}{R_3+\Delta R_3+R_4+\Delta R_4}\right)$$

the resistance changes in the strain gauges are very small :

$$\Delta R << R$$

the two halves of the bridge must have the same resistance:

$$R_1 = R_2 \text{ und } R_3 = R_4$$

Basics

$$\frac{U_a}{U_0} = \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}$$

with:

$$\frac{\Delta \mathrm{R}}{\mathrm{R}} = \mathrm{k} \cdot \varepsilon$$

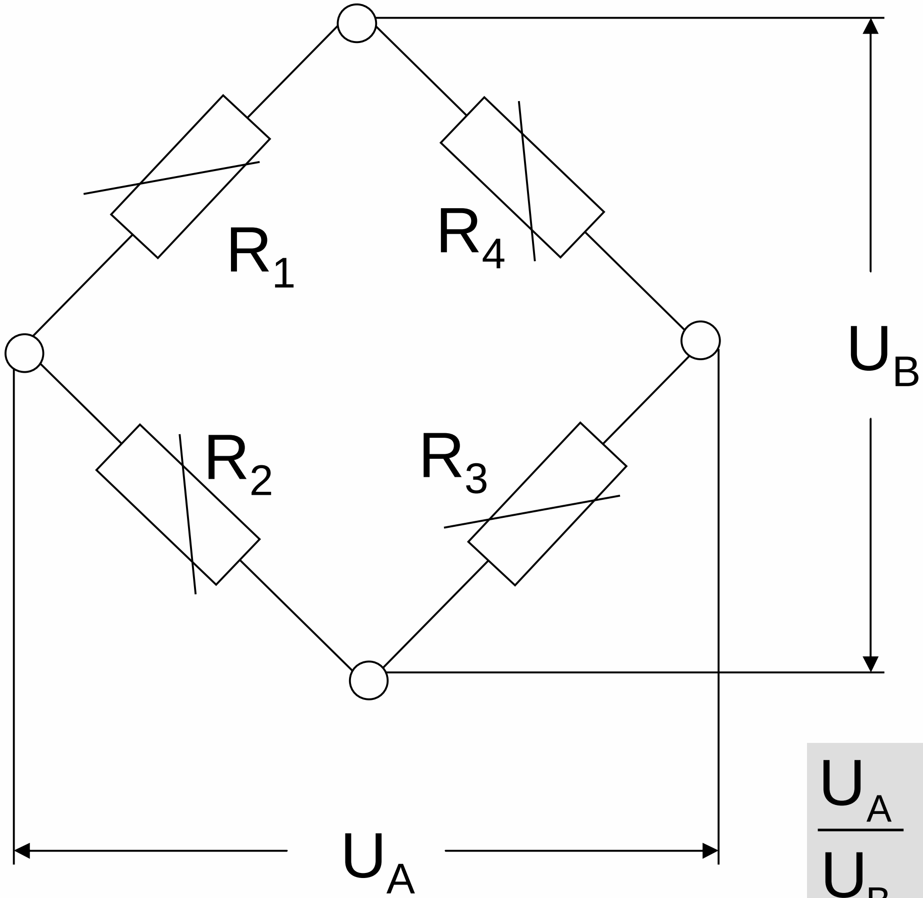
that implies:

$$\boxed{\frac{U_a}{U_0} = \frac{k}{4} \cdot \left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)}$$

Basics

- we use every time the Wheatstone Bridge Circuit with 4 resistors
- these 4 resistors could be 1, 2 or 4 strain gauges
- modern amplifiers completing “missing“ resistors in the Wheatstone Bridge Circuit
- with 4 active strain gauges we get the largest output value

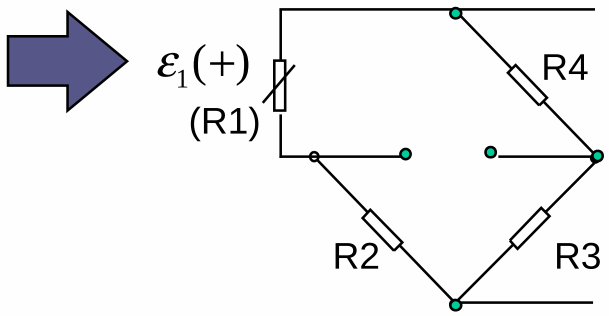
Basics

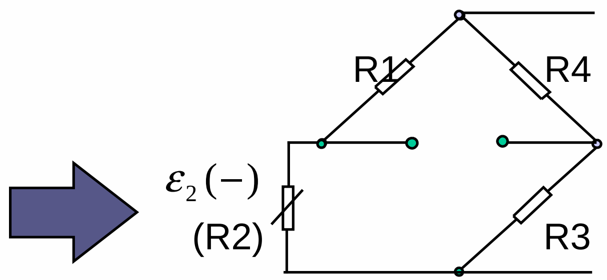


$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

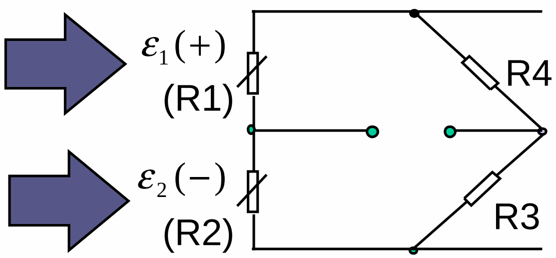
Elementary circuits with strain gauges

Quarter bridge 1 active strain gauge

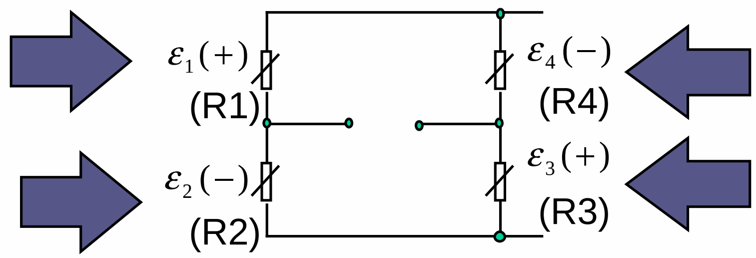




Half bridge 2 active strain gauges

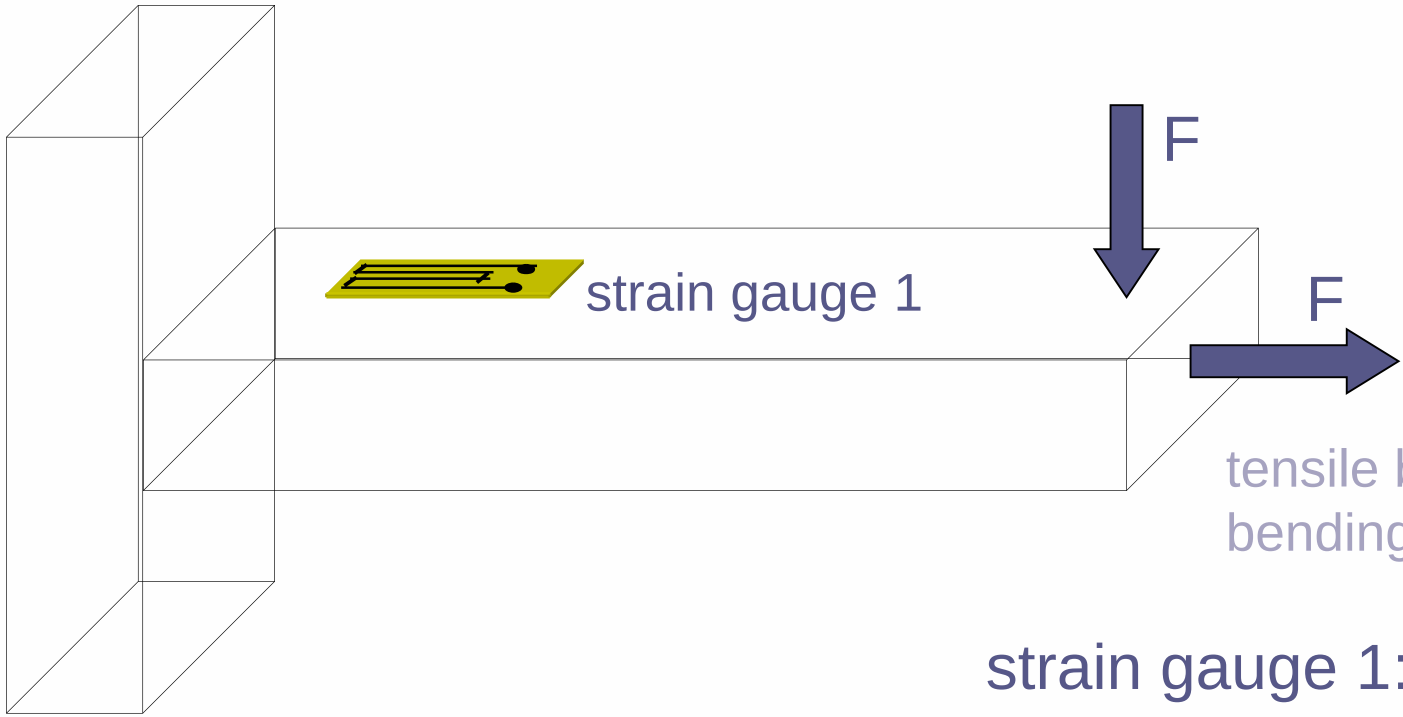


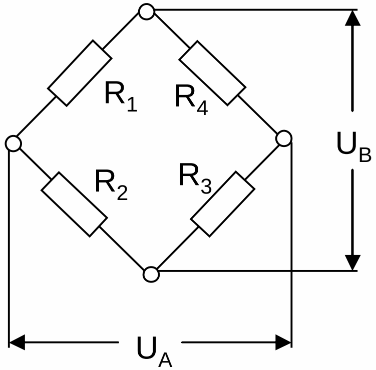
Full bridge 4 active strain gauges



Applying the Wheatstone Bridge Circuit

Measurements on a bending beam (Quarter Bridge)





tensile bar /
bending load

strain gauge 1: +

$$\frac{U_A}{U_B} = \frac{k}{4}(\varepsilon_1)$$

Temperature compensation: No!

