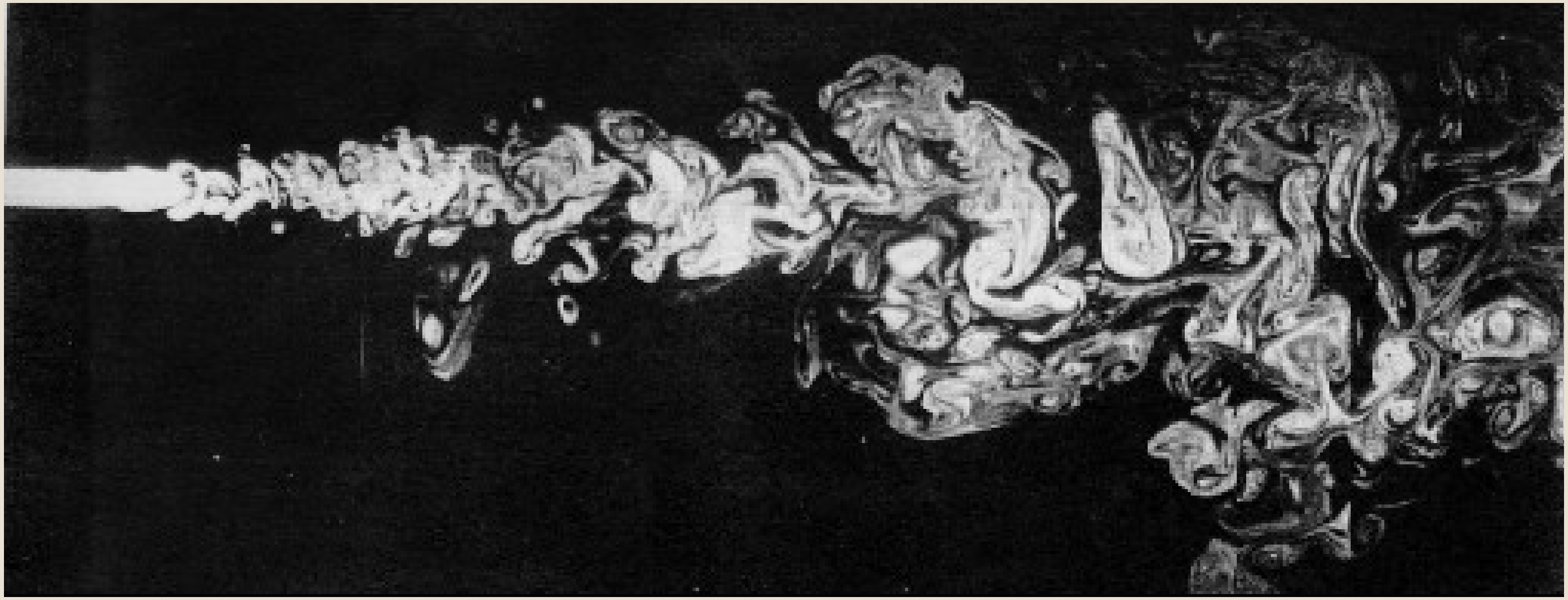
Turbulence Modelling: LES & DES

Dr Aleksey Gerasimov
Senior CFD Engineer
Fluent Europe Ltd

Outline

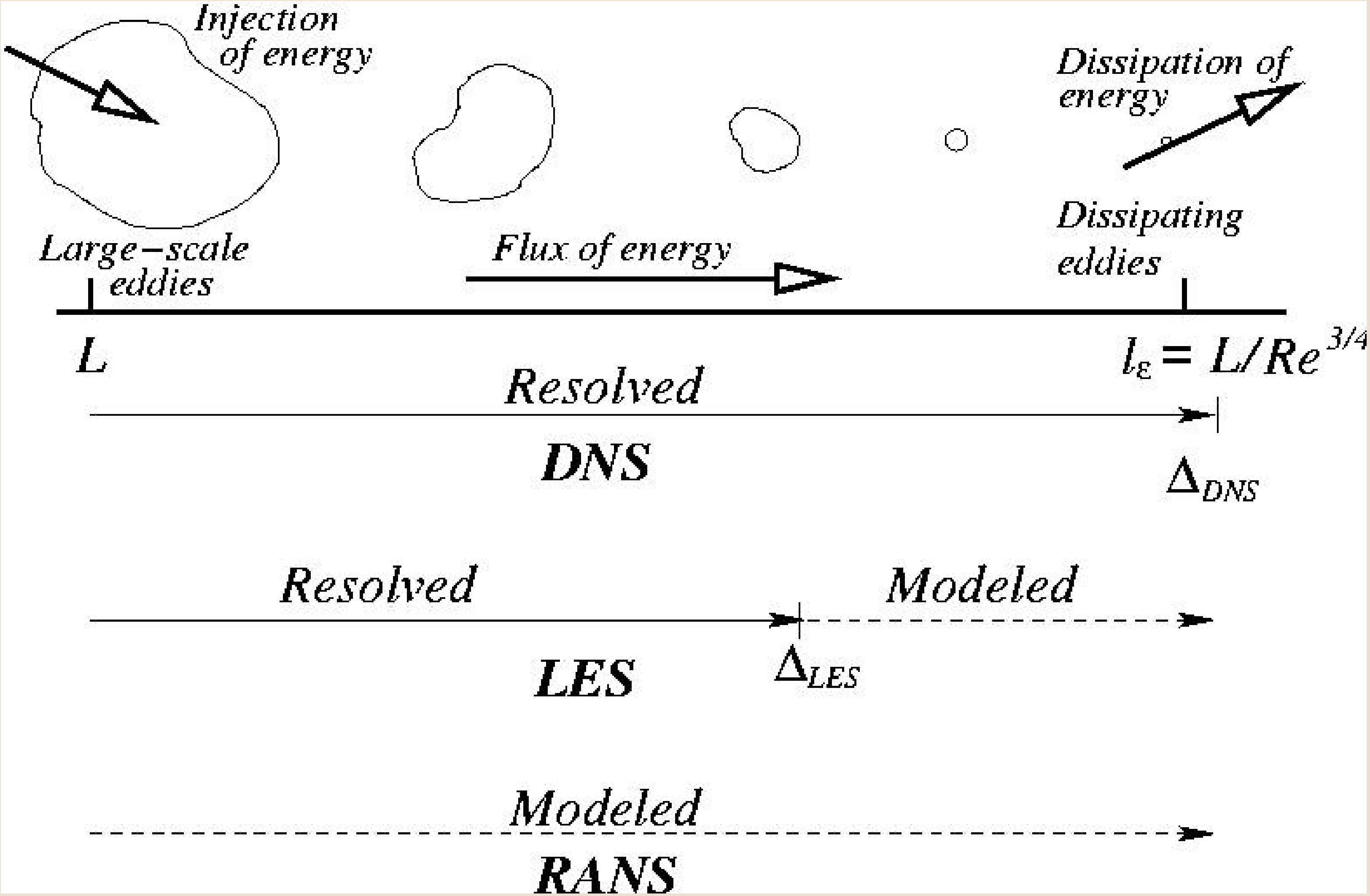
- What is LES?
- Why LES?
- What about the cost?
- What does FLUENT offer
- Best-practice LES with FLUENT
- Closing Remarks

What is LES?
LES: Basic Concepts (1)



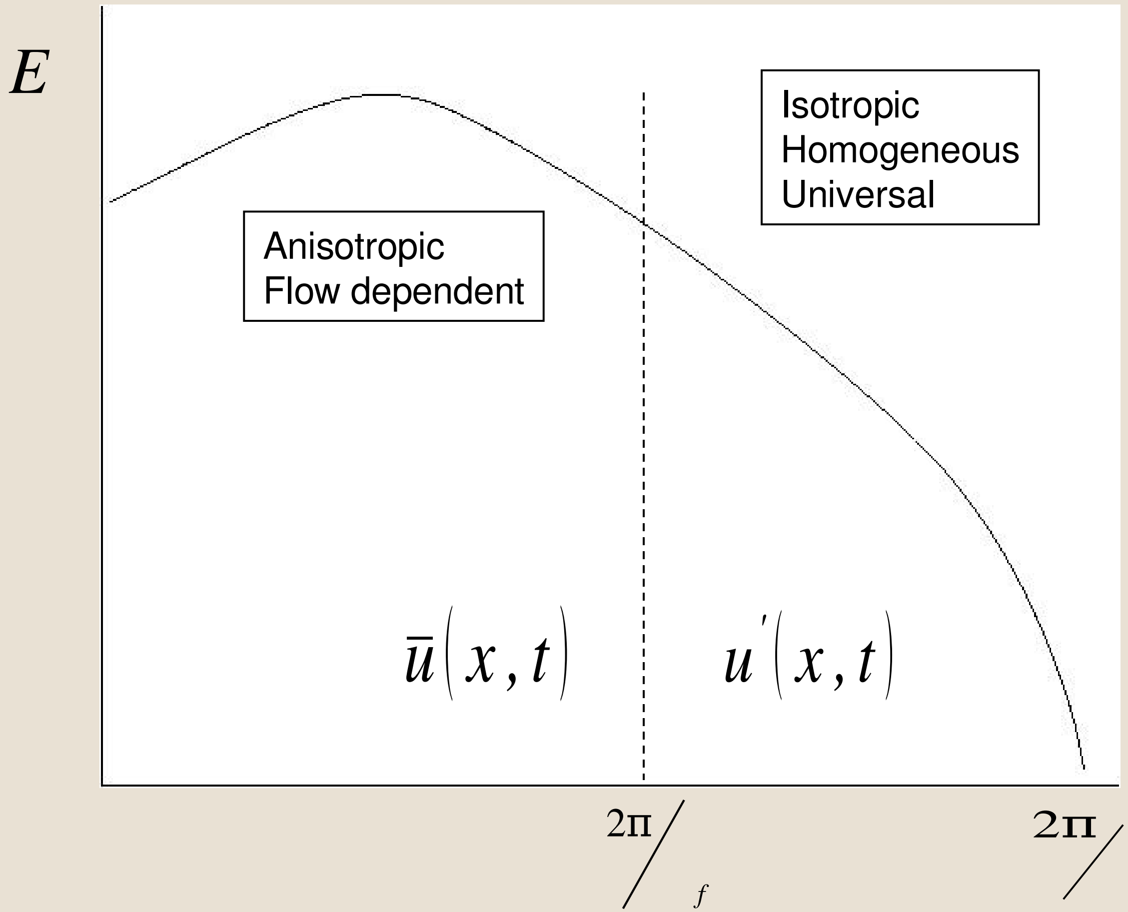
- Turbulent flow consists of eddies (structures) with wide range of length and time scales.
- In LES, large, energy-carrying eddies are directly computed (resolved), and the remaining smaller eddies are modeled.

LES: Basic Concepts (2)



LES: Filtering - Decomposition

$$u(x,t) = \underbrace{\bar{u}(x,t)}_{\text{resolved scale}} + \underbrace{u'(x,t)}_{\text{subgrid scale}}$$



Energy spectrum against the length scale

LES: Filtering of a Variable

- A random variable, ϕ (x), is filtered using a space-filter function, *G*

$$\bar{\varphi}(x)=\int_D \varphi(x')G(x,x')\,dx'$$

- With the top-hat filter (among others)

$$G(x,x')=\begin{cases} 1/V & \text{for } \{\boldsymbol{x}' \in \boldsymbol{\mathcal{V}} ¿ \\ 0 & \text{otherwise} ¿ \end{cases}$$

- The filtered variable becomes

$$\bar{\varphi}(x)=\frac{1}{V}\int_{\nu} \varphi(x')\,dx', \qquad \{\boldsymbol{x}' \in V ¿$$

LES: Filtered Navier-Stoke Equations

- Filtering the original Navier-Stokes equations gives filtered Navier-Stokes equations that are the governing equations in LES.

N-S equation

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial u_i}{\partial x_j}\right)$$

Filter

Filtered N-S equation

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial \bar{u}_i}{\partial x_j}\right)-\frac{\partial \tau_{ij}}{\partial x_j}$$

$$\tau_{ij} \equiv \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$ ← Needs modelling

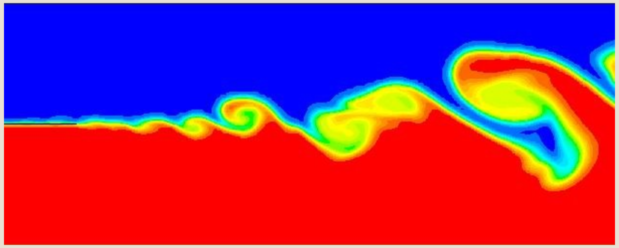
Sub-grid scale (SGS) stress

Why LES? - Rationales

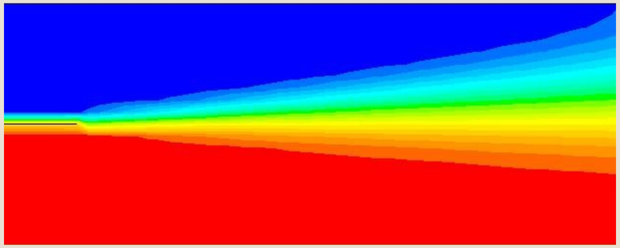
- Large eddies are responsible for the transports of momentum, energy, and other scalars.
- Large eddies are anisotropic, subjected to history effects, and are strongly dependent on boundary conditions, which makes their modelling difficult.
- Small eddies tend to be more isotropic and less flow-dependent (universal), which makes their modelling easier.

Why LES
Other Prediction Methods

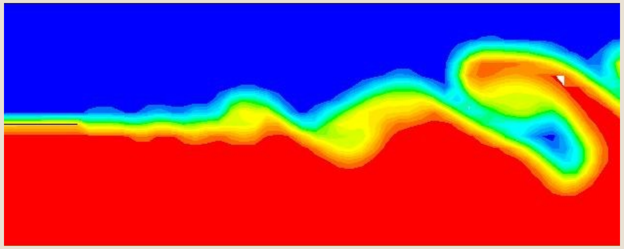
- Different approaches to make turbulence computationally tractable:
  - DNS: Direct Simulation.
  - RANS: Reynolds average (or time or ensemble)
  - LES: Spatially average (or filter)



DNS,
3D unsteady



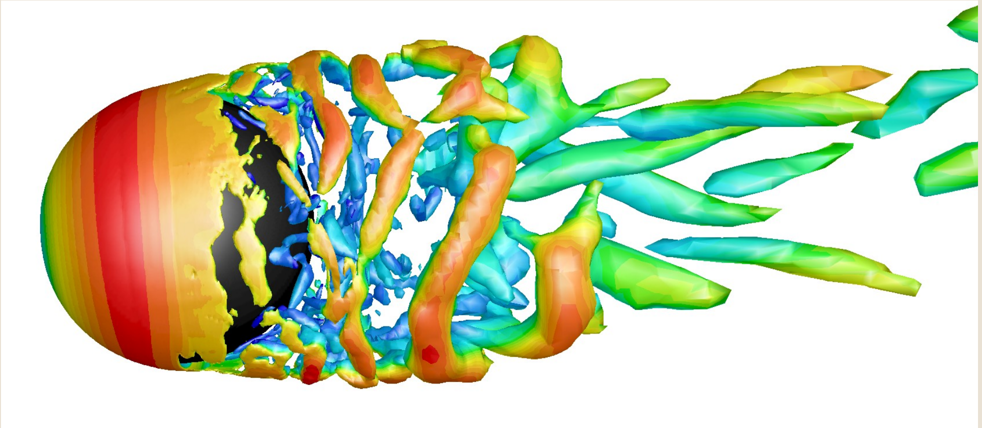
RANS, 2D or 3D
steady or unsteady



LES,
3D unsteady

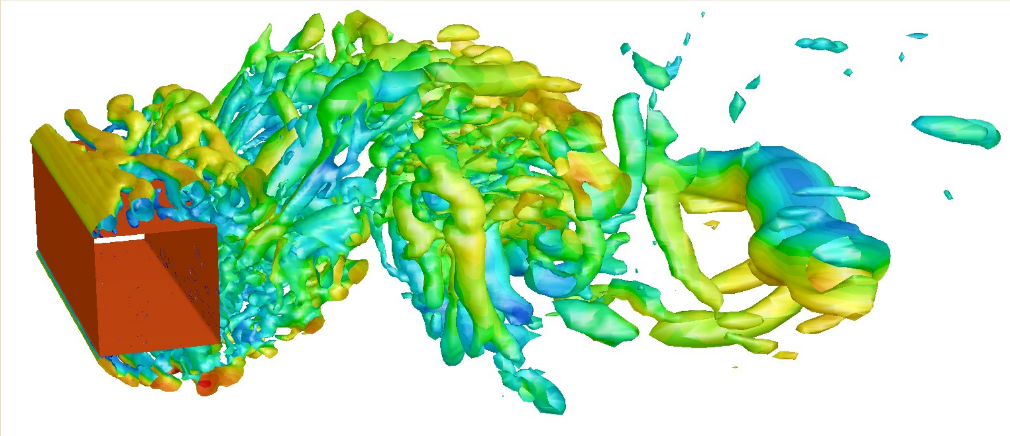
Why LES?

- Some applications need explicit computation of unsteady fields.
  - Bluff body aerodynamics
  - Aerodynamically generated noise (sound)
  - Fluid-structure interaction
  - Combustion instability
  - …
  - …



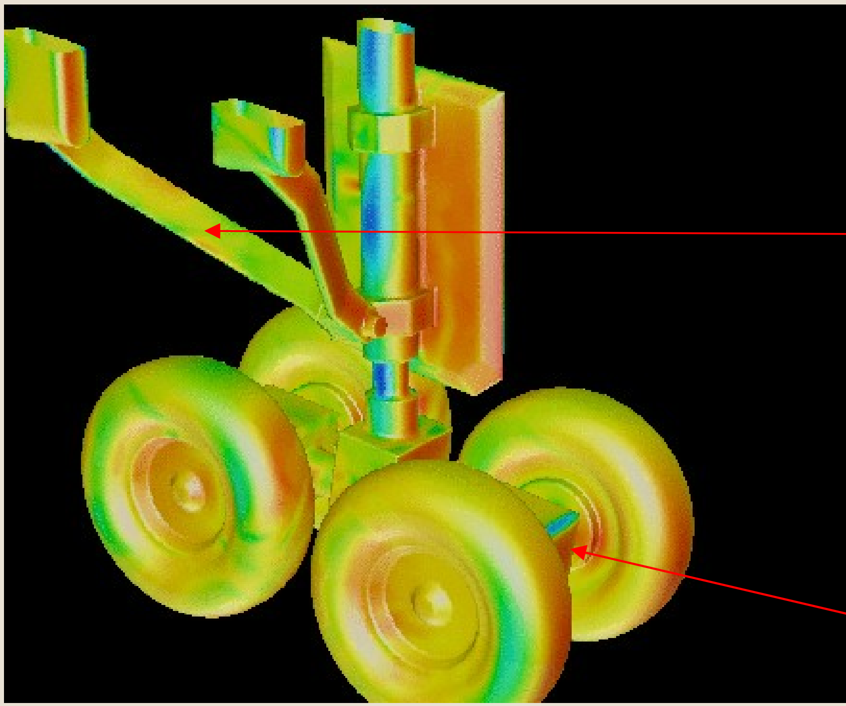
Why LES? - Examples
Bluff-Body Aerodynamics

- Unsteadiness in the flow due to large eddies (e.g., unsteady separation, vortex shedding) potentially impacts:
  - Maneuverability (handling quality) of vehicles
  - Safety & comfort (wake of buildings and ships)
  - Unsteady loading on structures (r.m.s. forces and moments, frequency contents)
    - Buildings
    - Bridges

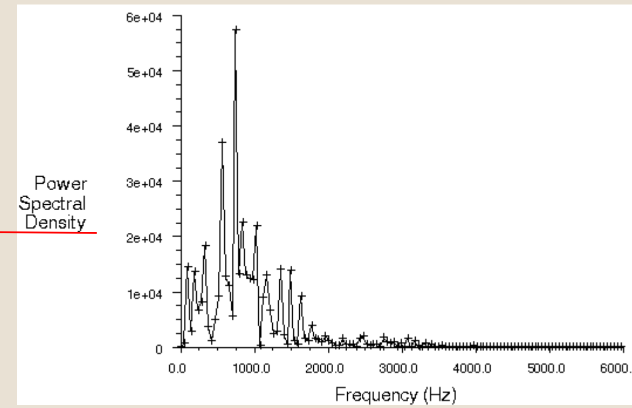


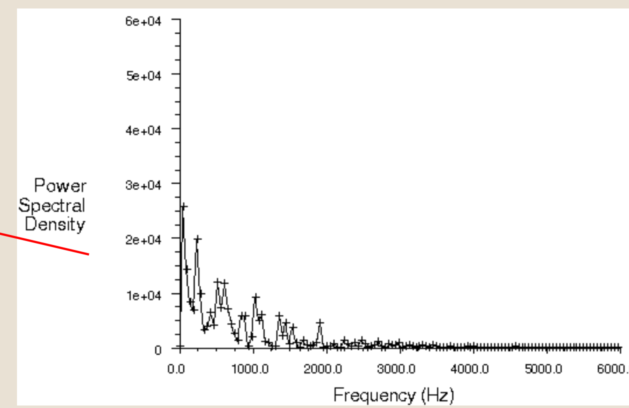
Why LES? – Examples
Aerodynamic Noise

- Flow unsteadiness is the very source of aerodynamic noise.



Contours of surface pressure on a landing gear

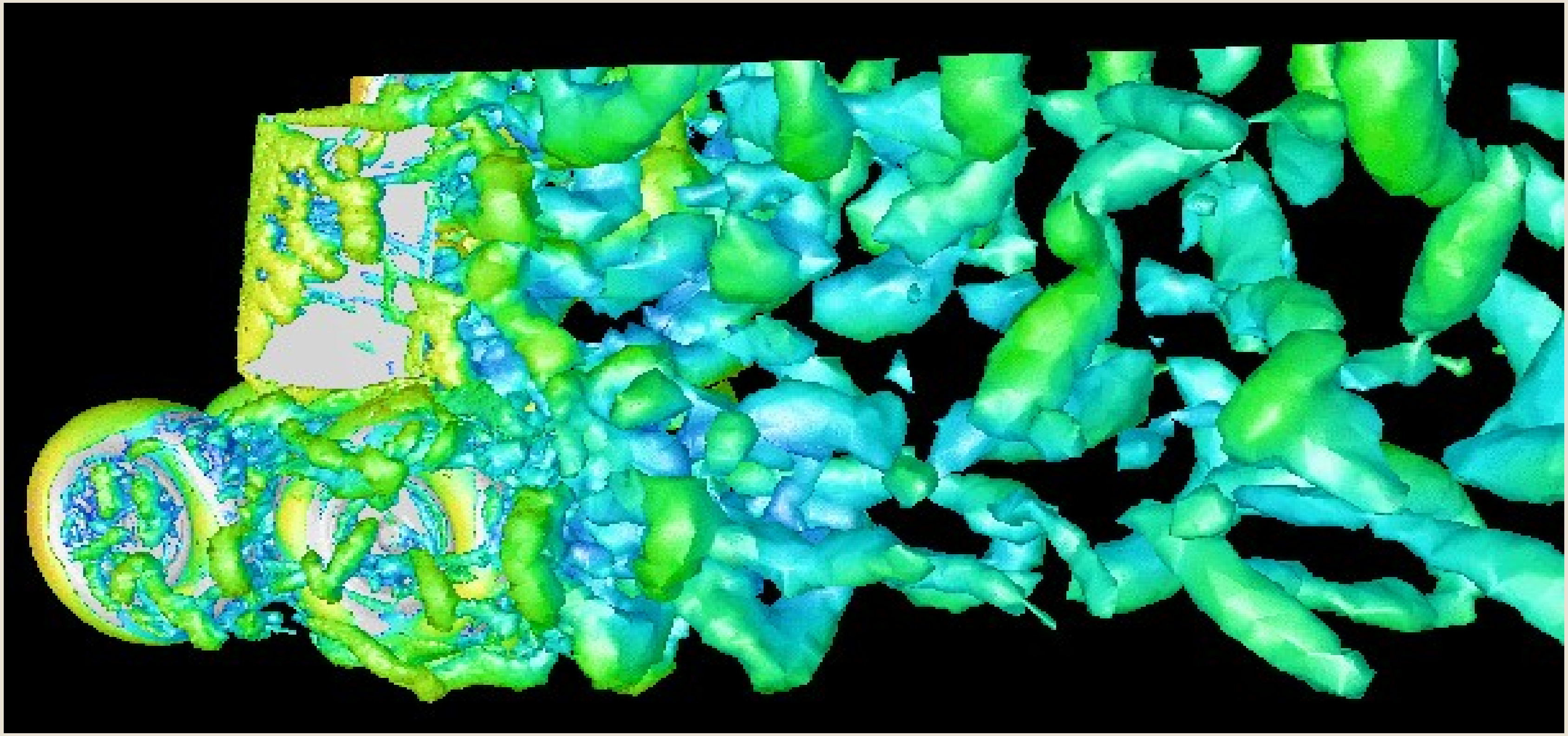




PSD of surface static pressure

Why LES? - Examples
Aerodynamic Noise – cont'd

flow

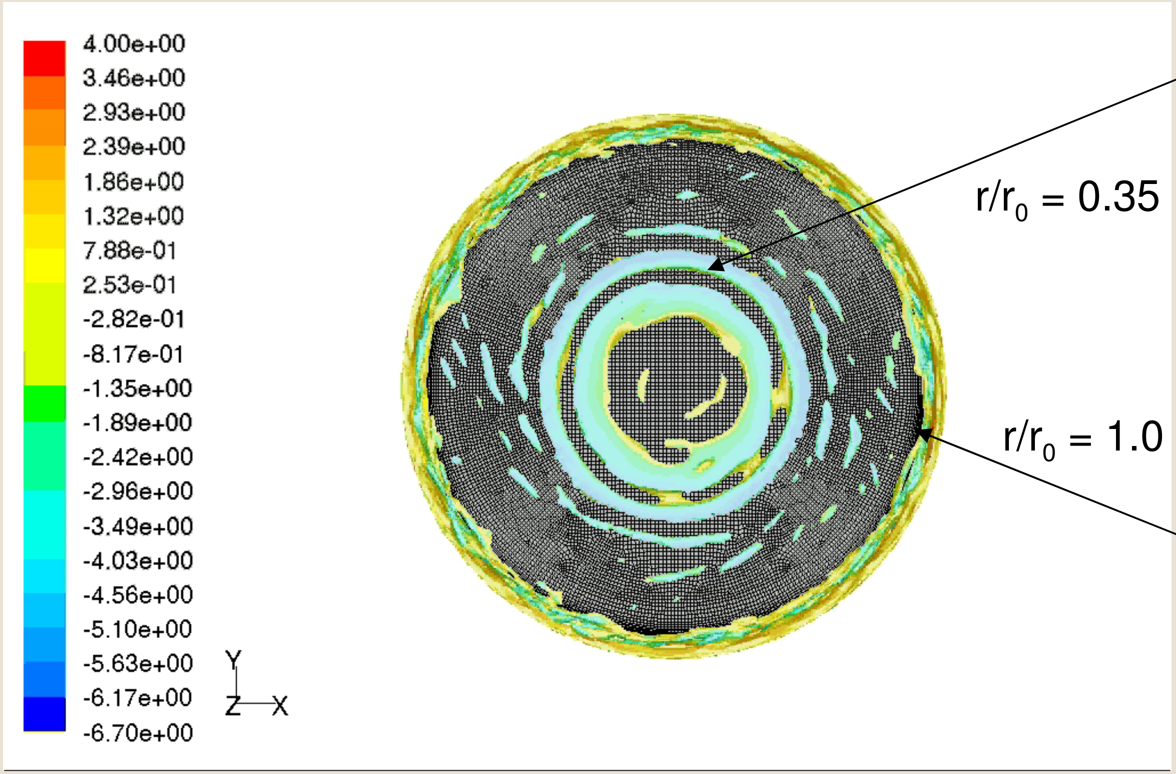


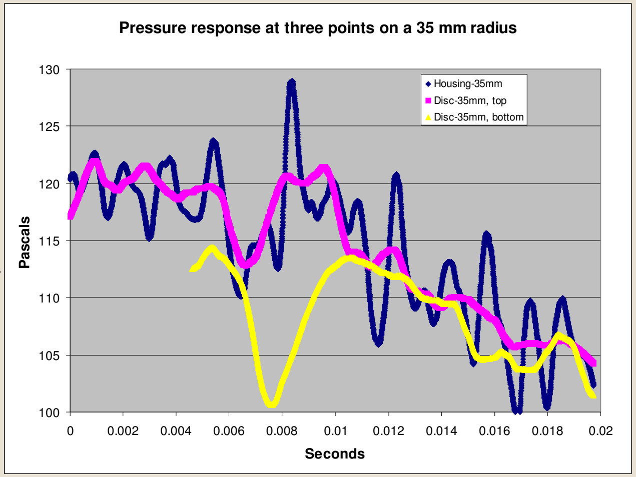
Iso-surface of the second-invariant of the deformation tensor
on a landing gear

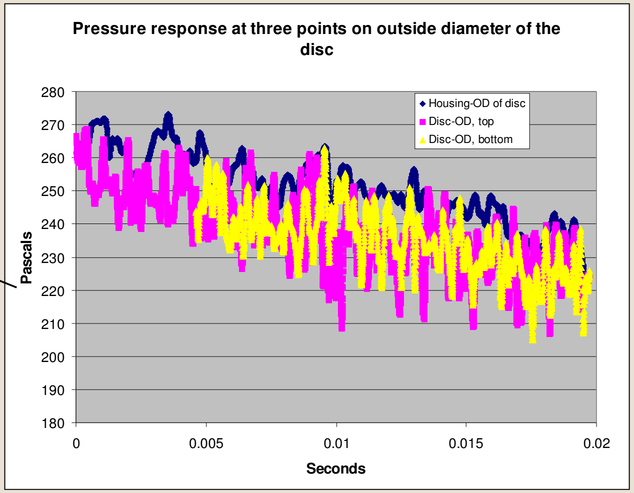
Why LES? – Examples
Fluid/Structure Interaction

- Unsteady fluid-dynamic loading causes dynamic response of structures

Spinning disk inside a cylindrical enclosure
(D = 95mm, N = 10,000 rpm)

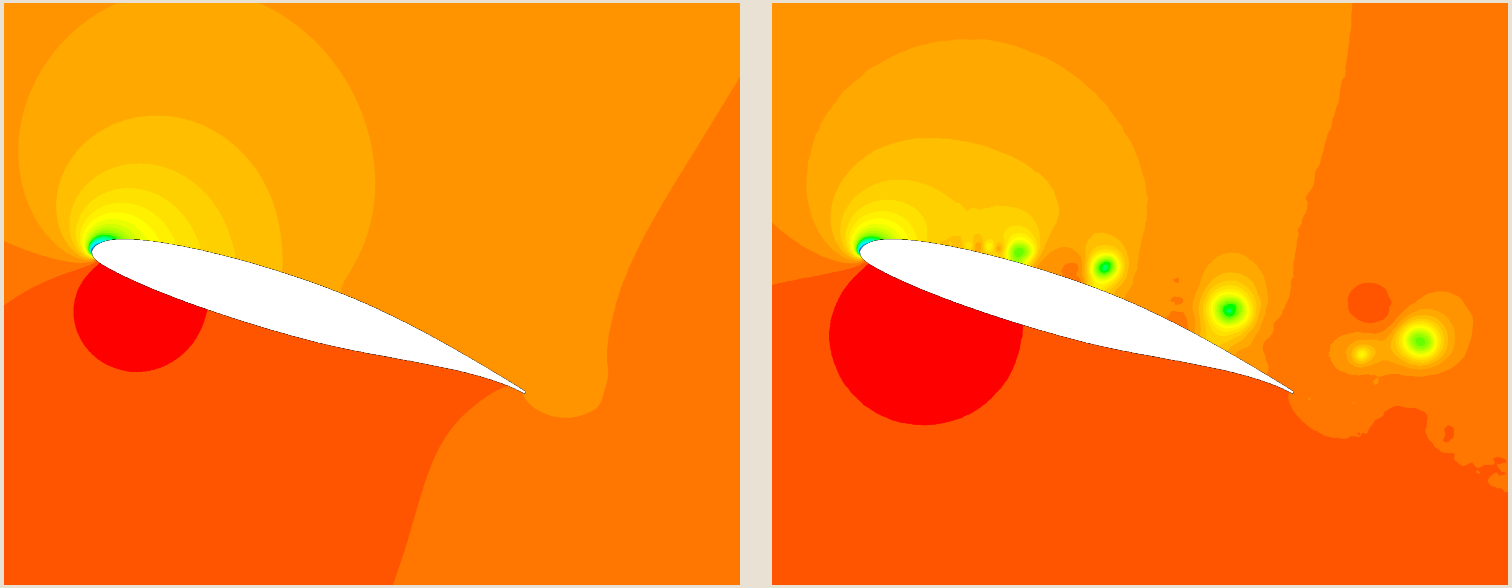






Why LES?
Why Not Unsteady RANS?

- URANS with “good” turbulence models can occasionally predict a regular vortex-shedding (“largest” scale), offering essentially VLES.
- Yet URANS fall sort of capturing the remaining large scales.