Calibrating measurement equipment (Quarter Bridge)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

k = 2 gauge factor (written on the strain gauge package)

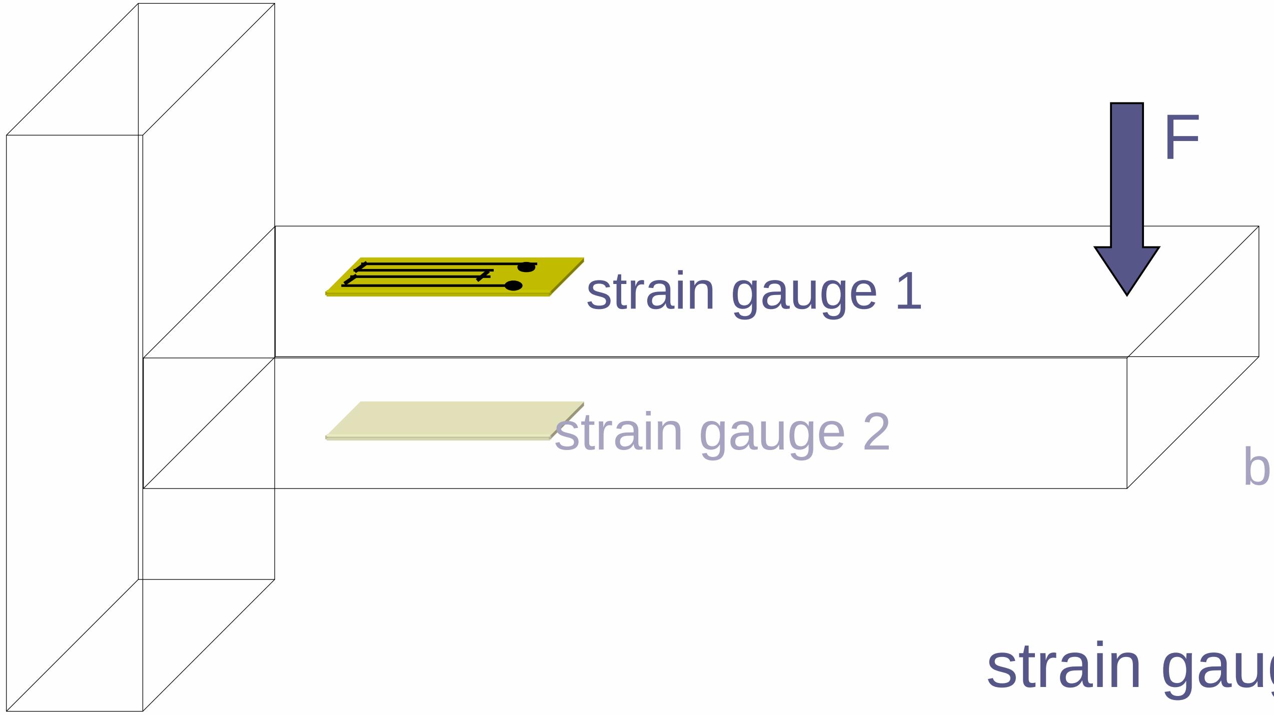
estimated strain level ε = 1000 µm/m (estimation!)

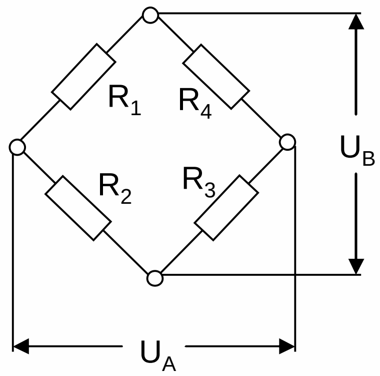
$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \cancel{\varepsilon_2} + \cancel{\varepsilon_3} - \cancel{\varepsilon_4}\right)$$

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1\right) = \frac{2}{4}(1000\mu m/m) = 0{,}5 \cdot 1000 \cdot 10^{-6} = 0{,}5 \cdot 10^{-3}$$

$$\underline{\underline{\frac{U_A}{U_B} = 0{,}5 mV/V}}$$

Measurements on a bending beam (Half Bridge)





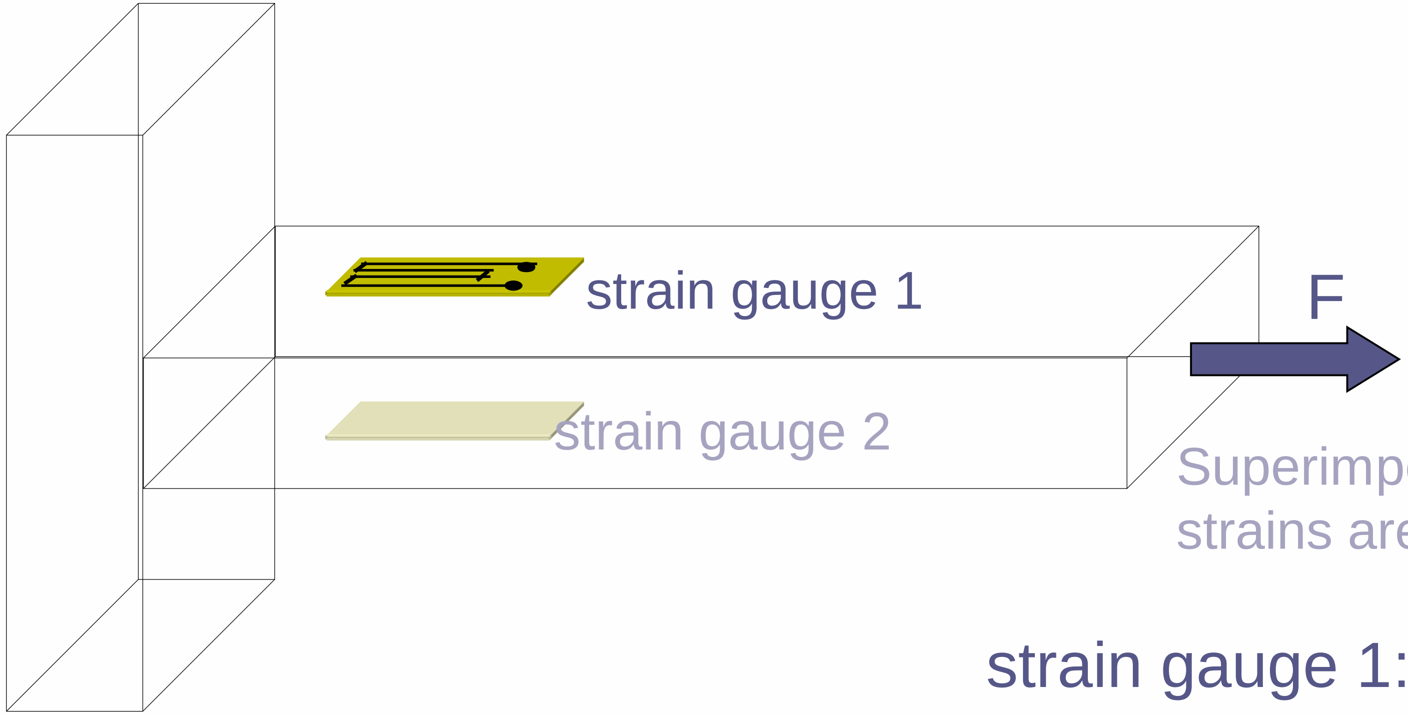
bending load

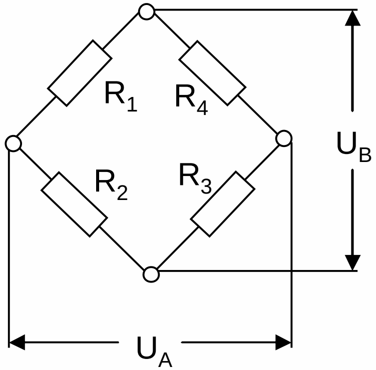
strain gauge 1: +
strain gauge 2: -

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2\right)$$

Temperature compensation: Yes!

Measurements on a bending beam (Half Bridge)





Superimposed normal strains are compensated

strain gauge 1: +
strain gauge 2: +

$$\frac{U_A}{U_B} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2)$$

Temperature compensation: Yes!

HBM

Calibrating measurement equipment (Half Bridge)

k = 2 gauge factor (written on the strain gauge package)

estimated strain level ε = 1000 µm/m (estimation!)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \cancel{\varepsilon_3} - \cancel{\varepsilon_4}\right)$$

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2\right) = \frac{2}{4}\left(1000\mu m/m - [-1000\mu m/m]\right)$$

$$\frac{U_A}{U_B} = 0{,}5 \cdot 2000 \cdot 10^{-6} = 1 \cdot 10^{-3}$$

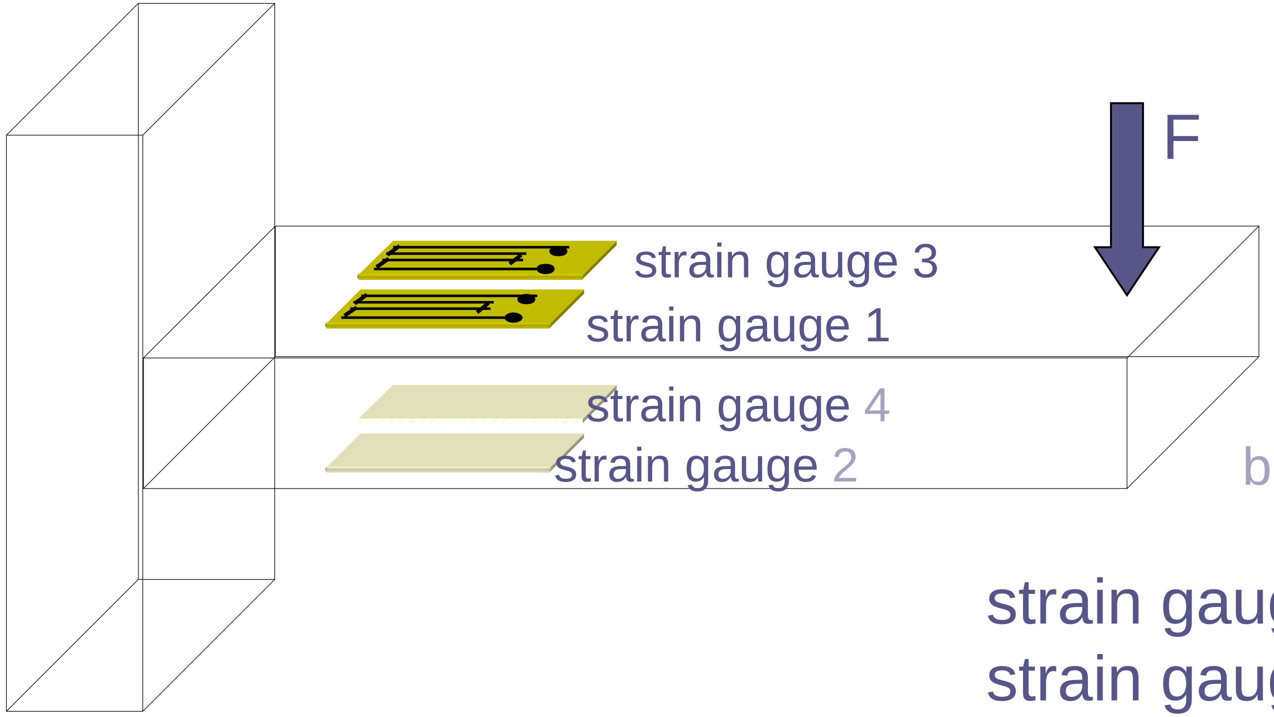
$$\underline{\underline{\frac{U_A}{U_B} = 1\,mV/V}}$$

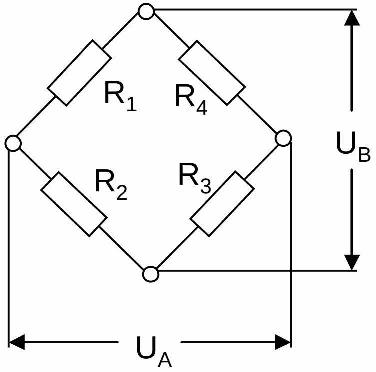
2 times the output value of a 1/4-bridge

Applying the Wheatstone Bridge Circuit

Optimisation of a wish mop

Measurements on a bending beam (Full Bridge)





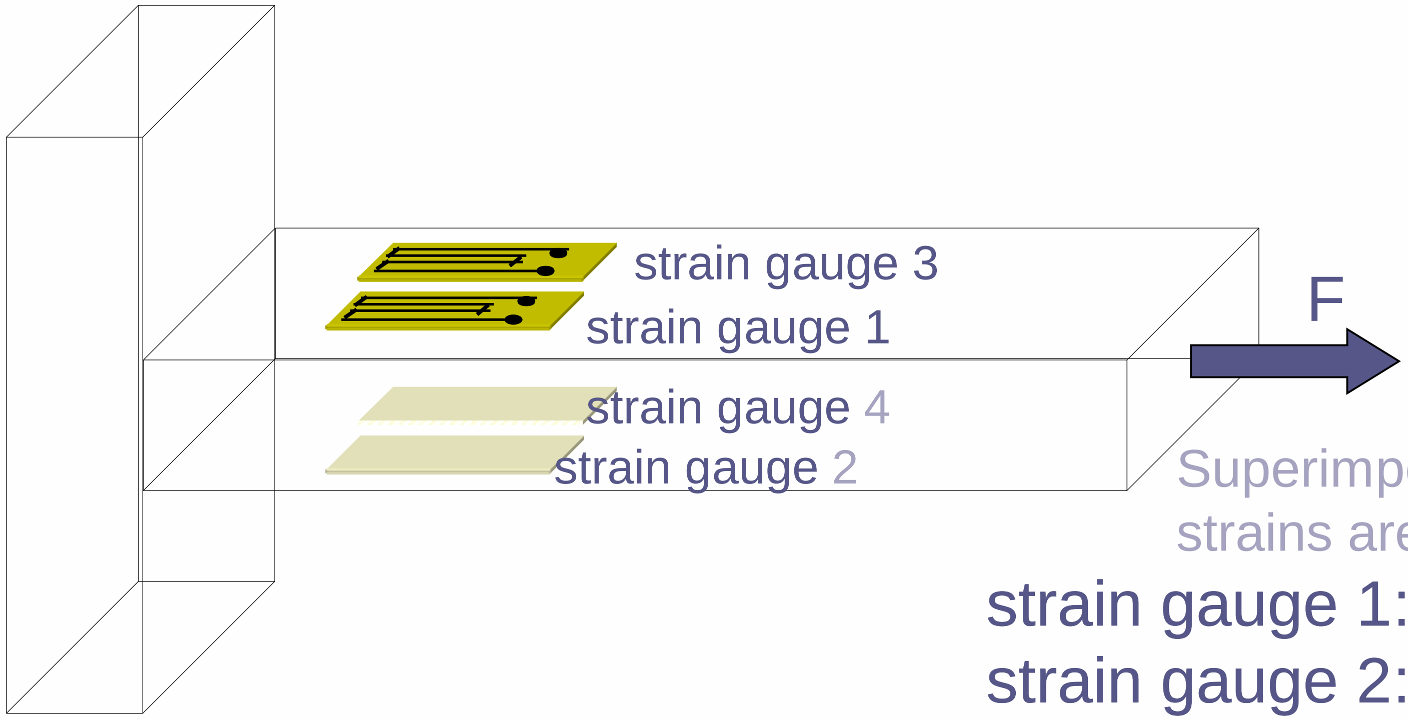
bending load

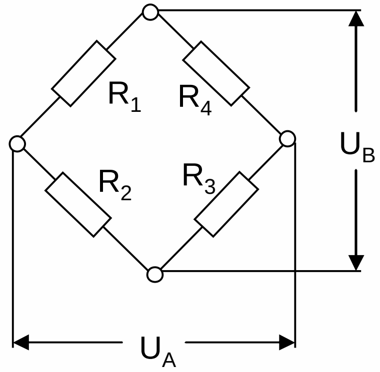
$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

strain gauge 1: +
strain gauge 2: -
strain gauge 3: +
strain gauge 4: -

Temperature compensation: Yes!

Measurements on a bending beam (Full Bridge)





Superimposed normal strains are compensated

strain gauge 1: +
strain gauge 2: +
strain gauge 3: +
strain gauge 4: +

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

Temperature compensation: Yes!

HBM

Calibrating measurement equipment (Full Bridge)

k = 2 gauge factor (written on the strain gauge package)

estimated strain level ε = 1000 µm/m (estimation!)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

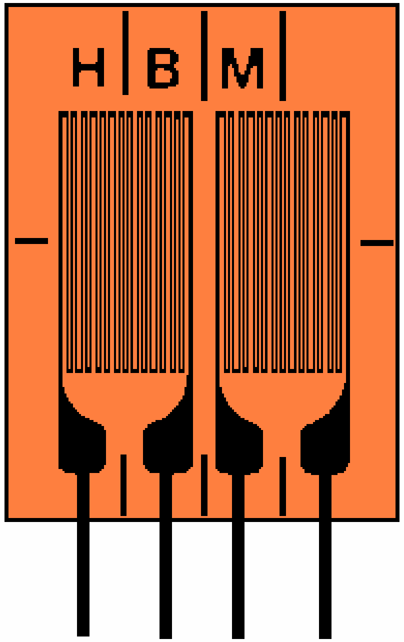
$$\frac{U_A}{U_B} = \frac{2}{4}\left(1000\mu m/m - [-1000\mu m/m] + 1000\mu m/m - [-1000\mu m/m]\right)$$

$$\frac{U_A}{U_B} = 0{,}5 \cdot 4000 \cdot 10^{-6} = 2 \cdot 10^{-3}$$

$$\underline{\underline{\frac{U_A}{U_B} = 2\,mV/V}}$$

4 times the output value of a 1/4-bridge

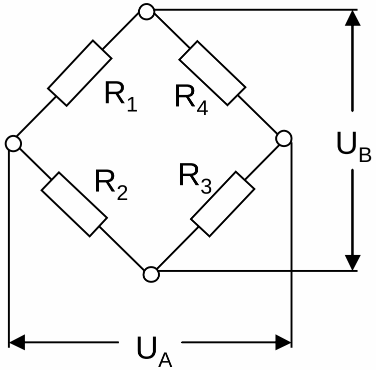
... and the right strain gauge for this application



DY11...

Measurements on a tension/compression bar

(Half Bridge)



$$-\nu = \frac{\varepsilon_q}{\varepsilon_l} \quad \Rightarrow \quad \varepsilon_q = -\nu\varepsilon_l \quad \Rightarrow \quad \varepsilon_2 = -\nu\varepsilon_1$$

ν…Poisson's ratio (steel $\approx$ 0,3)

$$\frac{U_A}{U_B} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2) = \frac{k}{4}[\varepsilon_1 - (-\nu\varepsilon_1)]$$

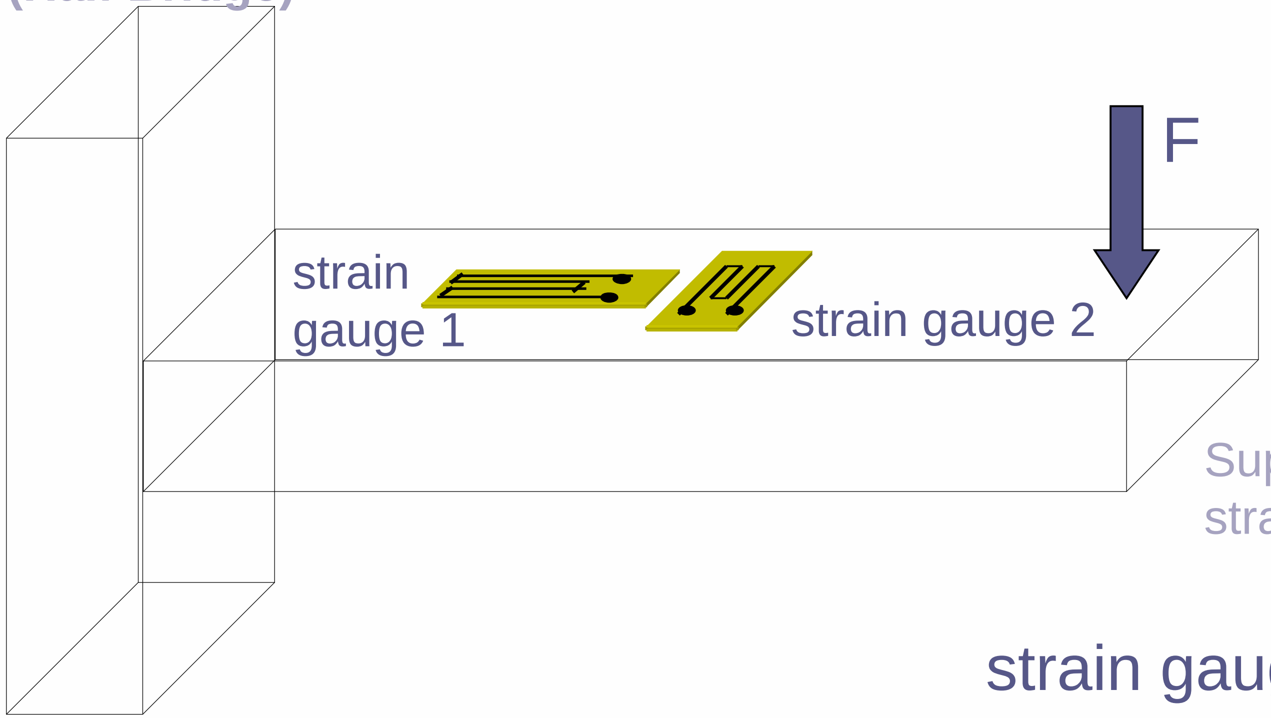
strain gauge 1: +

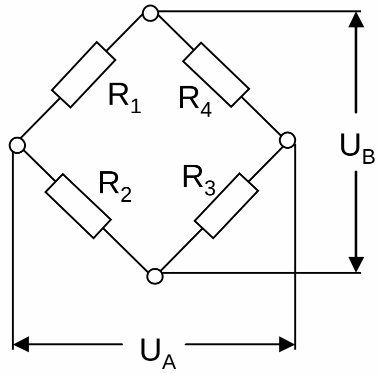
strain gauge 2: -

Temperature compensation: Yes!

Measurements on a tension/compression bar

(Half Bridge)





Superimposed bending strains occur in the result

strain gauge 1: +

strain gauge 2: -

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2\right) = \frac{k}{4}\left[\varepsilon_1 - \left(-\nu\varepsilon_1\right)\right]$$

Temperature compensation: Yes!