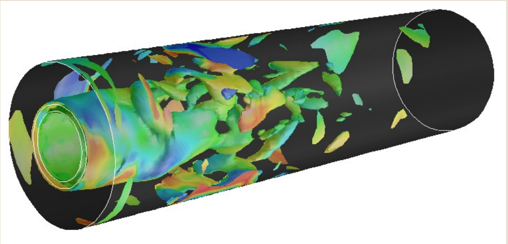


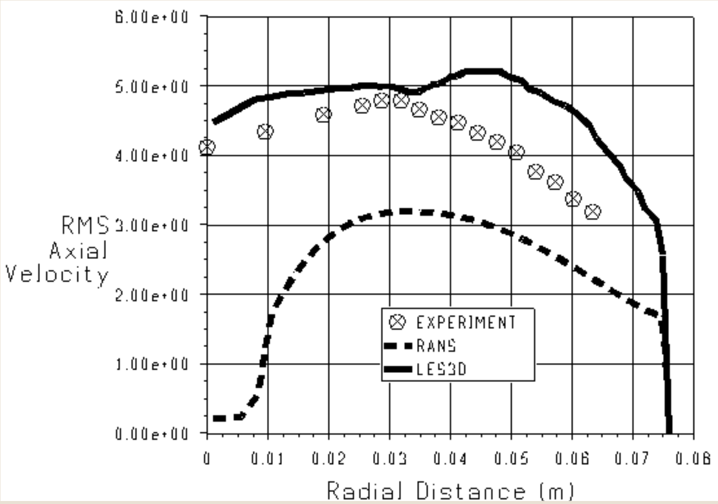
URANS with SST k-ω model LES

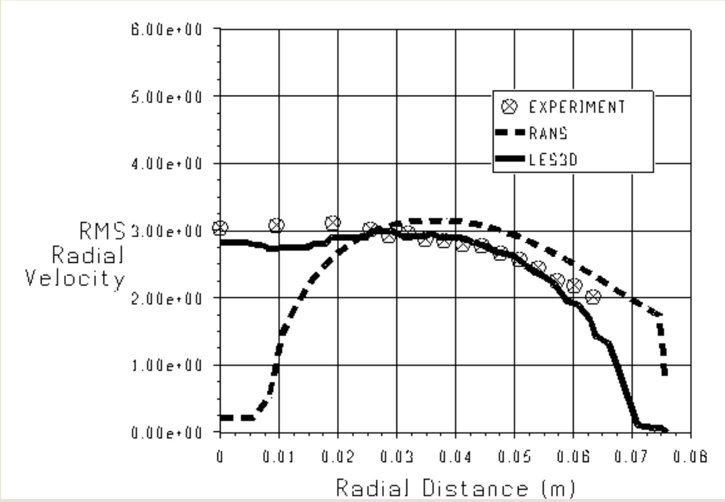
Why LES?
Difficulties with RANS (1)

- RANS-based models often fail to predict even time-averaged quantities.
  - Inability to model the physics of large-scale structures in transporting momentum and scalars

Dump combustor (Re_D = 10,000)

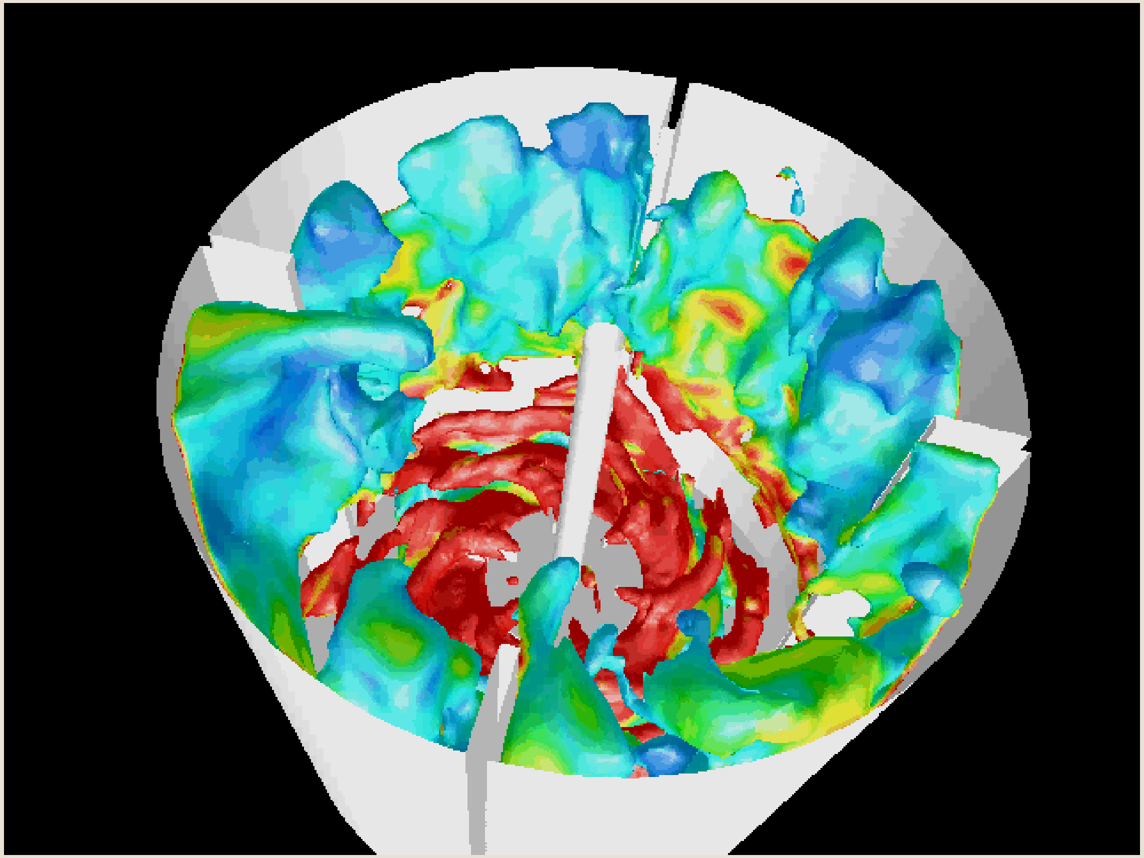






Why LES?
Difficulties with RANS (2)

- Large-scale structures mainly contributes to the mixing



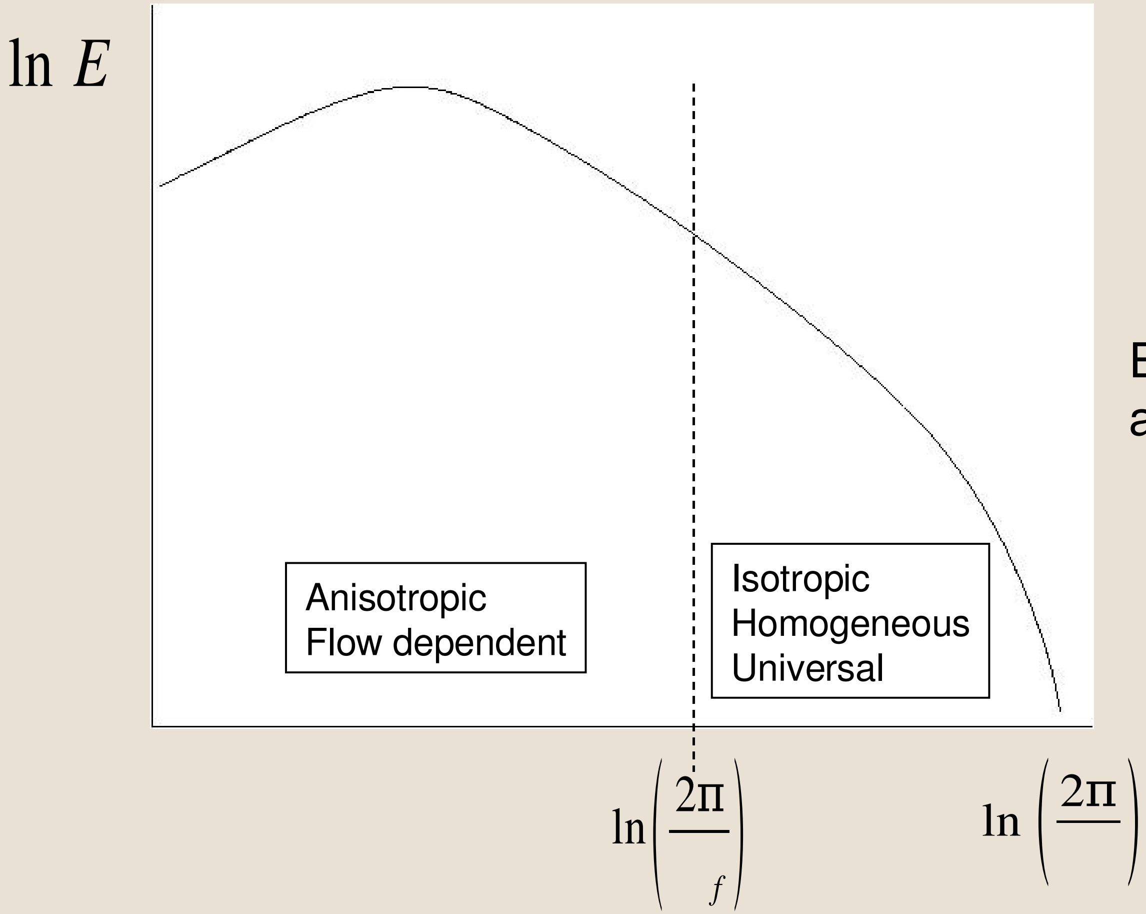
Iso-surface of velocity magnitude colored by strain-rate

What About the Cost?
LES is More Expensive

- LES always requires transient 3-D solutions to the filtered N-S equations
- Needs the mesh and the time-step sizes sufficiently fine to resolve the energy-containing eddies
- The cost of resolving near-wall region in high-Re wall-bounded flows is very high
- Compared to DNS ($\sim Re_t^3$), the cost of LES increases much more slowly with Re_t or remains largely independent of Re_t depending on whether or not near-wall region is resolved

What About the Cost?
Mesh Resolution (1)

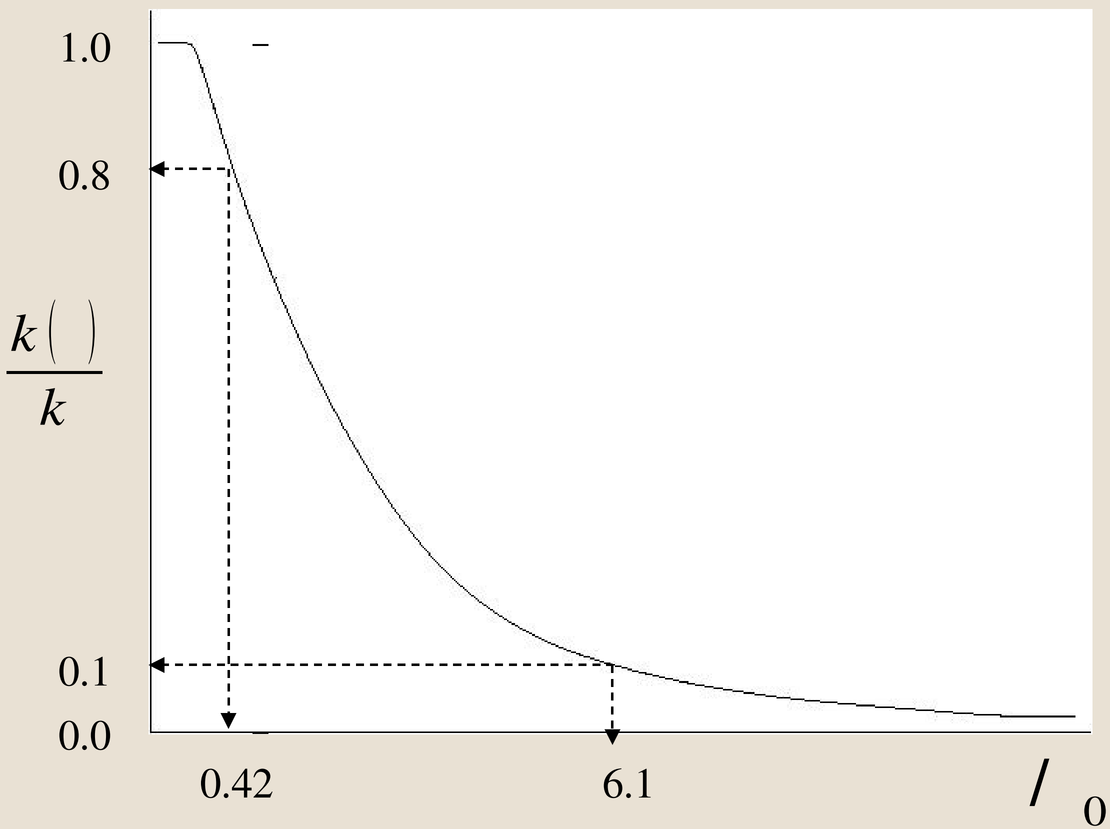
- The mesh resolution determines the fraction of turbulent kinetic energy directly resolved



Energy spectrum against the length scale

What About the Cost? Mesh Resolution (2)

- Suppose you want to resolve 80% of TKE.
- Then, we need to resolve the eddies whose sizes are larger than roughly half the size of the integral length scale ( ). $_0$



	$/\ _0$
$k(\ )=0.1\ k$	6.10
$k(\ )=0.5\ k$	1.6
$k(\ )=0.8\ k$	0.42
$k(\ )=0.9\ k$	0.16

Cumulative TKE against length-scale of eddies based on the Kolmogorov's energy spectrum

What About the Cost?
Time-Step Size

- The time-step size, Δt, should be small enough to resolve the time-scale, τ, of the smallest resolved eddies

- As $\tau \sim \Delta x / U$, it correspond to approximate value of Courant numer CFL = 1 (where U is the velocity scale of the flow)

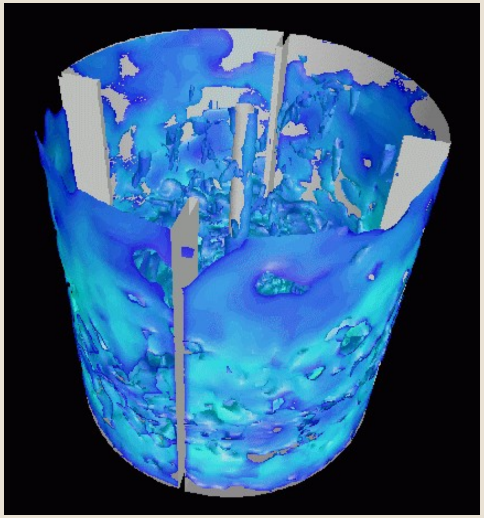
$$CFL = \frac{U \quad t}{x} \approx 1.0 \qquad \Rightarrow \qquad t = \frac{x}{U}$$

- New in 6.2: CFL post-treatment

What About the Cost?
An Example

Lab-scale mixing tanks

- Tank dimension : $L = 300\ mm$
- Integral length scale $l_0 \sim L/5 \sim 60$ mm
- Suppose you have a 100^3 cell mesh
  - This gives an average cell size of $\Delta x = 3\ mm$.
  - $l_0 = 20\,\Delta x$

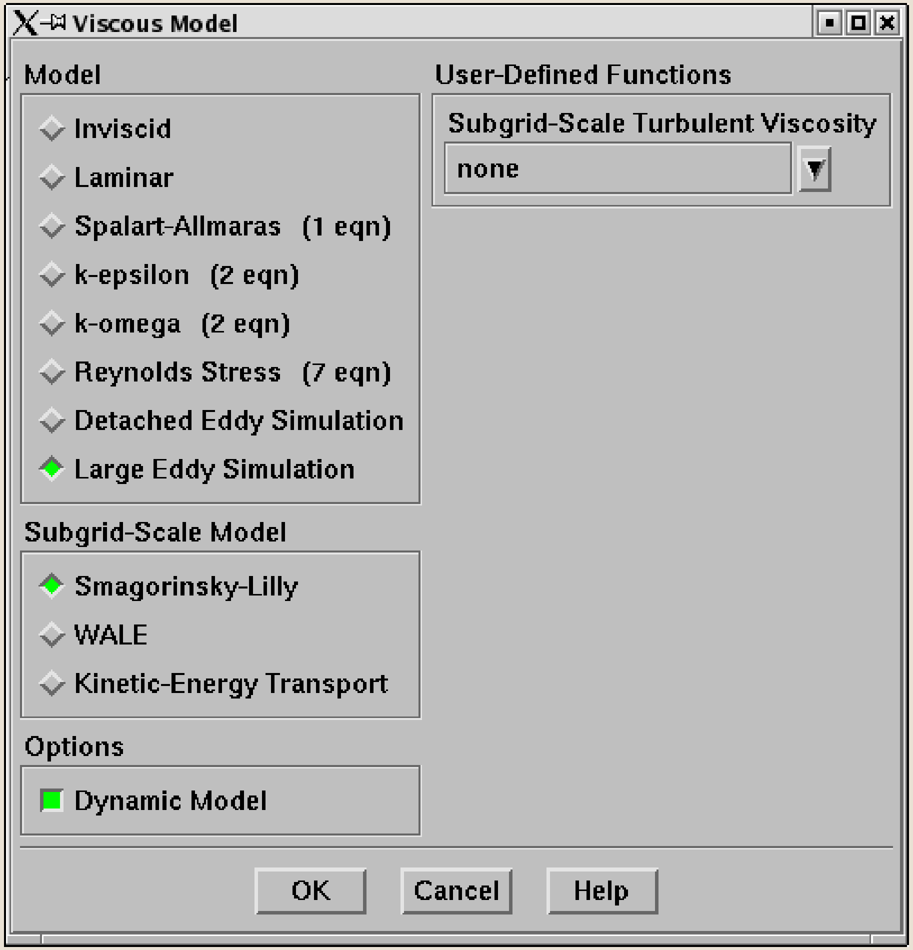


What About the Cost?
Wall-Bounded Flows (Pope, 2000)

- "Near-wall resolving" approach
  - Turbulence length scale becomes smaller in near-wall region.
  - Resolving the bulk of the energy requires a very fine mesh near the wall.
  - The cost increases as a power of the Reynolds number.
  - Too expensive for high-Re wall-bounded flows
- "Near-wall modelling" approach – An Alternative
  - Wall functions (e.g., laws-of-the-wall)
  - Zonal RANS/LES hybrid approaches (e.g., DES)

What Does FLUENT Offer?
LES Capability in FLUENT

- Underpinnings
  - Numerics
  - SGS turbulence models
  - Post-processing
- FLUENT's LES capability aimed at
  - Accuracy
  - Efficiency
  - Robustness
  - Usability



LES Capability in FLUENT
Numerics - Spatial Discretization (1)

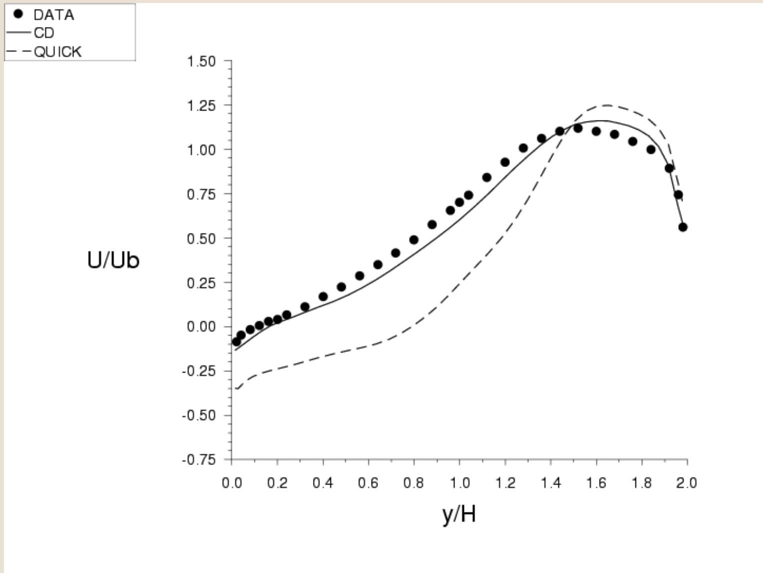
- Spatial accuracy has been greatly improved in recent releases.
  - Major upgrades
    - Node-based gradient scheme (6.1)
    - High-order reconstruction of “convecting” velocity (6.2)
  - Benefits mostly the cases involving unstructured (tet, hybrid) meshes
- Numerical diffusion ought to be minimized in LES.
  - Remember we set out to resolve eddies.
  - RANS vs. LES