Calibrating measurement equipment (Half Bridge)

k = 2; ν = 0,3

estimated strain level ε = 1000 µm/m (estimation!)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \cancel{\varepsilon_3} - \cancel{\varepsilon_4}\right)$$

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2\right) = \frac{k}{4}\left[\varepsilon_1 - \left(-\nu\varepsilon_1\right)\right] = \frac{2}{4}\left(1000\mu m/m - \left[-0{,}3 \cdot 1000\mu m/m\right]\right)$$

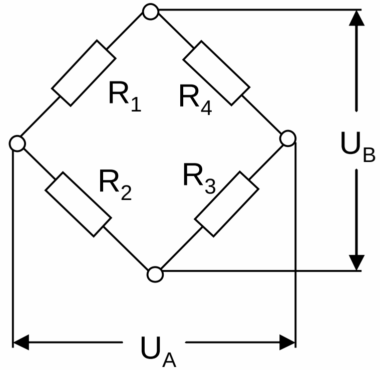
$$\frac{U_A}{U_B} = 0{,}5 \cdot \left(1000\mu m/m - \left[-300\mu m/m\right]\right) = 0{,}5 \cdot 1300 \cdot 10^{-6} = 0{,}65 \cdot 10^{-3}$$

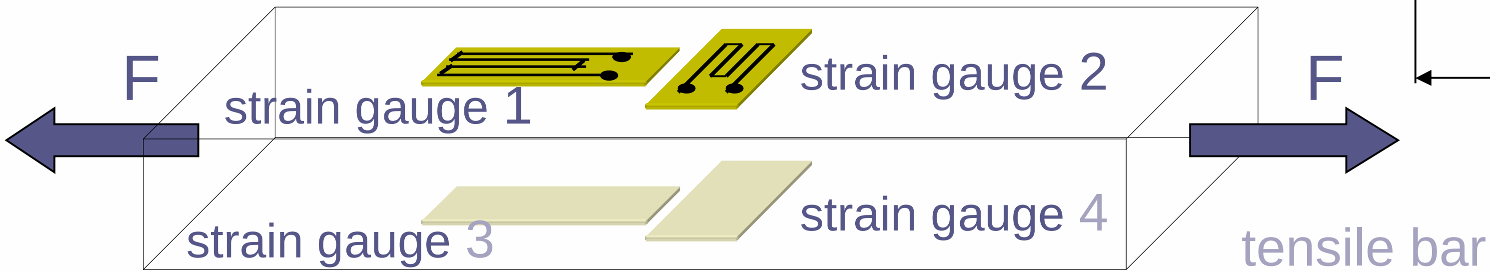
$$\frac{U_A}{U_B} = 0{,}65\ mV/V$$

1.3 times the output value of a 1/4-bridge

Measurements on a tension/compression bar

(Full Bridge)





$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

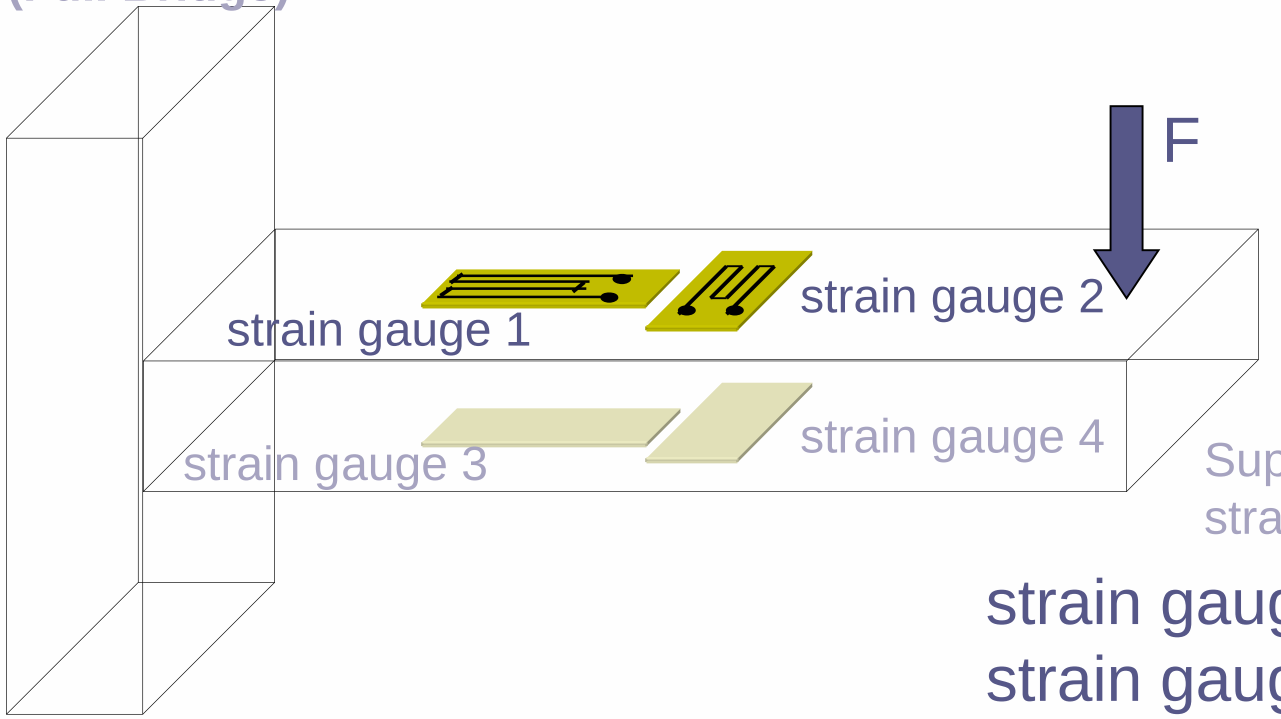
$$\frac{U_A}{U_B} = \frac{k}{4}\left[\varepsilon_1 - \left(-\nu\varepsilon_1\right) + \varepsilon_3 - \left(-\nu\varepsilon_3\right)\right]$$

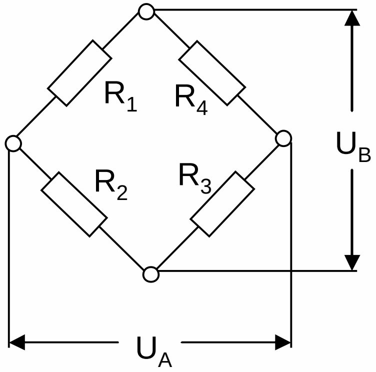
strain gauge 1: +
strain gauge 2: -
strain gauge 3: +
strain gauge 4: -

Temperature compensation: Yes!

Measurements on a tension/compression bar

(Full Bridge)





Superimposed bending strains are compensated

strain gauge 1: +

strain gauge 2: -

strain gauge 3: -

strain gauge 4: +

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

Calibrating measurement equipment (Full Bridge)

k = 2; ν = 0,3

estimated strain level ε = 1000 µm/m (estimation!)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

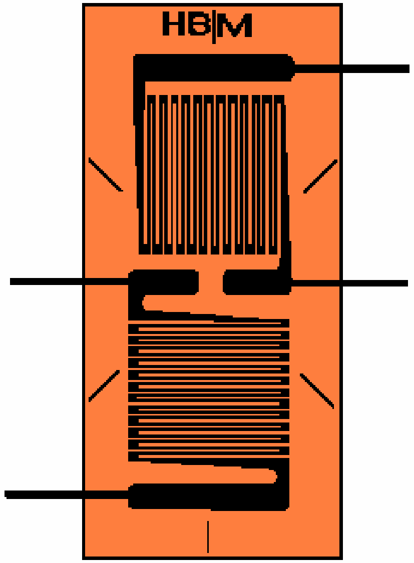
$$\frac{U_A}{U_B} = \frac{k}{4}\left[\varepsilon_1 - \left(-\nu\varepsilon_1\right) + \varepsilon_3 - \left(-\nu\varepsilon_3\right)\right]$$

$$\frac{U_A}{U_B} = \frac{2}{4}\left(1000\mu m/m - \left[-0{,}3 \cdot 1000\mu m/m\right] + 1000\mu m/m - \left[-0{,}3 \cdot 1000\mu m/m\right]\right)$$

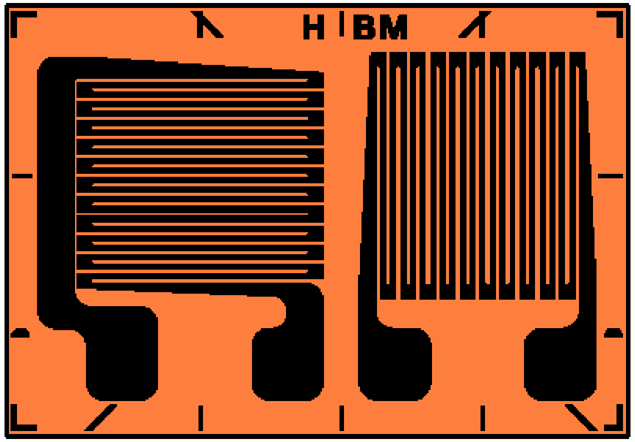
$$\frac{U_A}{U_B} = 0{,}5 \cdot 2600 \cdot 10^{-6} = 1{,}3 \cdot 10^{-3} = \underline{\underline{1{,}3\ mV/V}}$$

2.6 times the output value of a 1/4-bridge

... and the right strain gauge for this application

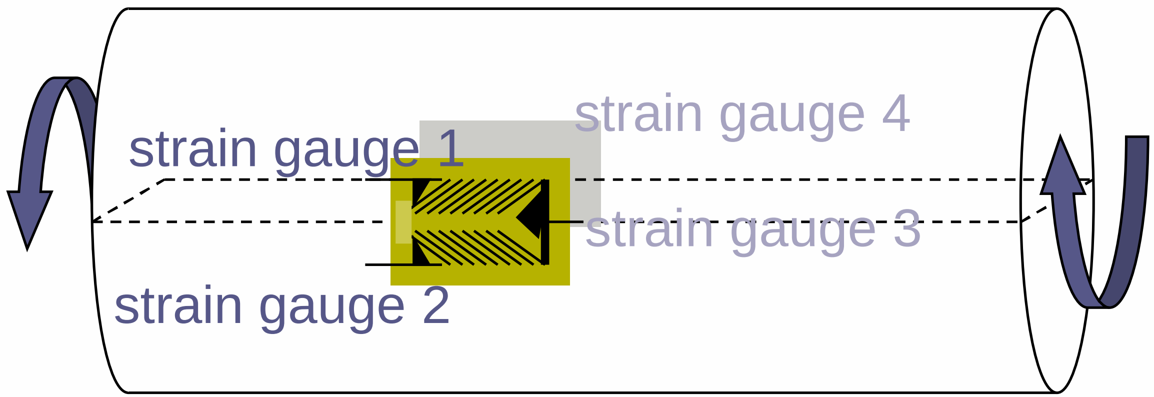


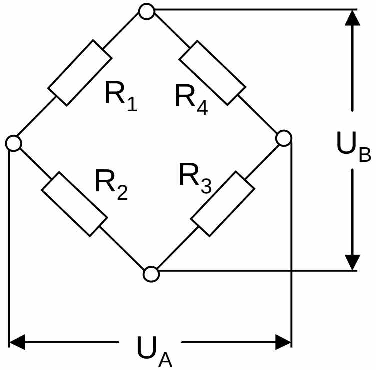
XY11...



XY31...

Measurements on a shaft under torsion (Full Bridge)





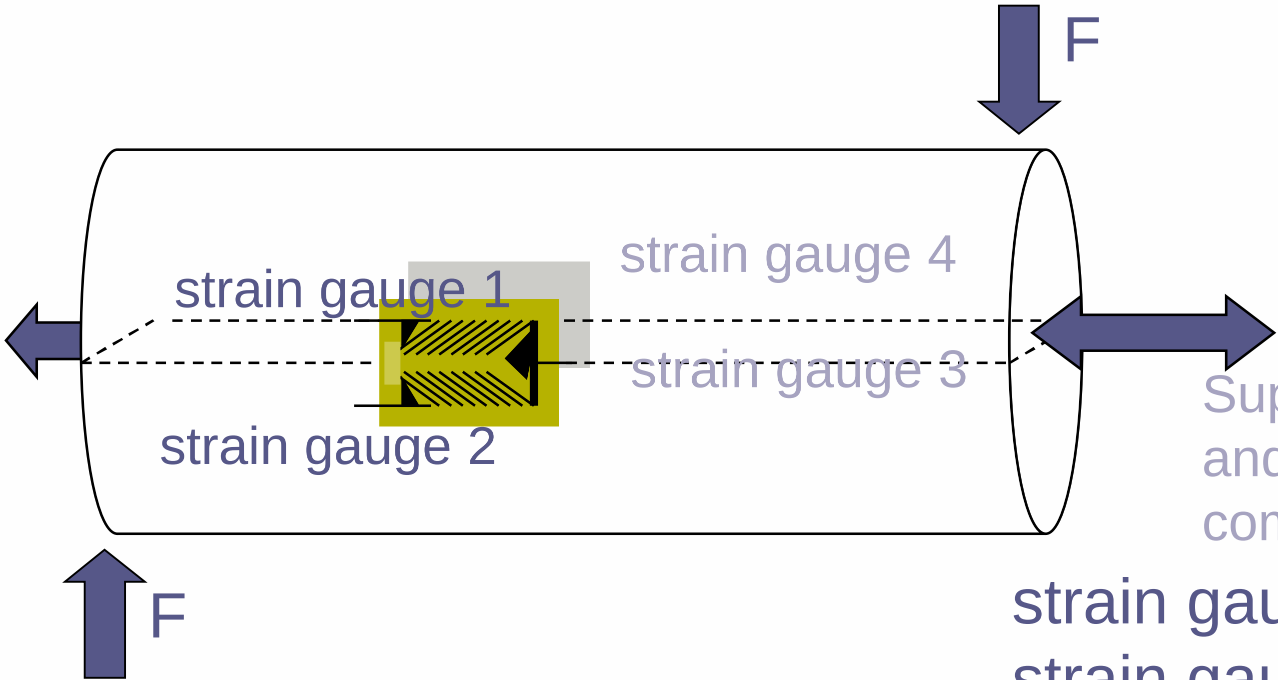
Torsion shaft

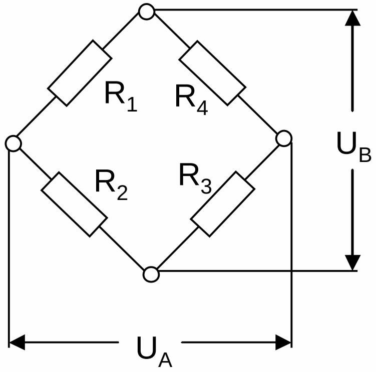
strain gauge 1: +
strain gauge 2: -
strain gauge 3: +
strain gauge 4: -

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

Temperature compensation: Yes!

Measurements on a shaft under torsion (Full Bridge)





Superimposed normal and bending strains are compensated

strain gauge 1: +
strain gauge 2: -
strain gauge 3: -
strain gauge 4: +

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

Temperature compensation: Yes!

Calibrating measurement equipment (Full Bridge)

k = 2

estimated strain level ε = 1000 µm/m (estimation!)

$$\frac{U_A}{U_B} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right)$$

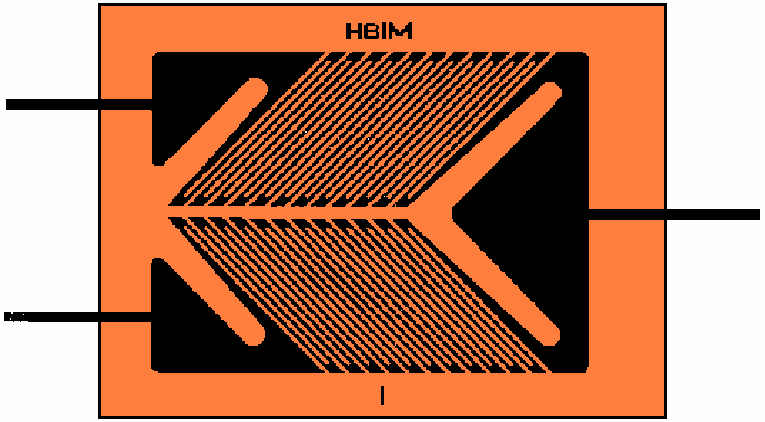
$$\frac{U_A}{U_B} = \frac{2}{4}\left(1000\mu m/m - [-1000\mu m/m] + 1000\mu m/m - [-1000\mu m/m]\right)$$

$$\frac{U_A}{U_B} = 0{,}5 \cdot 4000 \cdot 10^{-6} = 2 \cdot 10^{-3}$$

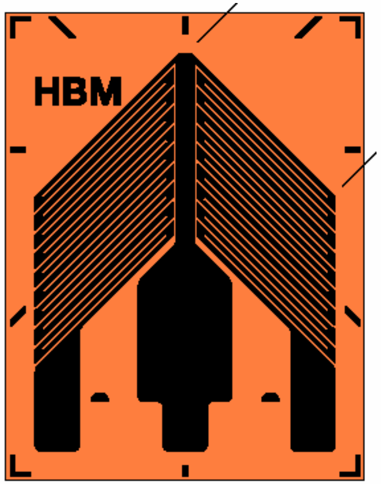
$$\underline{\underline{\frac{U_A}{U_B} = 2\,mV/V}}$$

4 times the output value of a 1/4-bridge

... and the right strain gauge for this application



XY11...



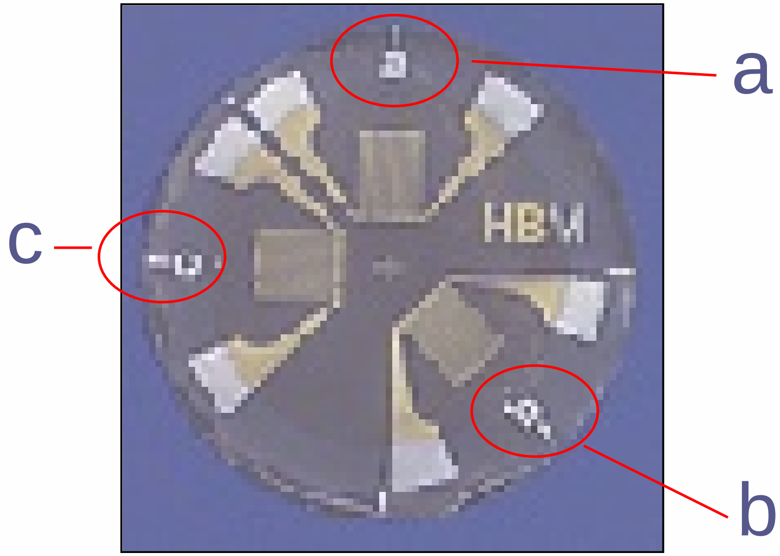
XY31...

strain gauge rosettes for stress analysis

two basic shapes (geometries):

0°/45°/90°-rosettes

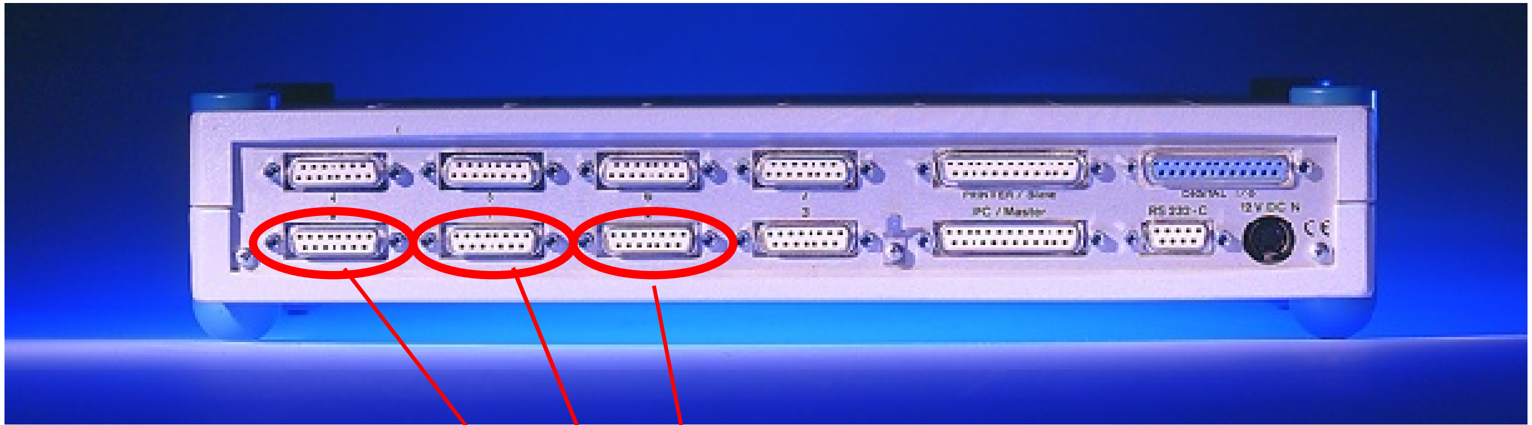
0°/60°/120°-rosettes



strain measurement in 3 directions (a, b, c)

strain gauge rosettes for stress analysis

Connection at the amplifier always as three different quarter bridges!

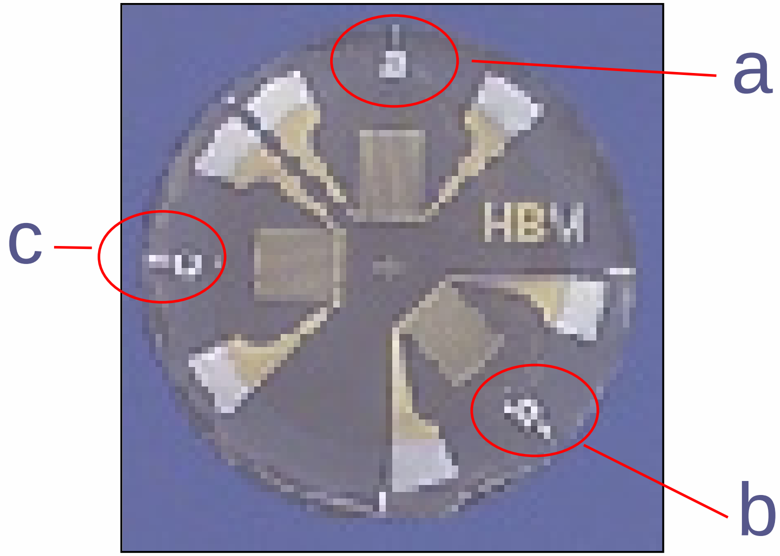


measuring grid a, b, c

e.g. Spider8-30

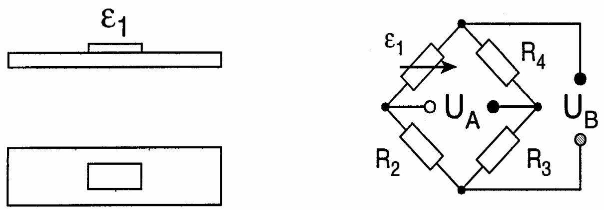
Calibrating measurement equipment
(3 different quarter bridges)

1000µm/m correspond to 2mV/V (at a gauge factor of 2),
valid for amplifier channel (measuring grid) a, b, c