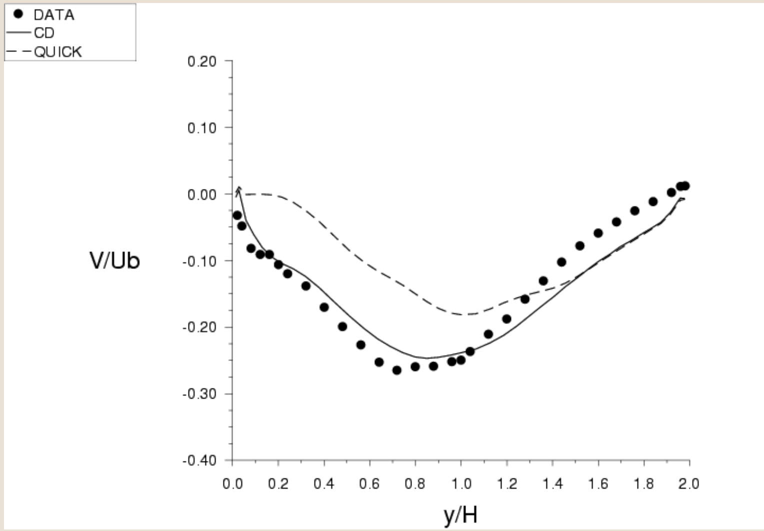
$$\nu_{t,\mathrm{RANS}} >> \nu_{t,\mathrm{SGS}}$$

LES Capability in FLUENT Numerics - Spatial Discretization (2)

- Discretization of convection terms is the most critical for LES
- FLUENT offers:
  - High-order upwind schemes (SOU, QUICK, MUSCL)
  - Bounded central differencing (BCD) scheme (*new in 6.2, seg. solver only, the default LES-convective scheme in 6.2*)
  - Low-diffusion flux (LDF) scheme (*new in 6.2, coupled solver only, default LES-convective scheme in 6.2*)
  - Central differencing (CD) scheme (*seg. solver only*)
- Central differencing (CD) gives the least numerical diffusion and thus the best accuracy
- Upwind schemes are too diffusive for LES and should be avoided whenever possible

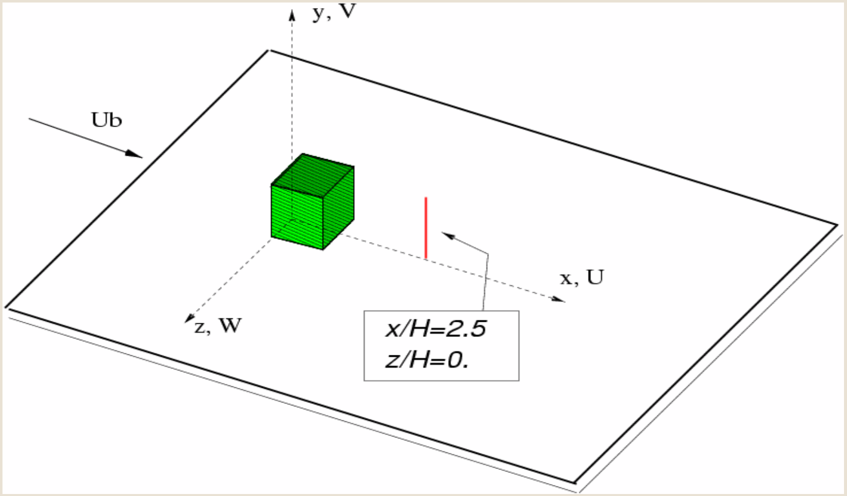
LES Capability in FLUENT
Example: CD *vs*. QUICK Scheme (1)

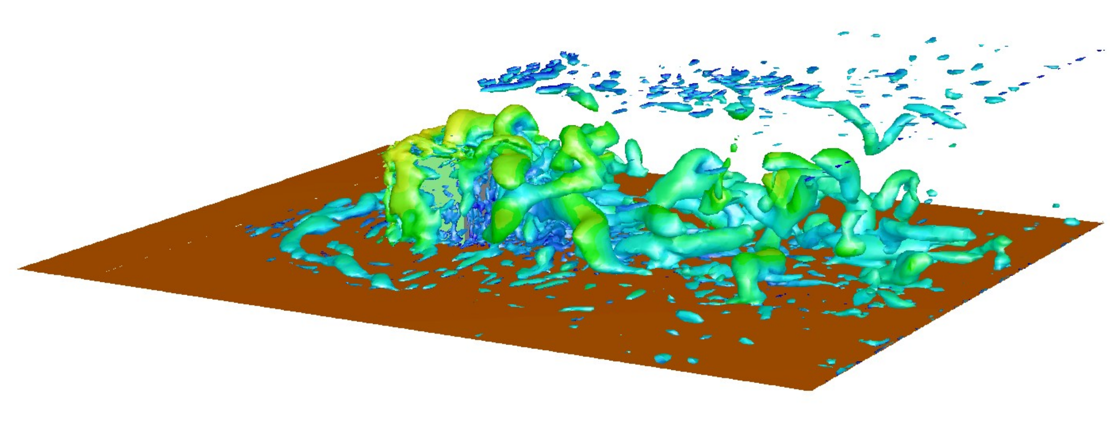




Surface-mounted cube in a channel

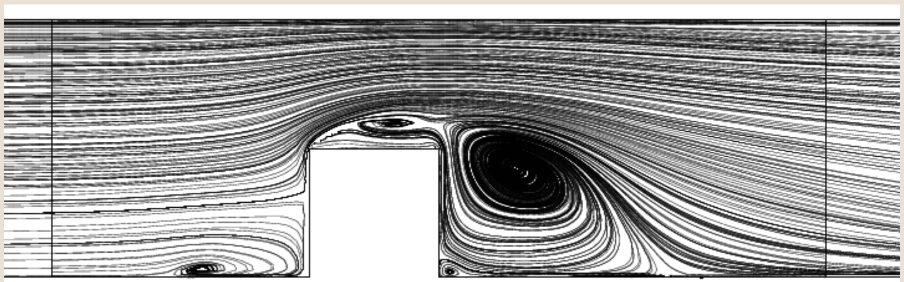
(Martinuzzi & Tropea, 1993)



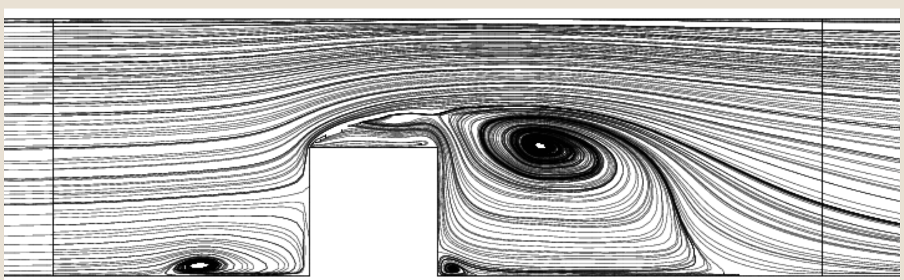


LES Capability in FLUENT
Example: CD *vs.* QUICK Scheme (2)

Central-differencing



QUICK



Time-averaged streamlines on the mid-plane

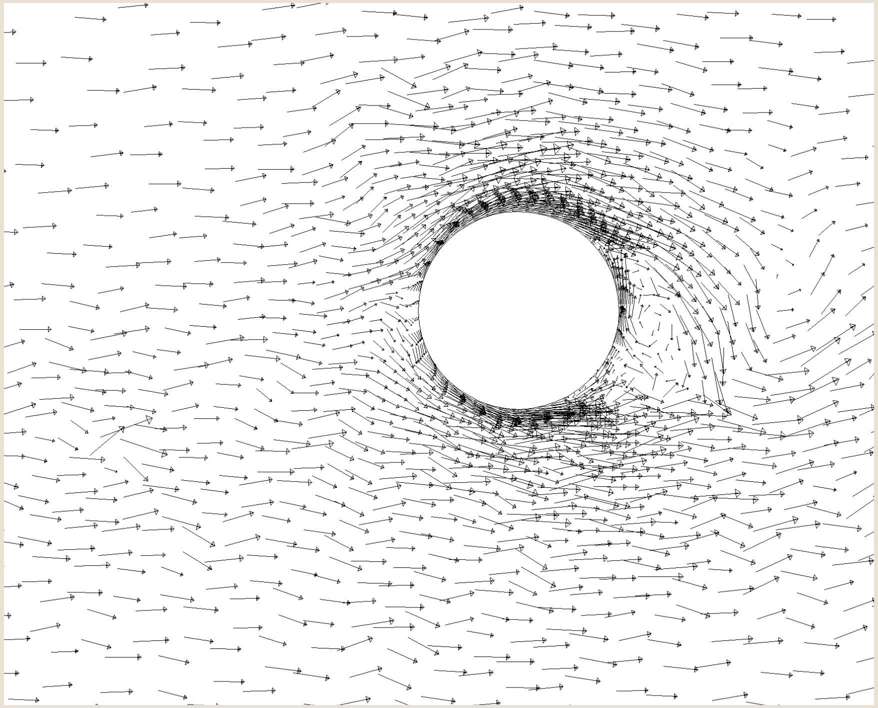
Separation and reattachment points predicted by FLUENT and others

	X_F	X_R
Exp. data (Martinuzzi & Tropea (1993)	1.04	1.61
FLUENT (LES + CD)	1.18	1.78
FLUENT (LES + QUICK)	1.26	2.40
Breuer *et al.* LES	1.23	1.70

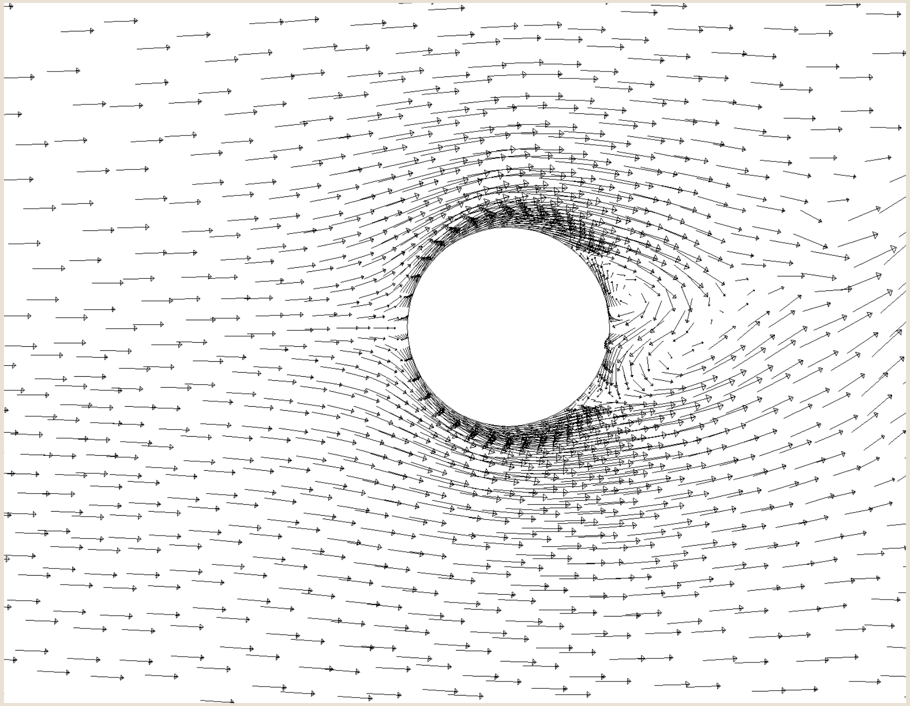
LES Capability in FLUENT
Numerics - Spatial Discretization (3)

- The CD scheme can become unstable, giving unphysical oscillations (wiggles) for high-Re flows.
- The BCD scheme detects and removes the wiggles.

Flow around 2-D section of a landing gear



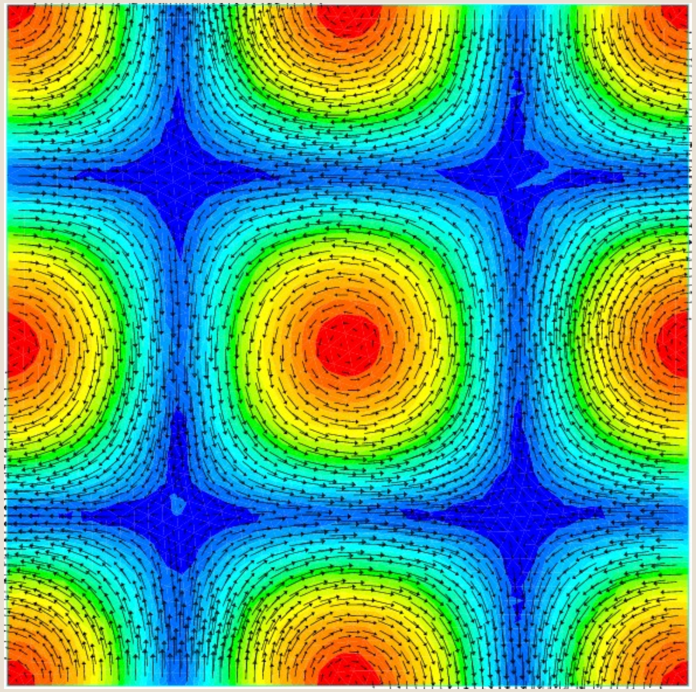
central differencing



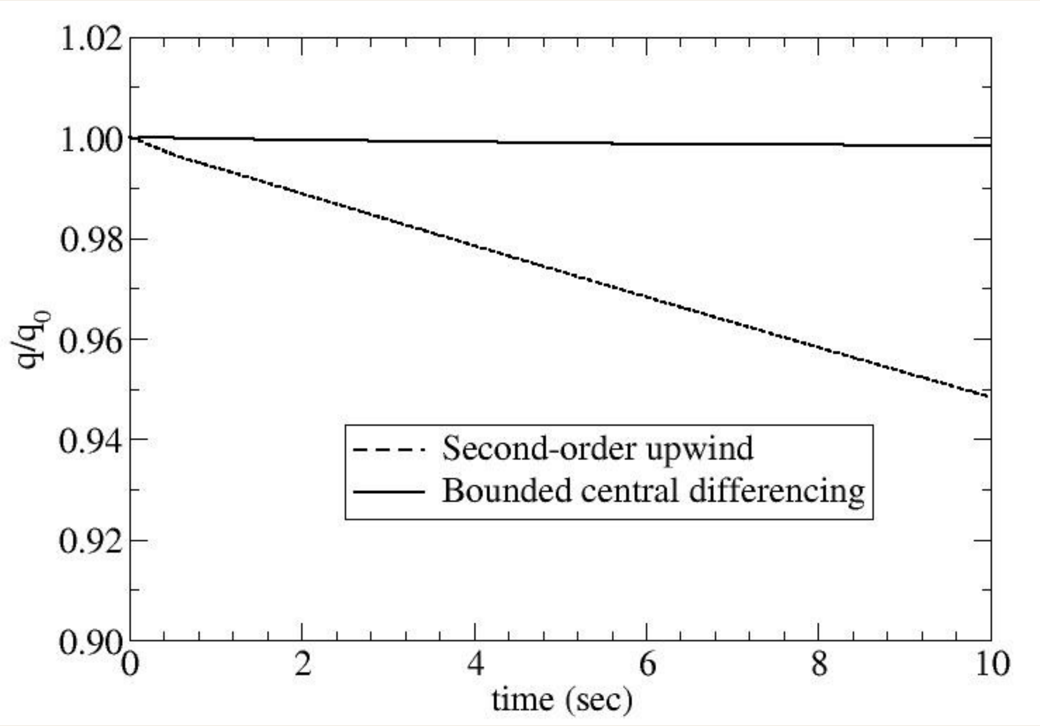
bounded central differencing

LES Capability in FLUENT
Numerics - Spatial Discretization (4)

- The BCD scheme has much lower numerical diffusion than high-order upwind schemes.