Applying the Wheatstone Bridge Circuit

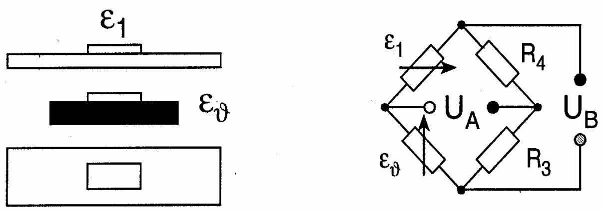
1



$$\varepsilon = \varepsilon_n + \varepsilon_b = \frac{4}{k} \cdot \frac{U_A}{U_B} - \varepsilon_\vartheta$$

T	F	M_b	M_d
1	1	1	0

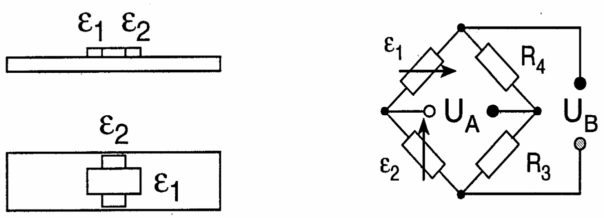
2



$$\varepsilon = \varepsilon_n + \varepsilon_b = \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	1	1	0

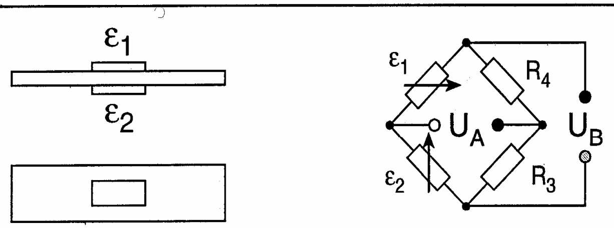
3



$$\varepsilon = \varepsilon_n + \varepsilon_b = \frac{1}{(1+\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

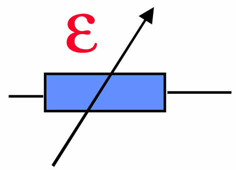
T	F	M_b	M_d
0	$1+\mu$	$1+\mu$	0

4

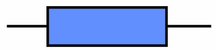


$$\varepsilon = \varepsilon_b = \frac{1}{2} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

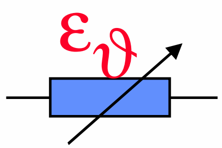
T	F	M_b	M_d
0	0	2	0



active strain gauge



passive strain gauge or resistor

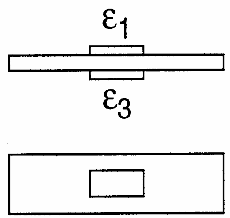


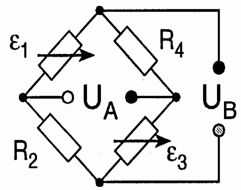
strain gauge for temperature compensation

M_b-bending moment; M_d-torque; T-temperature; F-normal force

Applying the Wheatstone Bridge Circuit

5

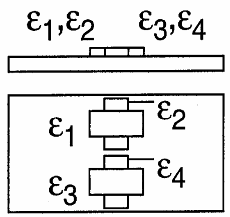


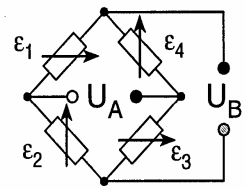


$$\varepsilon = \varepsilon_n = \frac{1}{2} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B} - \varepsilon_\vartheta$$

T	F	M_b	M_d
2	2	0	0

6

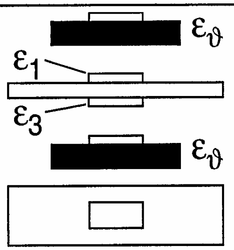


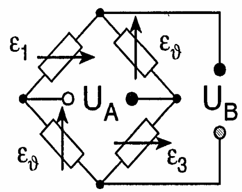


$$\varepsilon = \varepsilon_n + \varepsilon_b = \frac{1}{2(1+\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	$2(1+\mu)$	$2(1+\mu)$	0

7

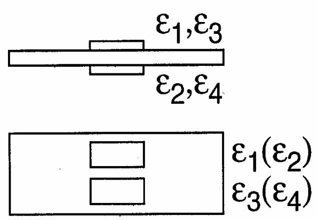


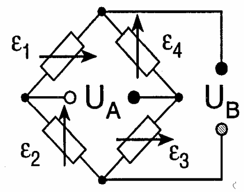


$$\varepsilon = \varepsilon_n = \frac{1}{2} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	2	0	0

8



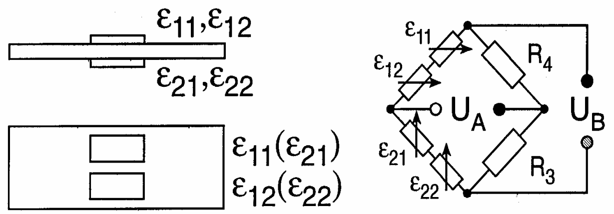


$$\varepsilon = \varepsilon_b = \frac{1}{4} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	0	4	0

Applying the Wheatstone Bridge Circuit

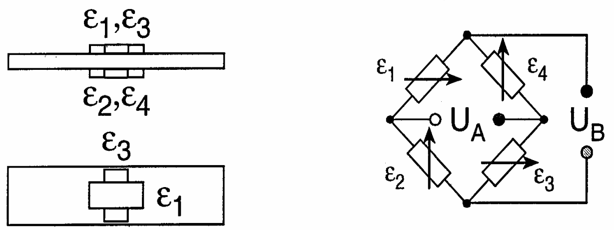
9



$$\varepsilon = \varepsilon_b = \frac{1}{2} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	0	2	0

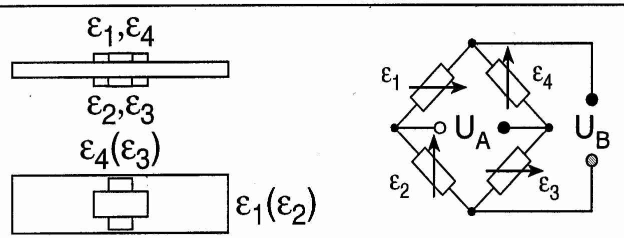
10



$$\varepsilon = \varepsilon_b = \frac{1}{2(1-\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	0	$2(1-\mu)$	0

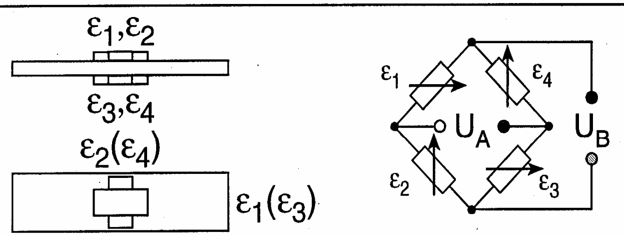
11



$$\varepsilon = \varepsilon_b = \frac{1}{2(1+\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	0	$2(1+\mu)$	0

12

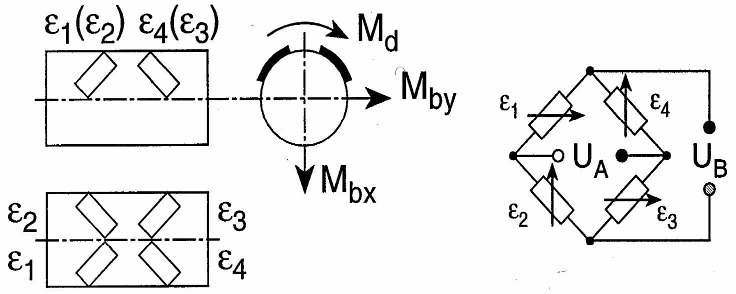


$$\varepsilon = \varepsilon_n = \frac{1}{2(1+\mu)} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_b	M_d
0	$2(1+\mu)$	0	0

Applying the Wheatstone Bridge Circuit

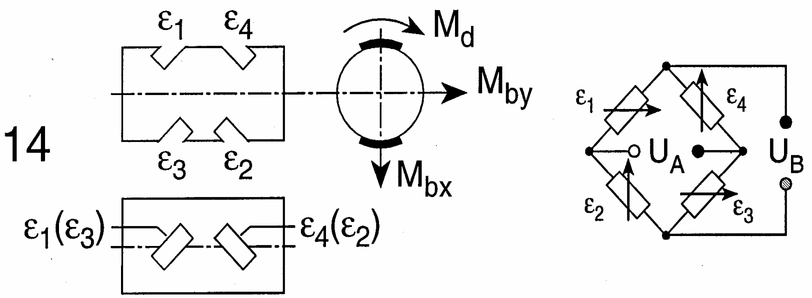
13



$$\varepsilon = \varepsilon_d = \frac{1}{4} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_{bx}	M_{by}	M_d
0	0	0	0	4

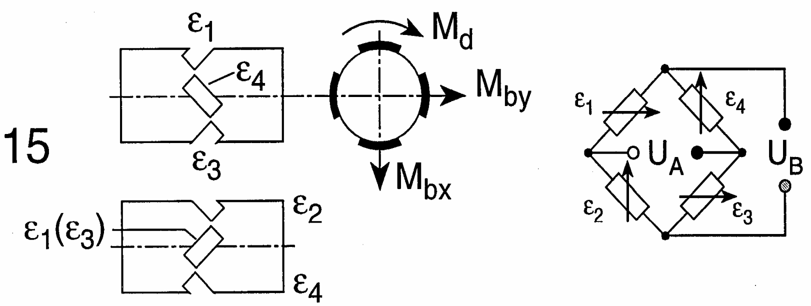
14



$$\varepsilon = \varepsilon_d = \frac{1}{4} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_{bx}	M_{by}	M_d
0	0	0	0	4

15



$$\varepsilon = \varepsilon_d = \frac{1}{4} \cdot \frac{4}{k} \cdot \frac{U_A}{U_B}$$

T	F	M_{bx}	M_{by}	M_d
0	0	0	0	4

Reduction and elimination of measurement errors, especially temperature effects

- *temperature error can be corrected mathematically*
- temperature compensation using Wheatstone bridge circuit

  -quarter bridge with compensating strain gauge

  -half and full bridges

Basics

Every strain gauge gives out a value when the temperature varies – without any mechanical load.