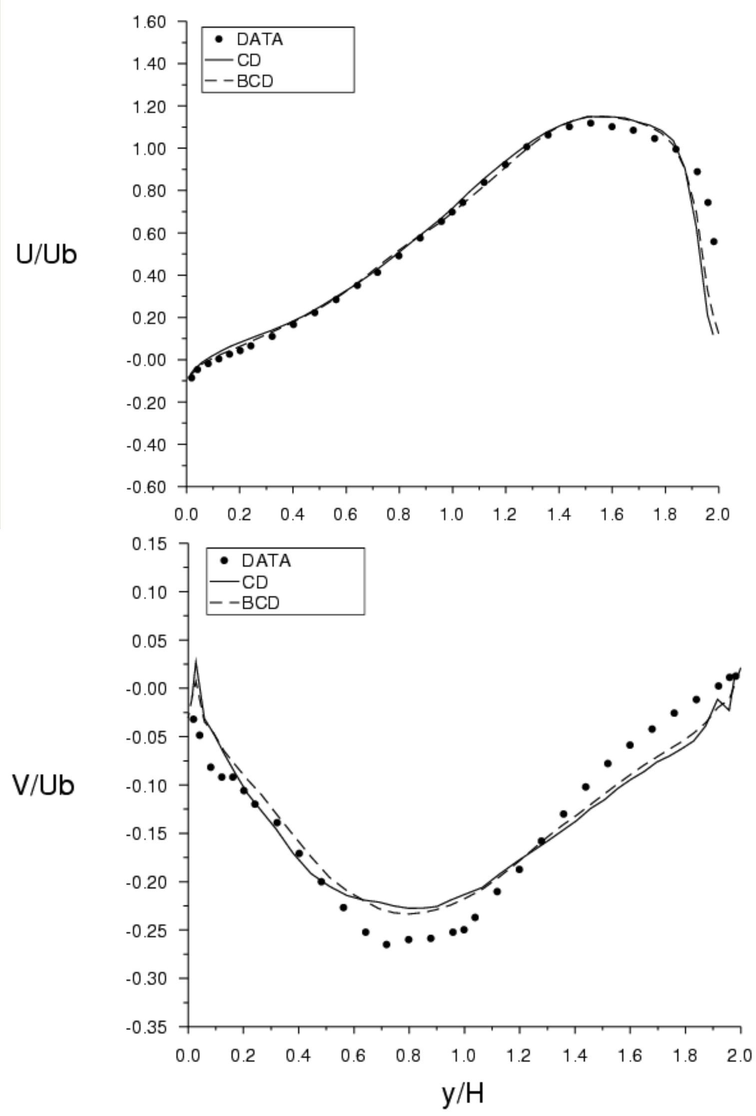


Taylor's inviscid vortex flow – 2-D periodic array of vortices

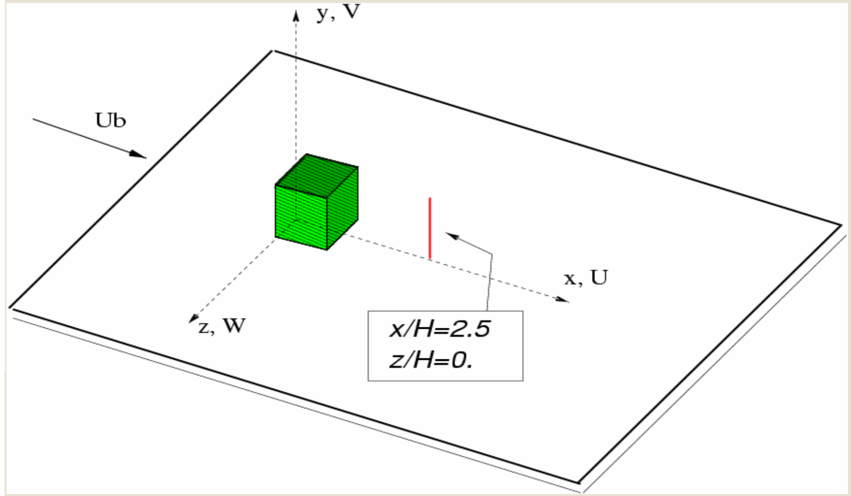


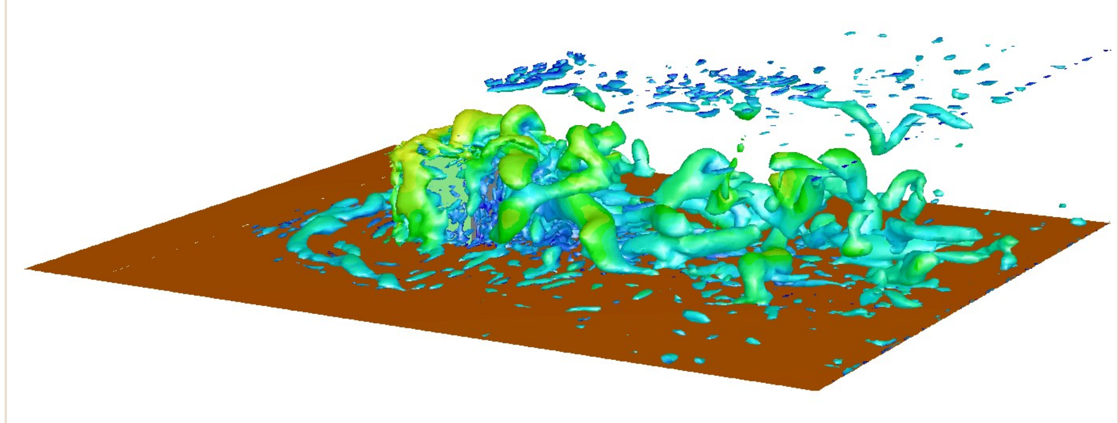
Evolution of total kinetic energy

LES Capability in FLUENT
Example: CD vs. BCD



Surface-mounted cube in a channel
(Martinuzzi & Tropea, 1993)





LES Capability in FLUENT
Numerics- Transient Algorithm (1)

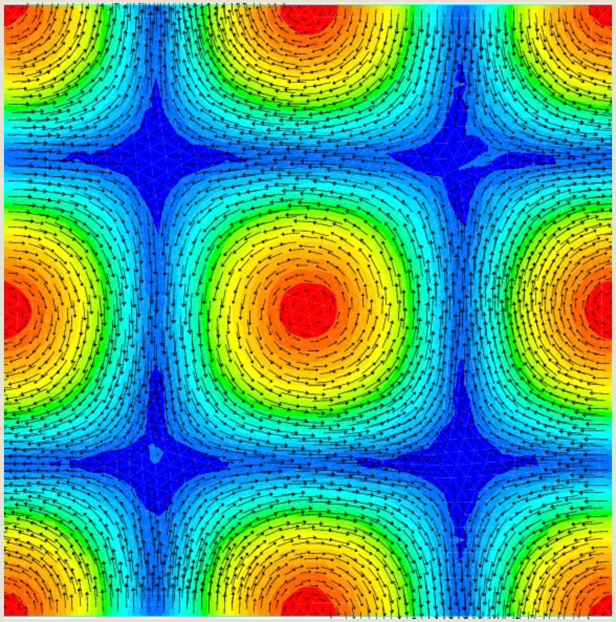
- Iterative Time-Advancement (ITA) schemes
  - Has been the workhorse for unsteady flow calculations in FLUENT segregated solver
  - No splitting error
  - Costly due to many outer (velocity-pressure) iterations
  - SIMPLE, SIMPLEC, PISO
- Non-Iterative Time-Advancement (NITA) schemes
  - New in 6.2 (*segregated solver only*)
  - Only one outer iteration per time-step
  - Splitting error commensurate with the truncation error (~ $O(\Delta t^2)$)
  - Two flavours
    - Fractional-step method
    - PISO

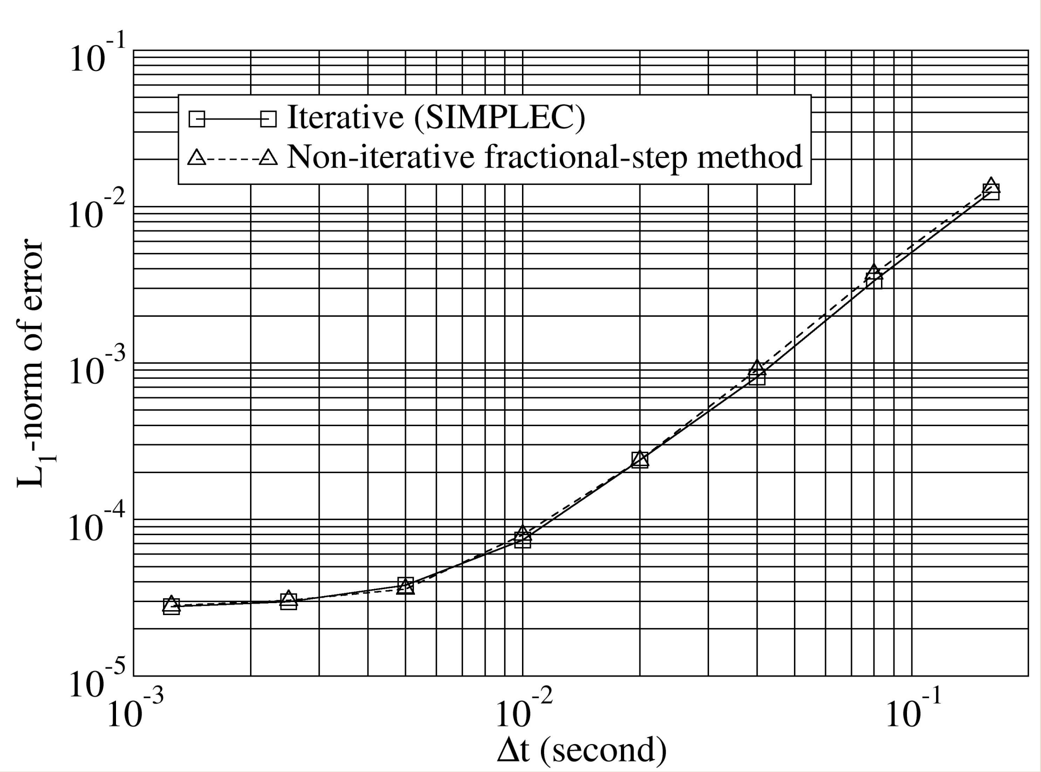
LES Capability in FLUENT
Transient Algorithm (2) – Example

Taylor's Vortex

Comparison of the fully-iterative (SIMPEC) and non-iterative time-advancement schemes

Re = 1

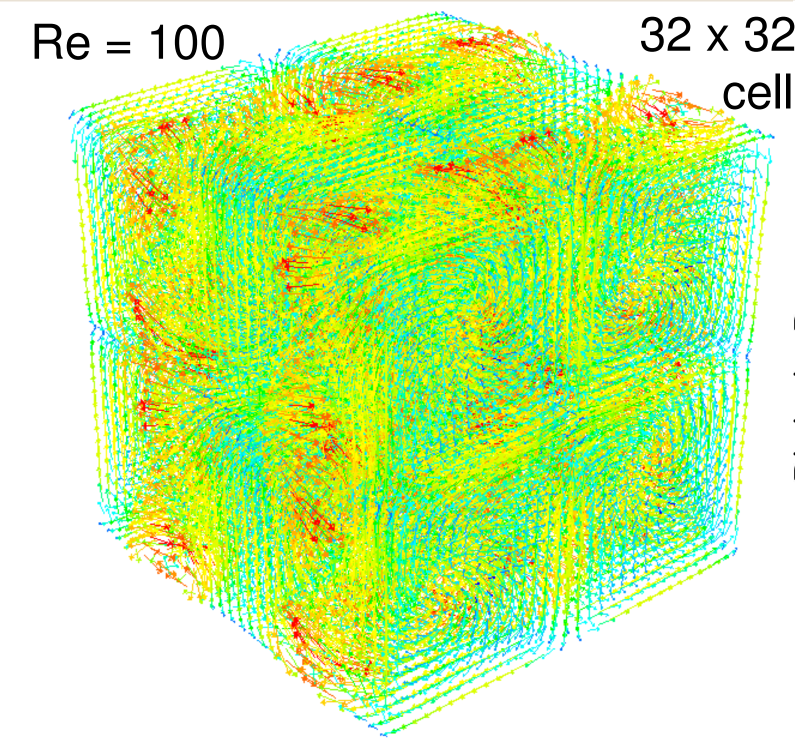


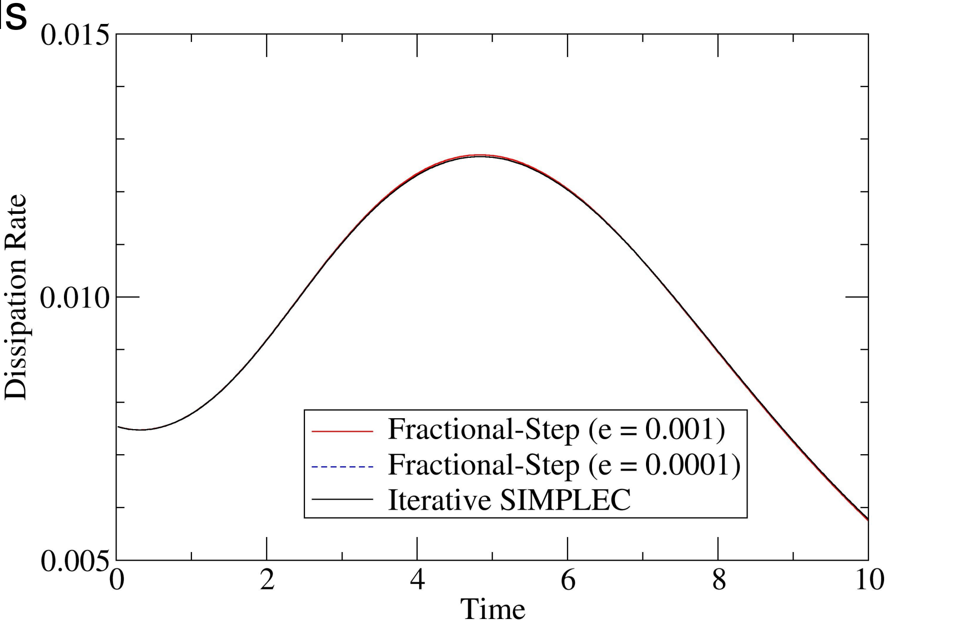


LES Capability in FLUENT Transient Algorithm (3) – Example

Green's Vortex

Comparison of NITA (SIMPEC) and NITA/FSM schemes





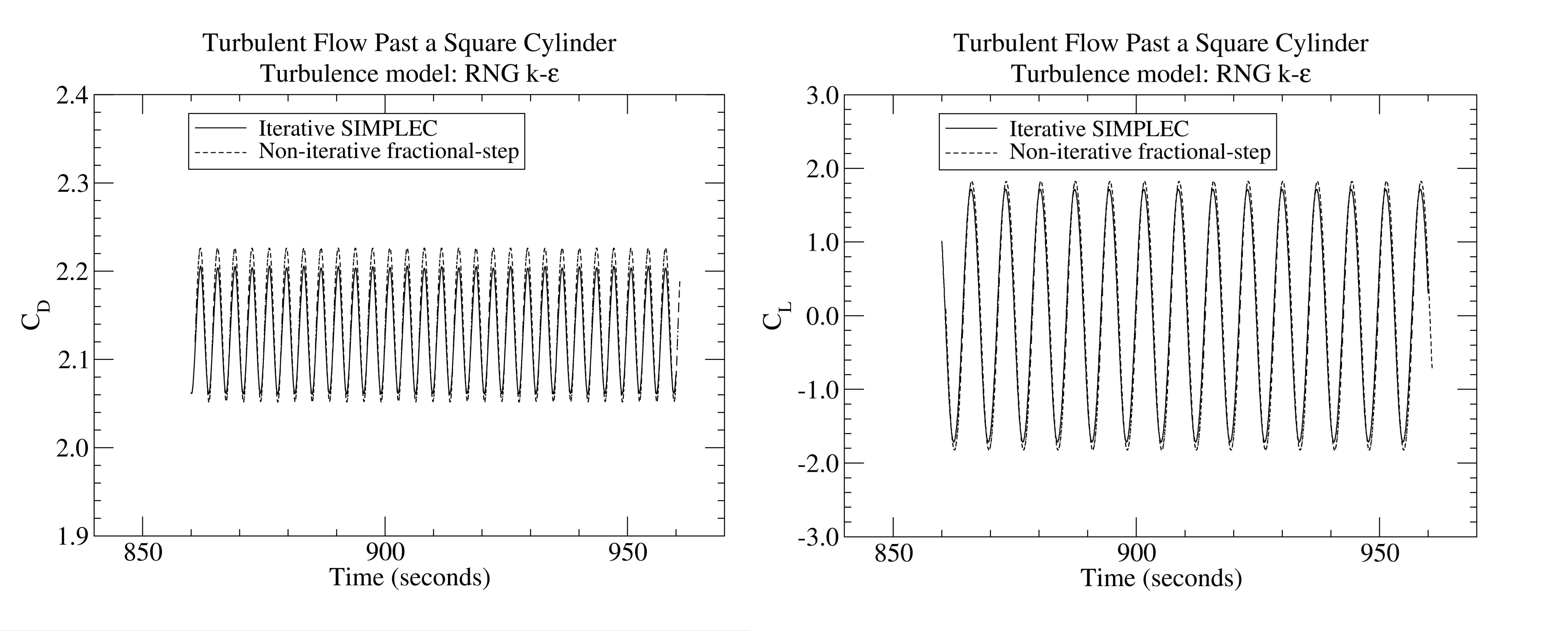
Dissipation rate integrated over the volume

LES Capability in FLUENT
Transient Algorithm (4) – Example

Vortex-Shedding from a Square Cylinder

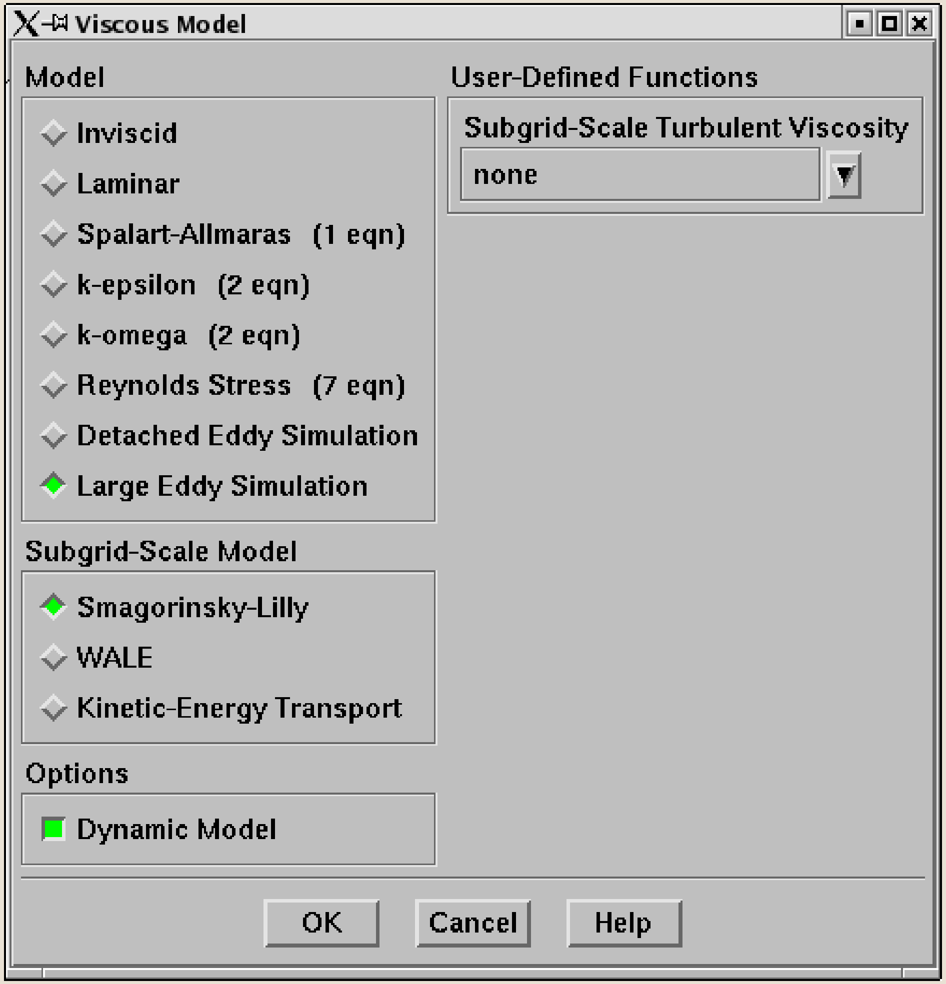
Comparison of NITA (SIMPEC) and NITA/FSM scheme

Re_H = 22,000 (Lyn, 1995), RNG k-ε model



LES Capability in FLUENT
Subgrid-Scale (SGS) Viscosity Models

- FLUENT 6.2 offers
  - Smagorinsky's model
  - WALE model (new in 6.2)
  - Dynamic Smagorinsky model (new in 6.2)
  - Dynamic subgrid kinetic energy transport model (new in 6.2)



LES Capability in FLUENT
Subgrid-Scale (SGS) Viscosity Models

Smagorinsky's Model

- Simple algebraic (0 - equation) model

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\left(C_s \bar{\ }\right)^2 |\bar{S}|\bar{S}_{ij}$$

with $\bar{\ } = \nabla^{1/3}, \quad \bar{S} \equiv \sqrt{2S_{ij}S_{ij}}$

- ***C_s = 0.1 ~ 0.2***
- The major shortcoming is that there is no ***C_s*** universally applicable to different types of flow
- Difficulty with transitional (laminar) flows
- An *ad hoc* damping is needed in near-wall region

WALE Model

- Wall-Adapting Local Eddy-Viscosity model
- Algebraic (0 - equation) model – retains the simplicity of Smagorinsky's model

$$\nu_{SGS} = \left(C_s \bar{\Delta}\right)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\underbrace{\left(\bar{S}_{ij}\bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}_{\text{near-wall modification}}}$$

- The WALE SGS model adapts to local near-wall flow structure
  - Wall damping effects are accounted for without using the damping function explicitly

WALE Model (cont'd)

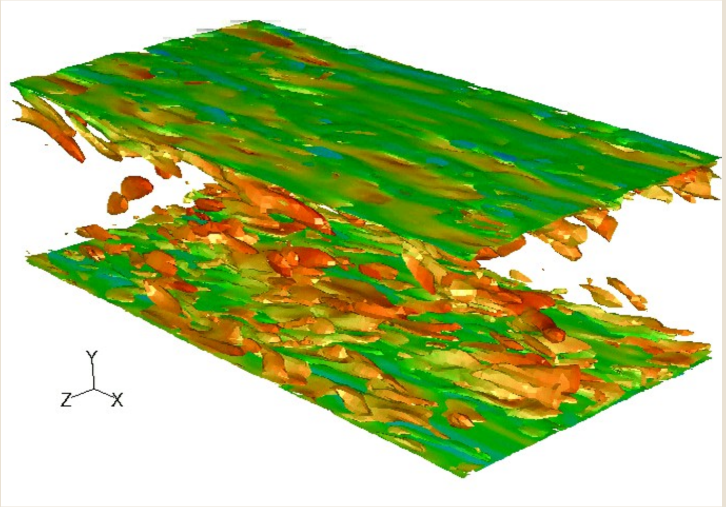
- Example
  - Simulate the mechanism associated with drag reduction using oscillating walls (Re_t=395, Re_H=20 000)
  - The wall is forced to oscillate in the span-wise direction with

$$W^+ = A^+ \sin(2\pi t^+/T^+)$$

  - Interaction between the stokes layer and the near-wall structures

Reduction rate (%) in wall-shear

	A^+=5	A^+=10	A^+=20
LES (FLUENT)	10.	22.	30.4
DNS (Choi *et al*)	9.4	22.8	30.6



LES Capability in FLUENT
Subgrid-Scale (SGS) Viscosity Models

Dynamic Smagorinsky's Model

- Based on the similarity concept and Germano's identity (Germano *et al*., 1991; Lilly, 1992)
- The model parameter (C_s) is automatically adjusted using the resolved velocity field
- FLUENT implementation
  - Locally dynamic model
  - Adapted for unstructured meshes (test-filter)
- Overcomes the shortcomings of the Smagorinsky's model
  - Can handle transitional flows
  - The near-wall (damping) effects are taken into account

Dynamic Subgrid KE Transport Model

- Kim and Menon (1997)
- One-equation (for SGS kinetic energy) model

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2C_k k_{sgs}^{1/2} \, \bar{\;} \, \bar{S}_{ij}$$

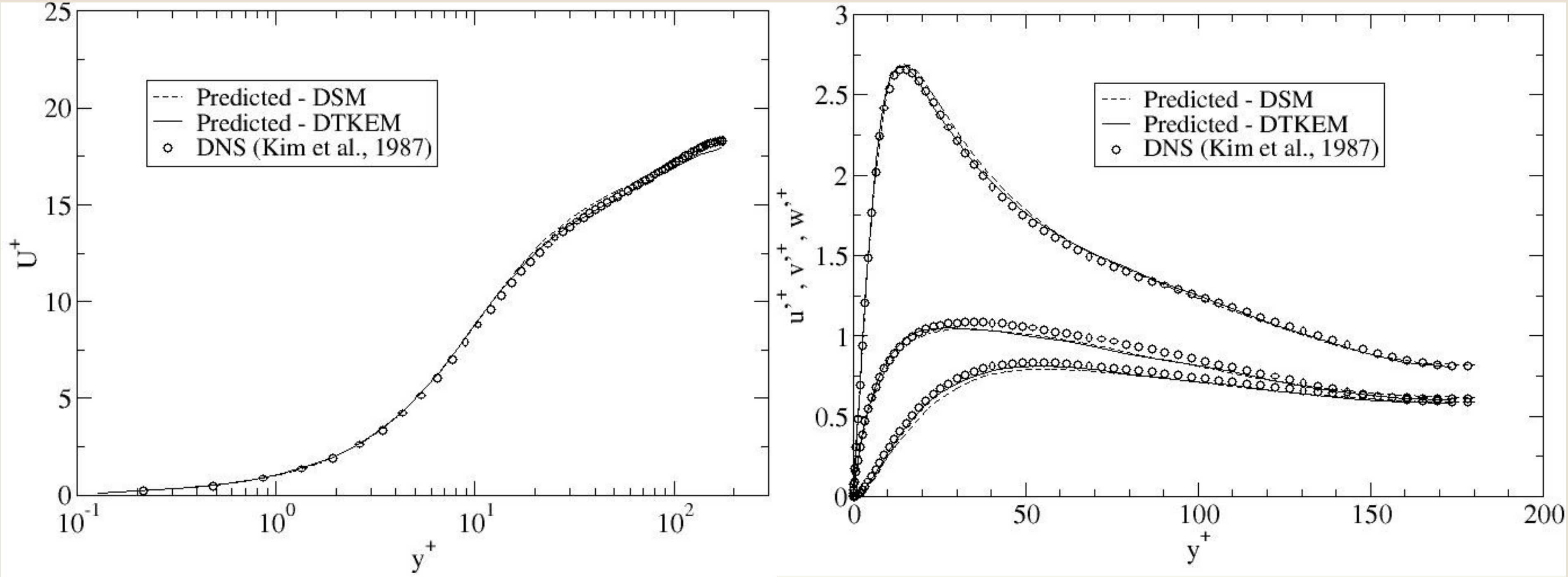
$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial \bar{u}_j k_{sgs}}{\partial x_j} = -\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j} - C_\varepsilon \frac{k_{sgs}^{3/2}}{\bar{\;}} + \frac{\partial}{\partial x_j}\left(\frac{\nu_{sgs}}{\sigma_k}\frac{\partial k_{sgs}}{\partial x_j}\right)$$

- Like the dynamic Sgamorinsky's model, the model constants (***C_k***, ***C_ε***) are automatically adjusted on-the-fly using the resolved velocity field.
- The dynamic procedure has stronger arguments than the dynamic Smagorinsky's model.

LES Capability in FLUENT
Subgrid-Scale (SGS) Viscosity Models

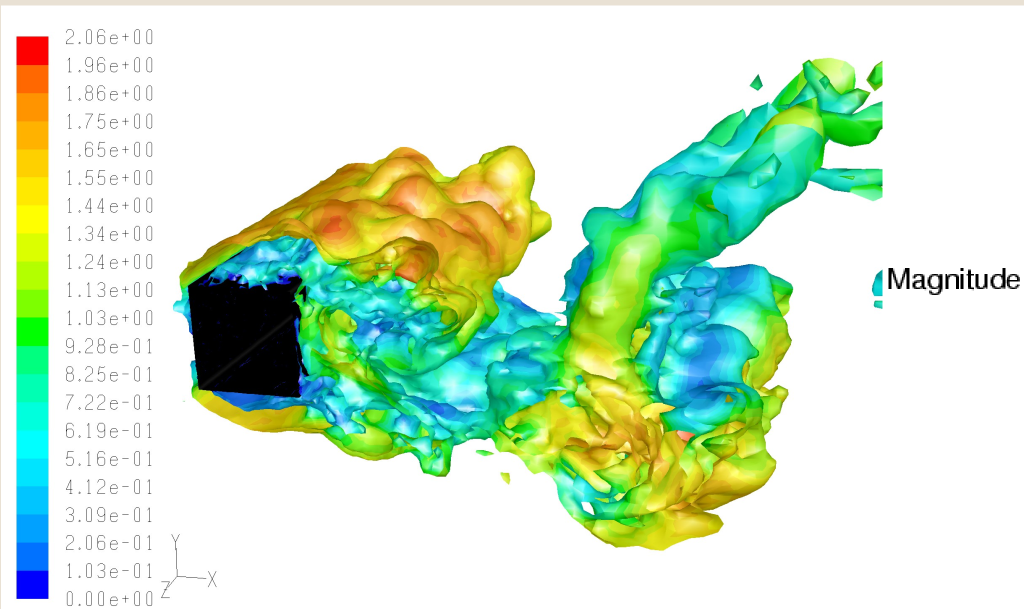
Channel Flow (Re_t = *180*)

72 x 72 x 72 cells

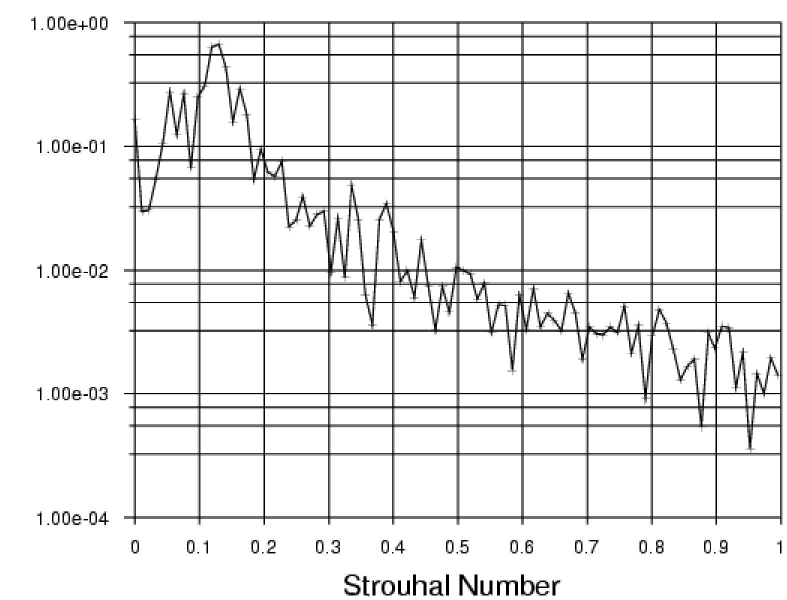


Example: LES of the Flow Past a Square Cylinder (Re_H = 22,000)

	C_D	St
Dynamic Smag.	2.28	0.130
Dynamic TKE	2.22	0.134
Exp.(Lyn *et al.*, 1992)	2.1 – 2.2	0.130

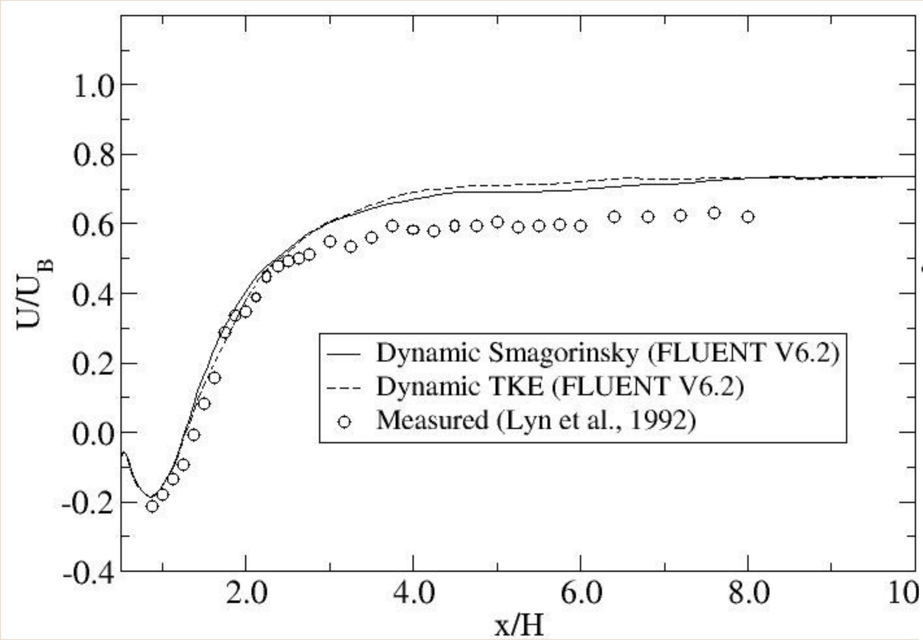


Iso-contours of instantaneous vorticity magnitude

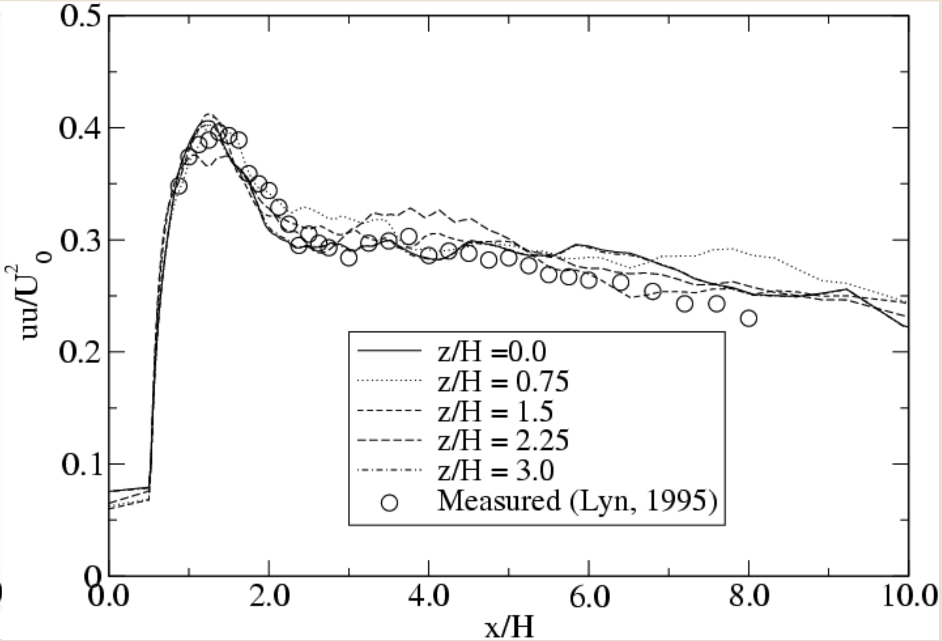


C_L spectrum

Example: LES of the Flow Past a Square Cylinder (Re_H = 22,000)