Cause:

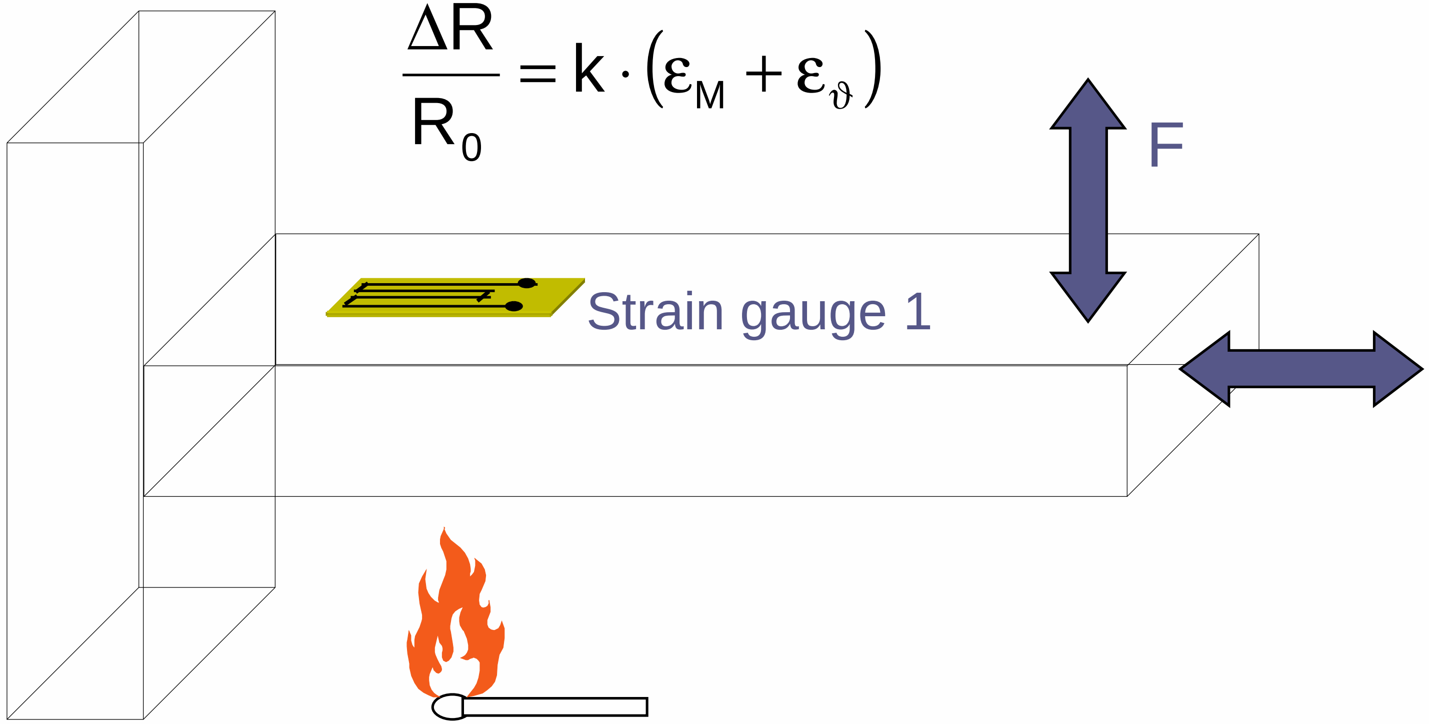
- Work piece expansion with temperature
- Change in specific resistance of strain gauge
- Strain gauge grid expansion with temperature

HBM, Thomas Kleckers (Januar 1999)

Basics

Thermal output of a quarter bridge

$$\frac{\Delta R}{R_0} = k \cdot (\varepsilon_M + \varepsilon_\vartheta)$$

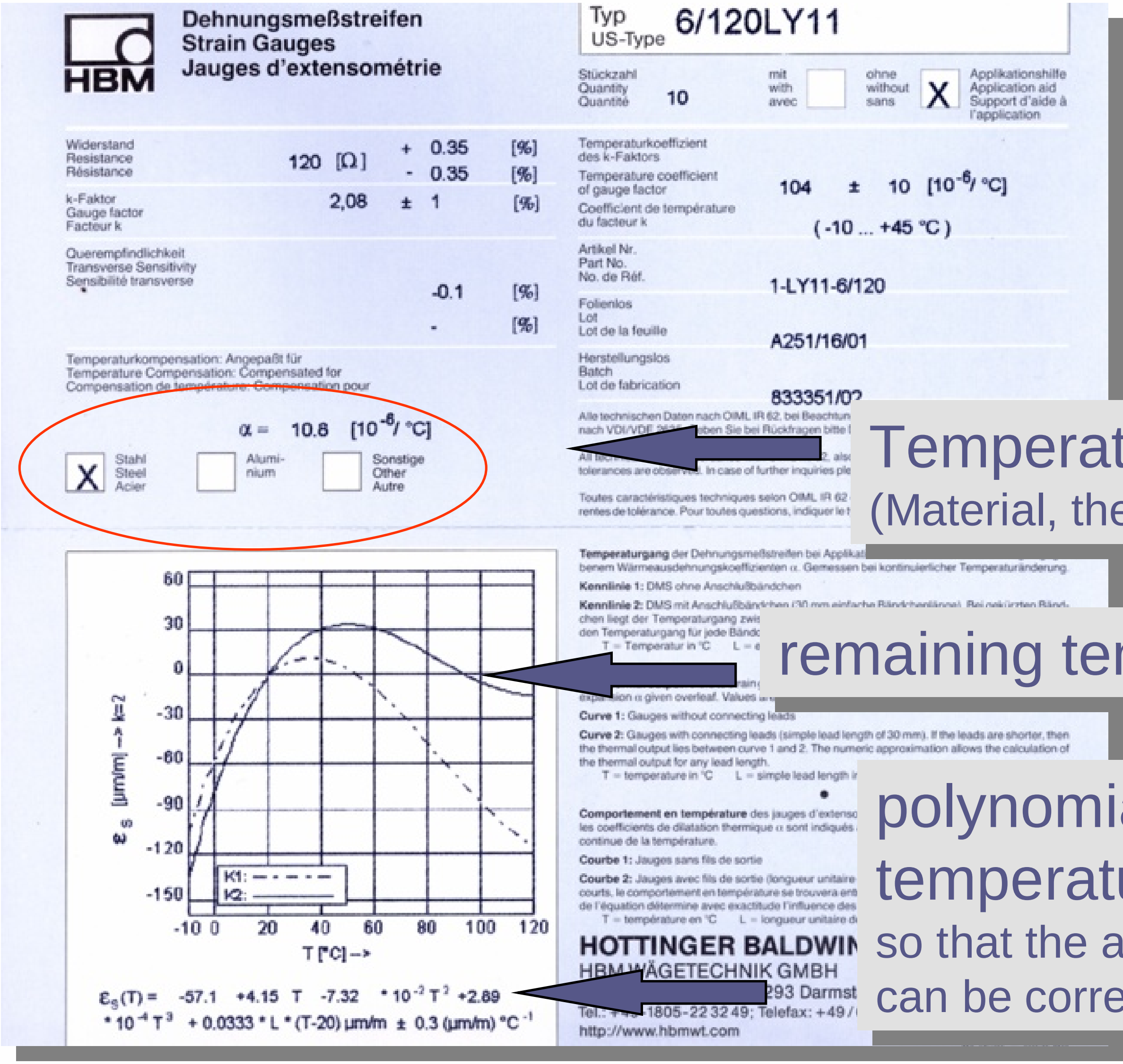


Temperature compensated strain gauges (series Y)

1	for ferritic steel	$\alpha = 10{,}8 \cdot 10^{-6}/\,K$
3	for aluminium	$\alpha = 23 \cdot 10^{-6}/\,K$
5	for austenitic steel	$\alpha = 16 \cdot 10^{-6}/\,K$
6*	for quartz	$\alpha = 0{,}5 \cdot 10^{-6}/\,K$
7*	for titanium /grey cast iron	$\alpha = 9 \cdot 10^{-6}\,/\,K$
8*	for plastic material	$\alpha = 65 \cdot 10^{-6}/\,K$
9*	for molybdenum	$\alpha = 5{,}4 \cdot 10^{-6}/\,K$

* not for all measuring grid length available

Applying the Wheatstone Bridge Circuit



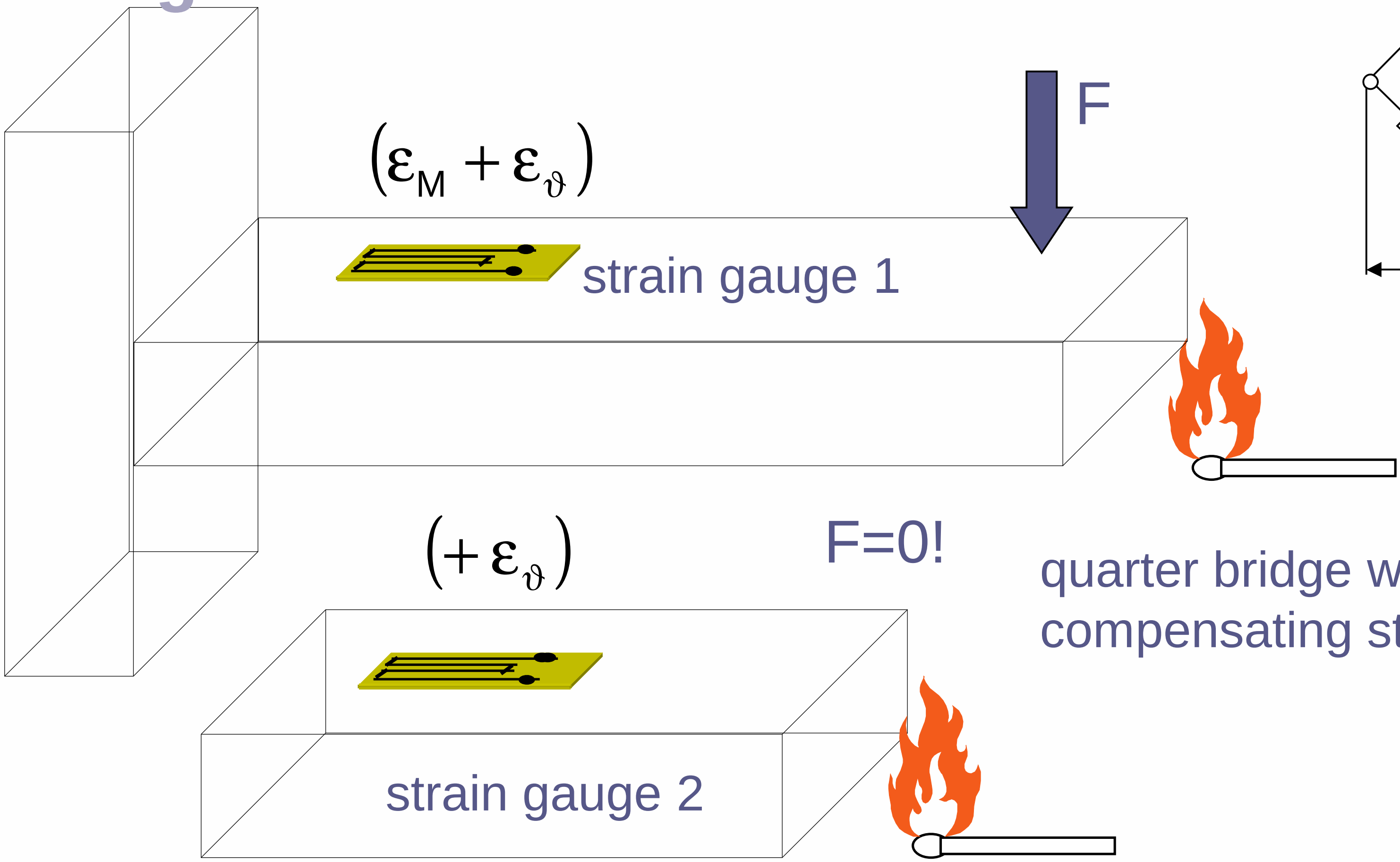
$\varepsilon_S(T) = -57.1 + 4.15\ T - 7.32 \cdot 10^{-2}\ T^2 + 2.89 \cdot 10^{-4}\ T^3 + 0.0333 \cdot L \cdot (T-20)\ \mu m/m \pm 0.3\ (\mu m/m)\ °C^{-1}$