Streamwise mean velocity along the wake centerline



Streamwise normal stress along the wake centerline

LES Capability in FLUENT
SGS Models: Summary

Sub-grid stress : turbulent viscosity

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\rho\nu_t\bar{S}_{ij}$$

- Smagorinsky model (*Smagorinsky, 1963*)
  - Need ad-hoc near wall damping

$$\nu_t = \left(C_s\bar{\Delta}\right)^2 \left|\bar{S}\right|$$

- WALE model (*Nicoud & Ducros 1999*)
  - Correct asymptotic near wall behaviour

$$\nu_t = \left(C_s\bar{\Delta}\right)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\bar{S}_{ij}\bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- Dynamic model (*Germano et al., 1991*)
  - Local adaptation of the Smagorinsky constant

$$\nu_t = \left(C_D\bar{\Delta}\right)^2 \left|\bar{S}\right|$$

- Dynamic sub-grid kinetic energy transport model (*Kim & Menon 2001*)
  - Robust constant calculation procedure
  - Physical limitation of backscatter

$$\nu_t = C_k k_{sgs}^{1/2} \bar{\ }$$

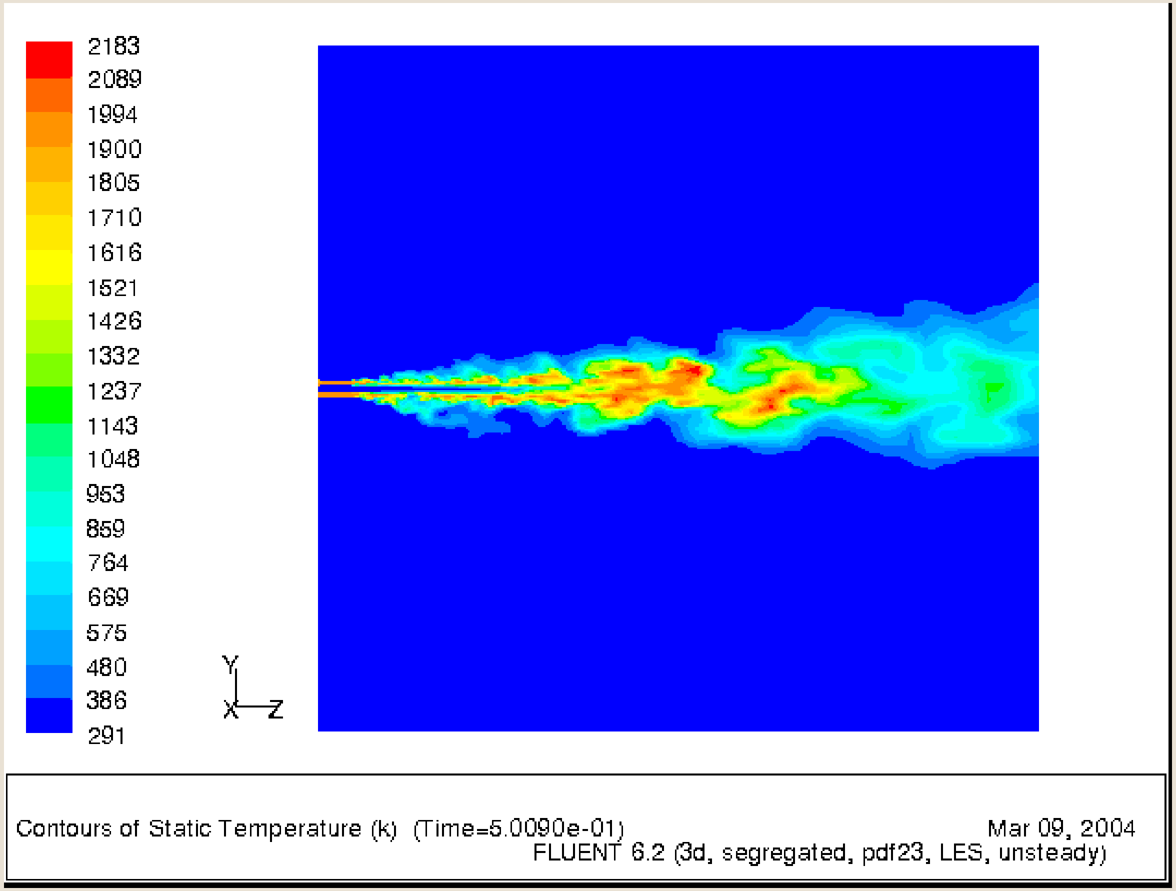
$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial \bar{u}_j k_{sgs}}{\partial x_j} = -\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j} - C_\varepsilon \frac{k_{sgs}^{3/2}}{\bar{\ }} + \frac{\partial}{\partial x_j}\left(\frac{\nu_{sgs}}{\sigma_k}\frac{\partial k_{sgs}}{\partial x_j}\right)$$

LES Capability in FLUENT
LES for Combustion Simulation

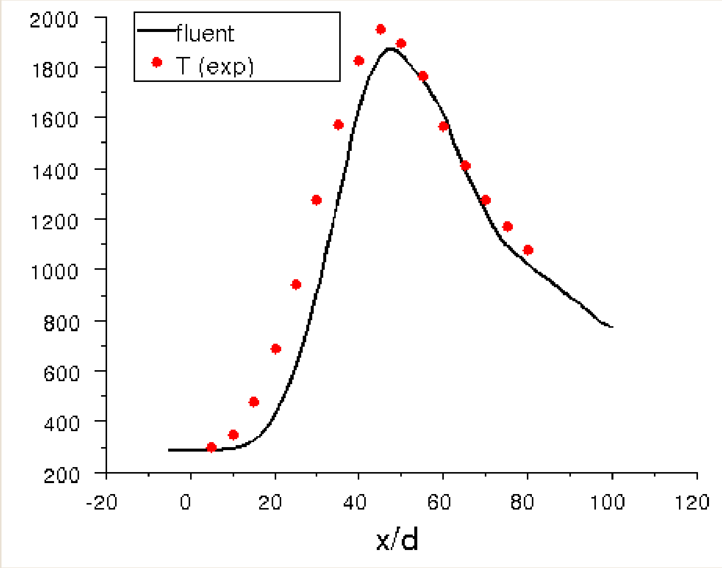
- LES is compatible with all FLUENT's turbulent combustion models
  - Eddy-dissipation/finite-rate
  - Non-premixed combustion model (mixture-fraction/PDF)
  - Partially-premixed or premixed combustion model (Reaction-progress variable)
  - PDF transport
- The dynamic procedure is not implemented yet for scalar transport equations
  - Subgrid-scale turbulent diffusivities are computed with Prandtl (Schmidt) number.

LES Capability in FLUENT
Example: Combustion Simulation

LES for Sandia Flame-D using mixture-fraction/PDF model



Instantaneous temperature field



Mean temperature (in K) along the centreline

LES Capability in FLUENT
Inflow Boundary Conditions

- It is often important to specify a realistic turbulent inflow velocity for accurate prediction of the (downstream) flow

$$u_i(x,t) = \underbrace{U_i(x)}_{time-averaged} + \underbrace{u_i'(x,t)}_{coherent+random}$$

- The random-number based inflow boundary condition in 6.1 is superseded by two new methods in 6.2
  - Spectral synthesizer
  - Vortex method

LES Capability in FLUENT
Inflow Boundary Conditions

Spectral Synthesizer (1)

- Largely based on the work of Celik *et al.*(2001)

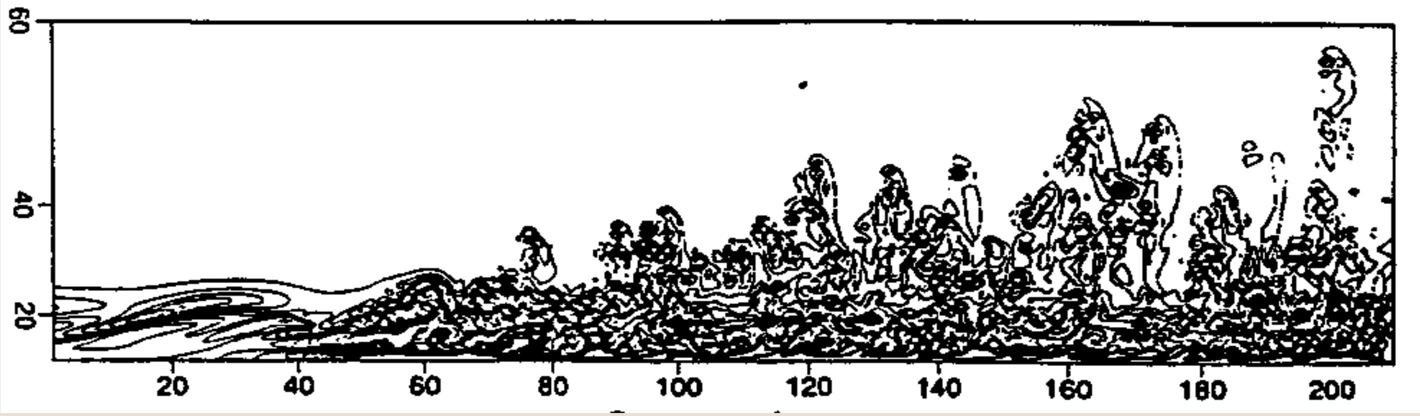
$$v_i(\vec{x},t) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} [p_i^n \cos(\tilde{k}_j^n \, \tilde{x}_j + \omega_n \tilde{t}) + q_i^n \sin(\tilde{k}_j^n \, \tilde{x}_j + \omega_n \tilde{t})]$$

- Able to synthesize anisotropic, inhomogeneous turbulence from RANS results (*k-ε*, *k-ω*, and RSM fields)

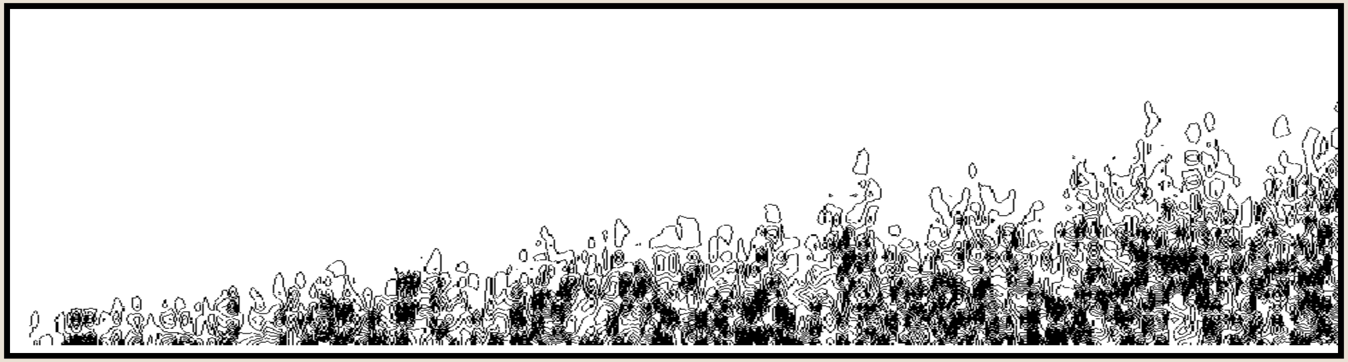
- The velocity-field satisfies the continuity by design.

LES Capability in FLUENT
Inflow Boundary and Conditions

Spectral Synthesizer (2)



Speziale, C.G., AIAA J., Vol.36, No.2, 1998



A.Smirnov,I.Celik,S.Shi, JFE, Vol.123, pp.359-371, 2001

Vortex Method

- In essence, vorticity-transport is modeled by distributing and tracking many point-vortices on a plane (Sergent, Bertoglio)

$$\omega(x,t)=\sum_{k=1}^{N}\Gamma_k(t)\,\eta\left(|x-x_k|,t\right)$$

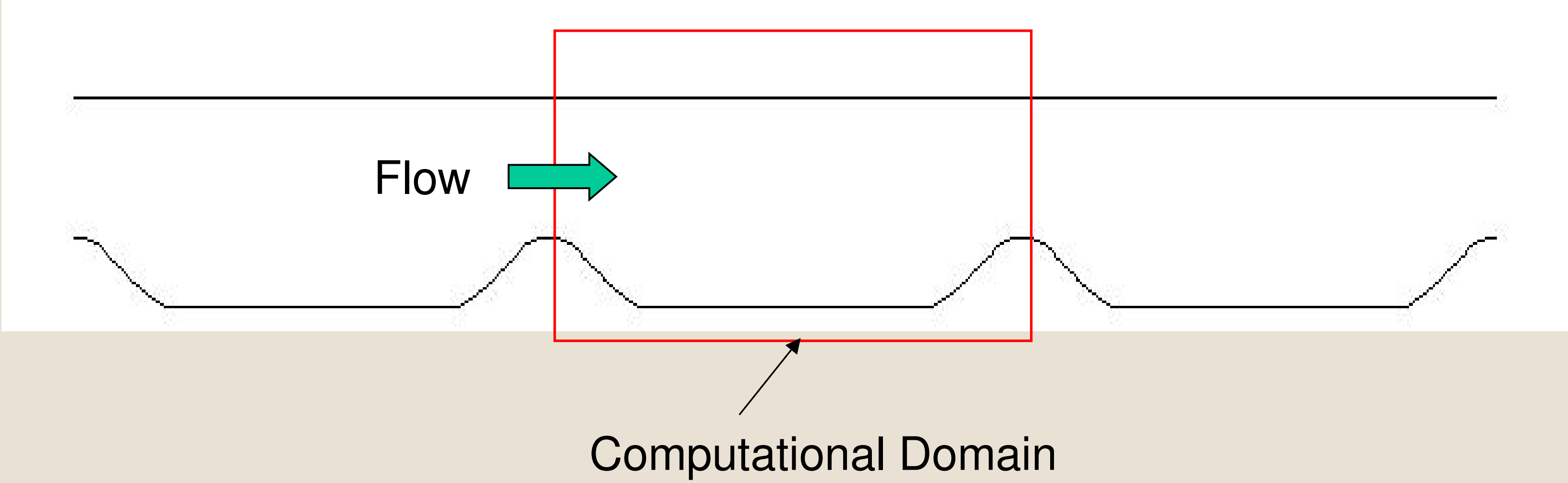
- Velocity field computed using the Biot-Savart's law

$$u(x,t)=-\frac{1}{2\pi}\iint\frac{(x-x')\times\omega(x')\,e_z}{|x-x'|^2}\,dx'$$

LES Capability in FLUENT
Inflow Boundary Conditions

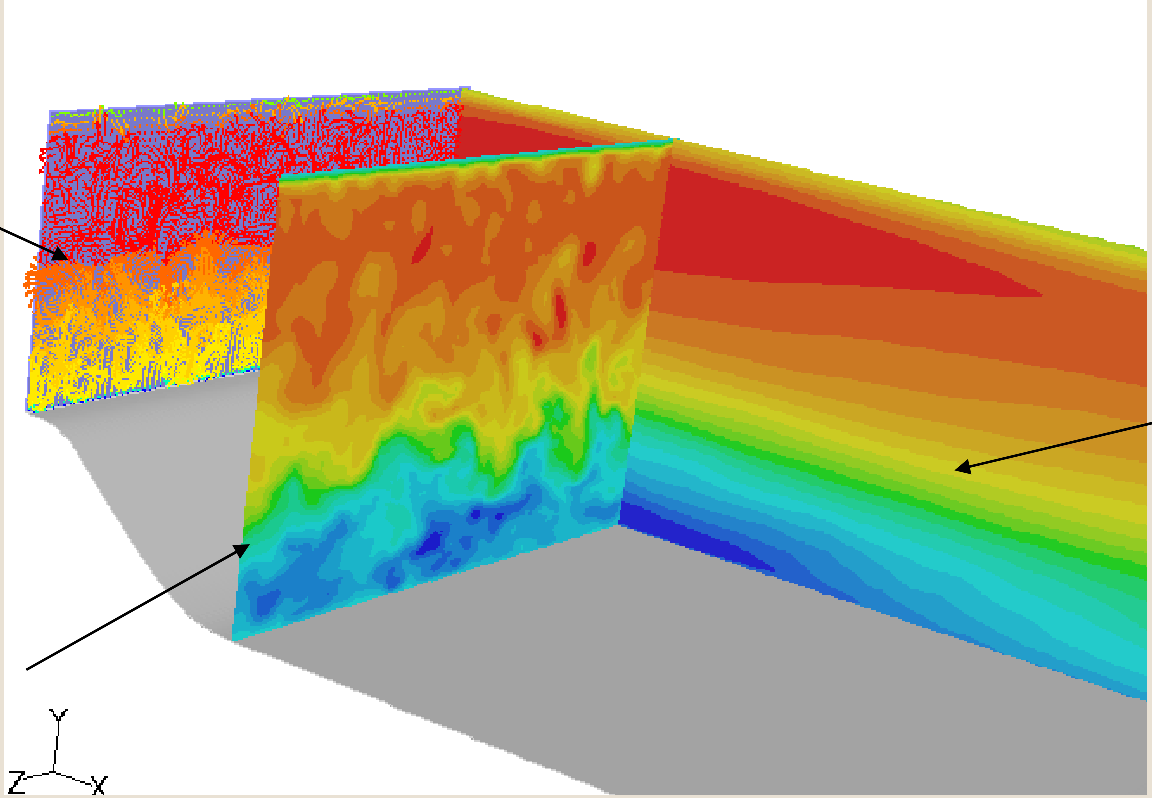
Vortex Method - Example

3-D Wavy Channel ($Re_H = 10{,}600$)



Vortex Method – Example (cont'd)

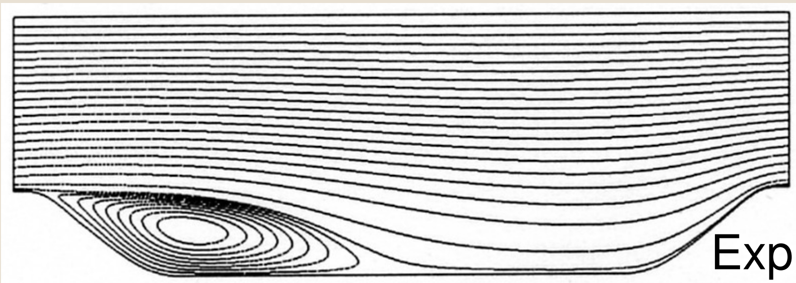
Vortex Inlet
- U_{mean}
- k, ε (...)