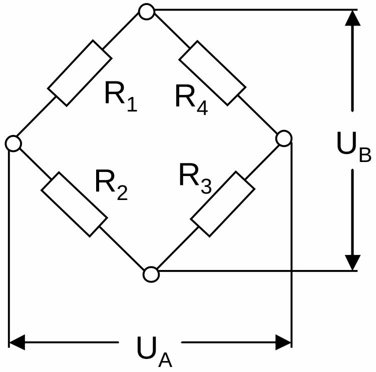
Temperature Compensation
(Material, thermal expansion coefficient

remaining temperature variation

polynomial (of the remaining temperature variation)
so that the arising measurement fault can be corrected mathematically

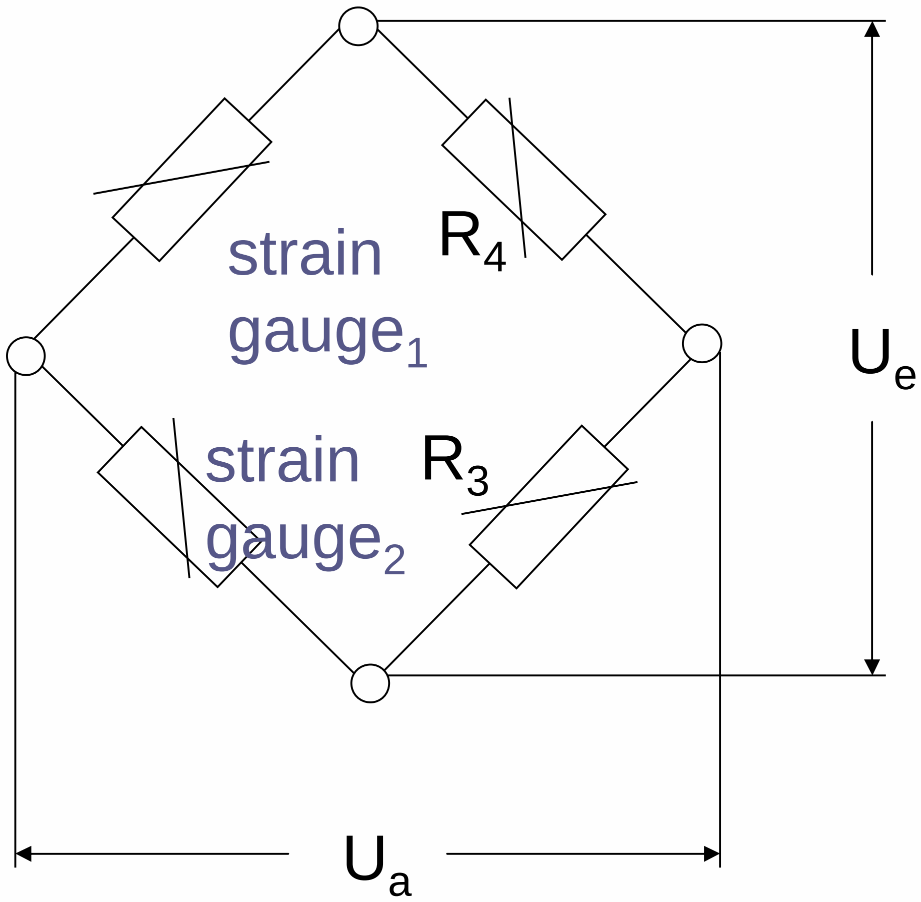
temperature compensation using Wheatstone bridge circuit





quarter bridge with compensating strain gauge

temperature compensation using Wheatstone bridge circuit



quarter bridge with
compensating strain gauge

$$\text{strain gauge}_1 \Rightarrow (\varepsilon_M + \varepsilon_\vartheta)$$

$$\text{strain gauge}_2 \Rightarrow (+\varepsilon_\vartheta)$$

$$\frac{U_a}{U_e} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2)$$

quarter bridge with compensating strain gauge

The compensating strain gauge:

- must be applied in a spot where it will be subjected to the same interference effect as the active strain gauge

- must have the same physical properties as the active strain gauge (same package or batch)

- must only be subjected to the interference effect and never to the quantity to be measured ε_M or its side effects

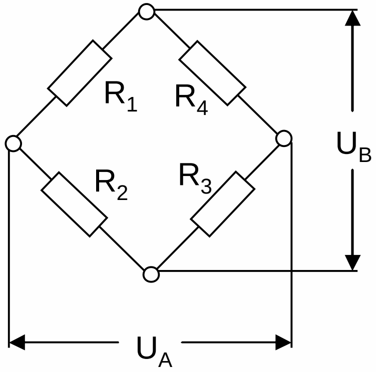
- must be applied at the same material (e.g. steel) as the active strain gauge

temperature compensation using Wheatstone bridge circuit

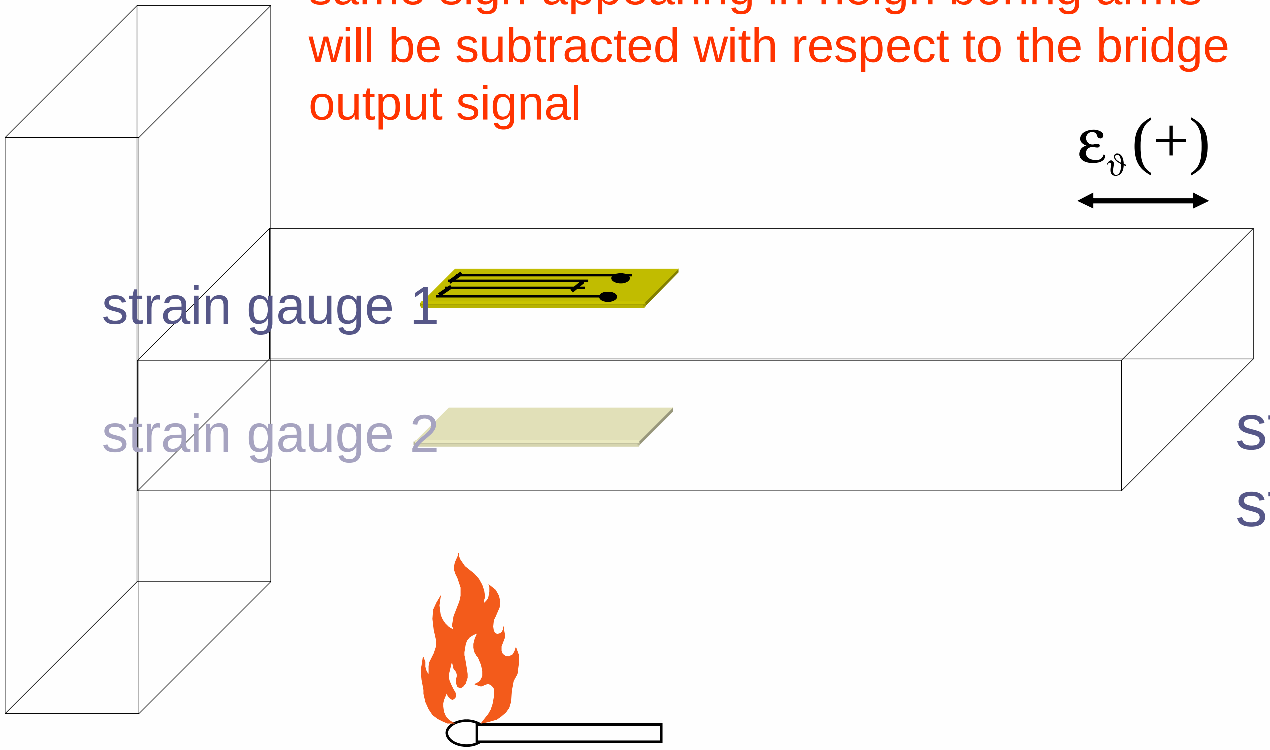
Advantages:

- it is not necessary to measure the temperature during the strain measurement
- it is not absolutely necessary that the strain gauges are adjusted to the material

temperature compensation using Wheatstone bridge circuit



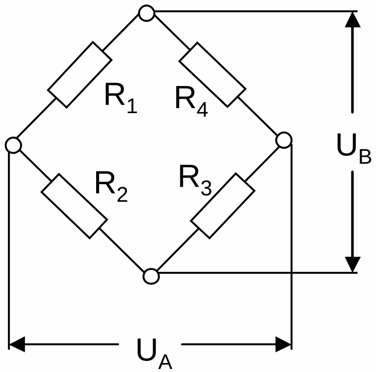
Changes of the strain gauge resistance of the same sign appearing in neigh boring arms will be subtracted with respect to the bridge output signal



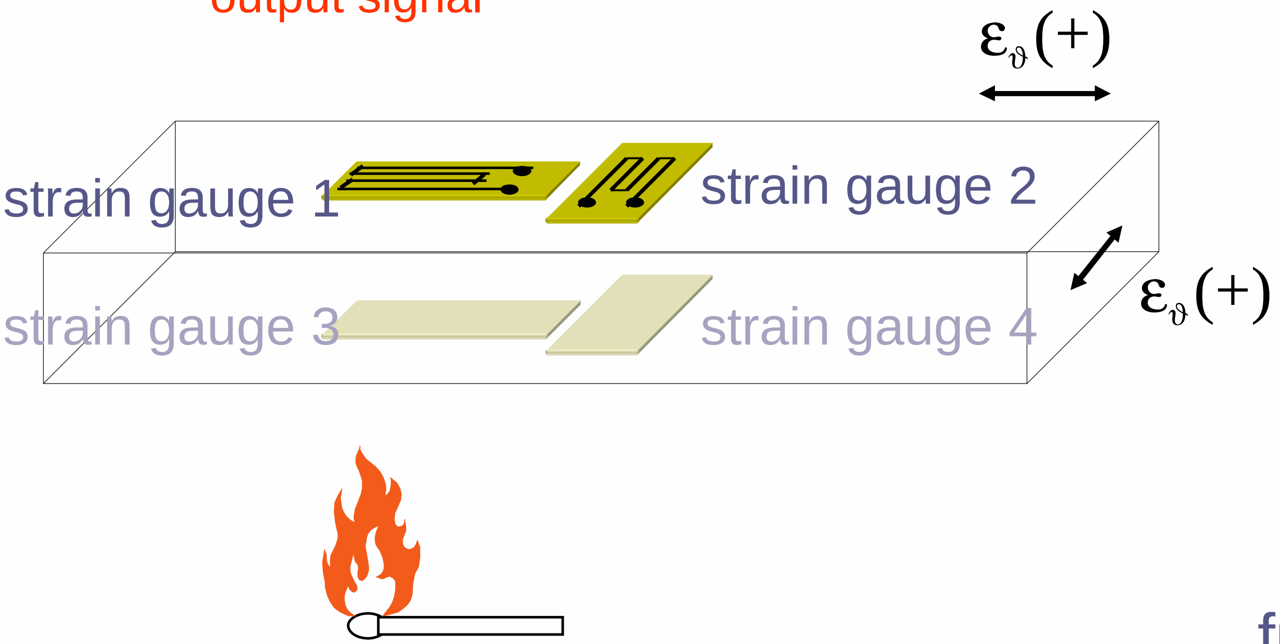
strain gauge 1: +
strain gauge 2: +

half bridge

temperature compensation using Wheatstone bridge circuit



Changes of the strain gauge resistance of the same sign appearing in neigh boring arms will be subtracted with respect to the bridge output signal



s. g. 1: +
s. g. 2: +
s. g. 3: +
s. g. 4: +

full bridge

thank you...

... for your attention

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measurement with confidence