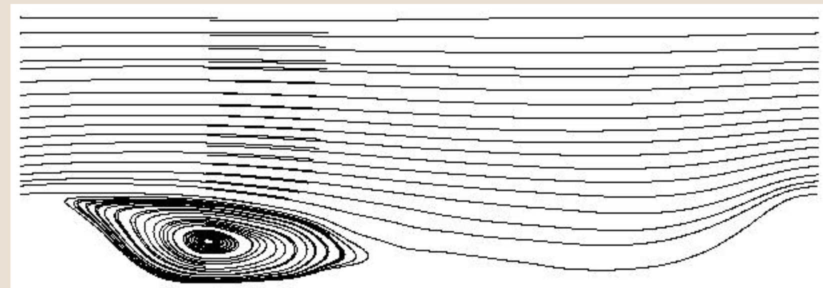
Mean field

Instantaneous field

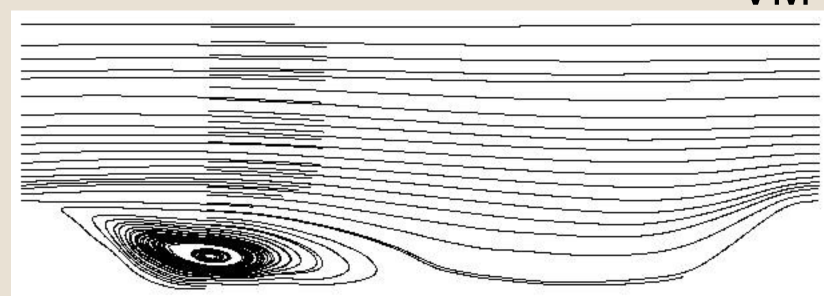
Vortex Method – Example (cont'd)



Periodic



VM



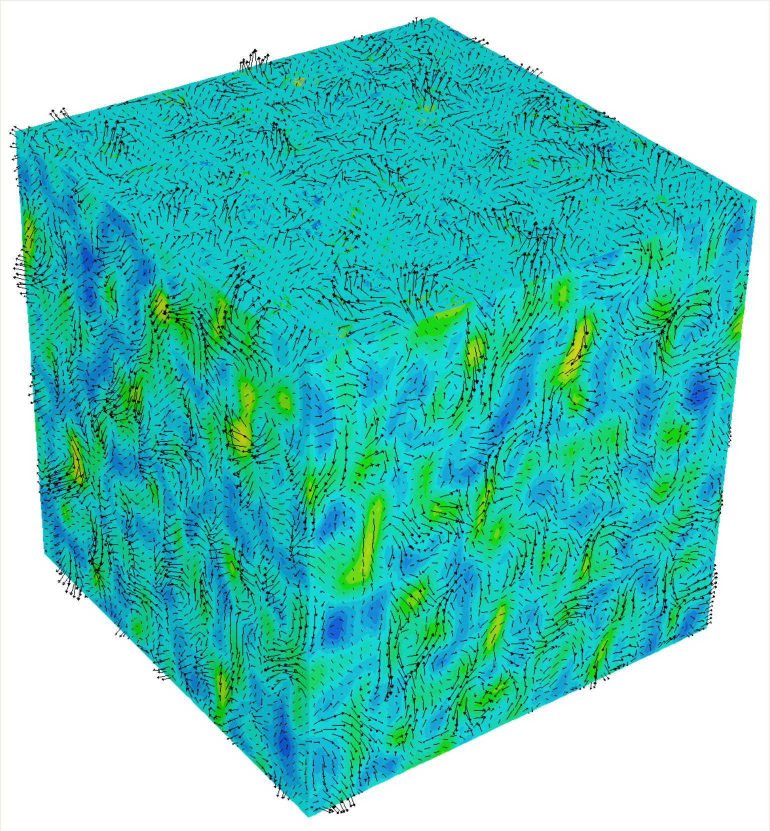
Random number

LES predictions of the reattachment point by different methods

	x_r
Exp.	$4.7\ H$
Periodic	$5.0\ H$
VM	$5.2\ H$
Random	$7.7\ H$

LES Capability in FLUENT
Initial Conditions

- Initial condition for velocity field is generally not important for statistically steady-steady flows
- Patching a realistic turbulent velocity field can however help shorten the simulation time substantially to get to a statistically steady state
- The same spectral synthesizer can be used to superimpose turbulence on top of the mean velocity field (new in 6.2).



Velocity field generated by turbulence synthesizer for a homogeneous turbulence

LES Capability in FLUENT
Detached Eddy Simulation (DES)

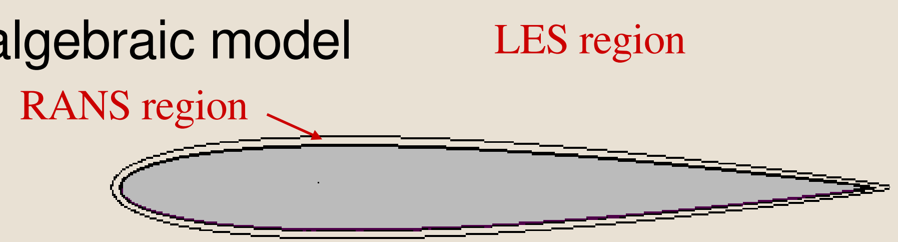
- Motivation
  - Near-wall-resolving LES is prohibitively expensive for high-Re wall bounded flows
  - Using RANS in near-wall regions would significantly mitigate the mesh resolution requirement, reducing the cost
- FLUENT offers a RANS/LES hybrid model based on S-A turbulence model (Spalart *et al.*, 1997)

$$\frac{D\tilde{\nu}}{Dt} = C_{b1}\tilde{S}\tilde{\nu} - C_{w1} f_w \left(\frac{\tilde{\nu}}{\bar{d}}\right)^2 + \frac{1}{\sigma_{\tilde{\nu}}}\left[\frac{\partial}{\partial x_j}\left\{\left(\quad + \rho\tilde{\nu}\right)\frac{\partial \tilde{\nu}}{\partial x_j}\right\} + \ldots\right]$$

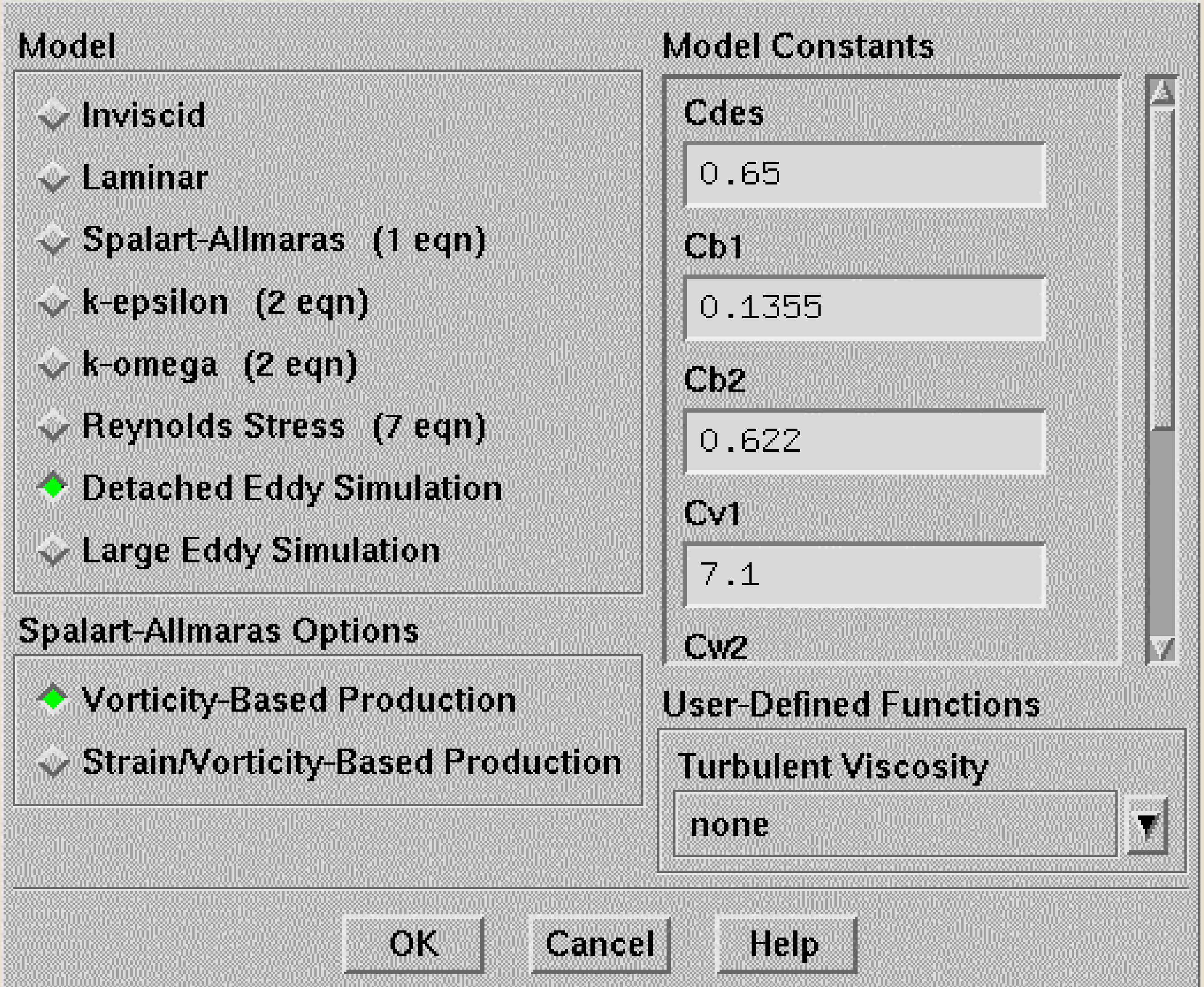
$$\bar{d} = \min\left(d_w, C_{DES} \quad \right)$$

- One-equation SGS turbulence model
  - In equilibrium, it reduces to an algebraic model

$$\tilde{\nu} \approx \left(C_{DES} \quad \right)^2 S$$

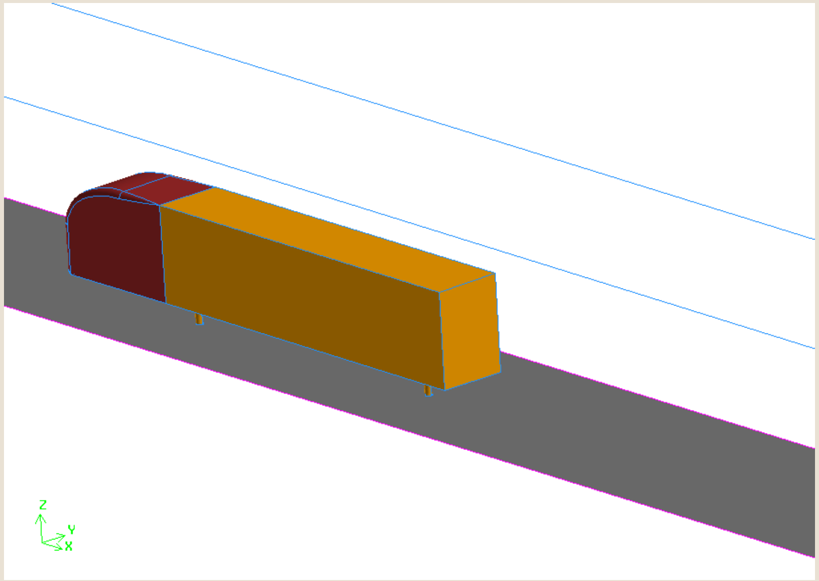


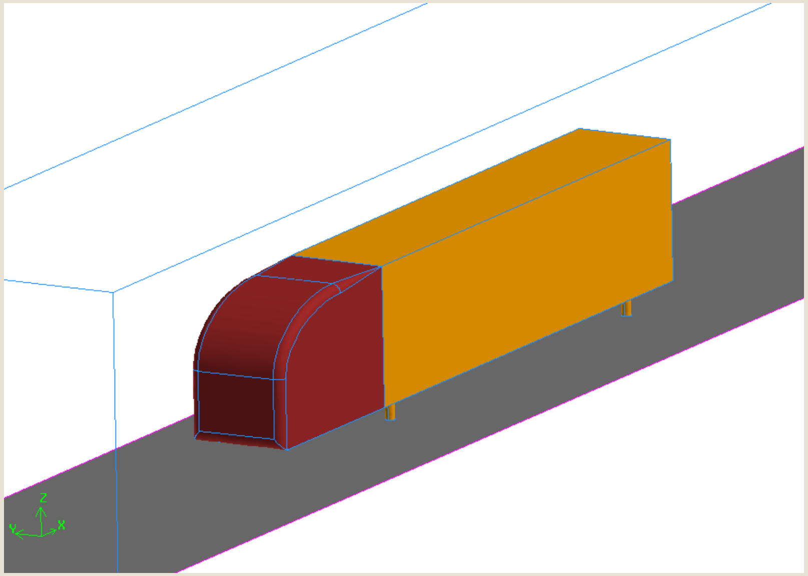
LES Capability in FLUENT
Detached Eddy Simulation (DES)



LES Capability in FLUENT
DES Example: GTS Body (1)

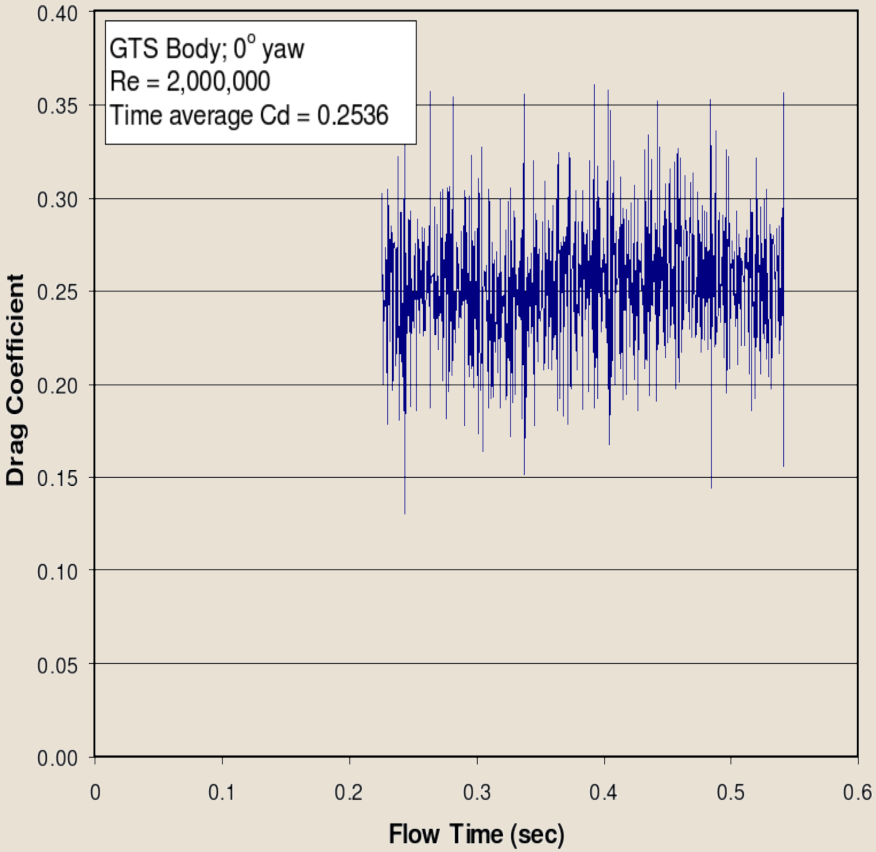
- Simplified truck configuration tested at the NASA Ames wind tunnel.
- $Re_L = 2 \cdot 10^6$
- Computed with 11M-cell hex mesh
- Zero-yaw (head-wind)



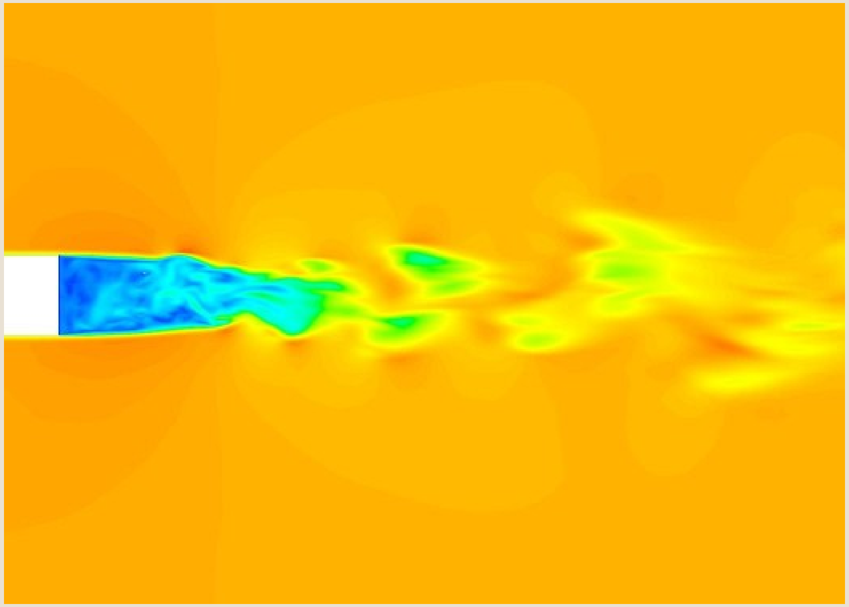


LES Capability in FLUENT
DES Example: GTS Body (2)

- Drag prediction

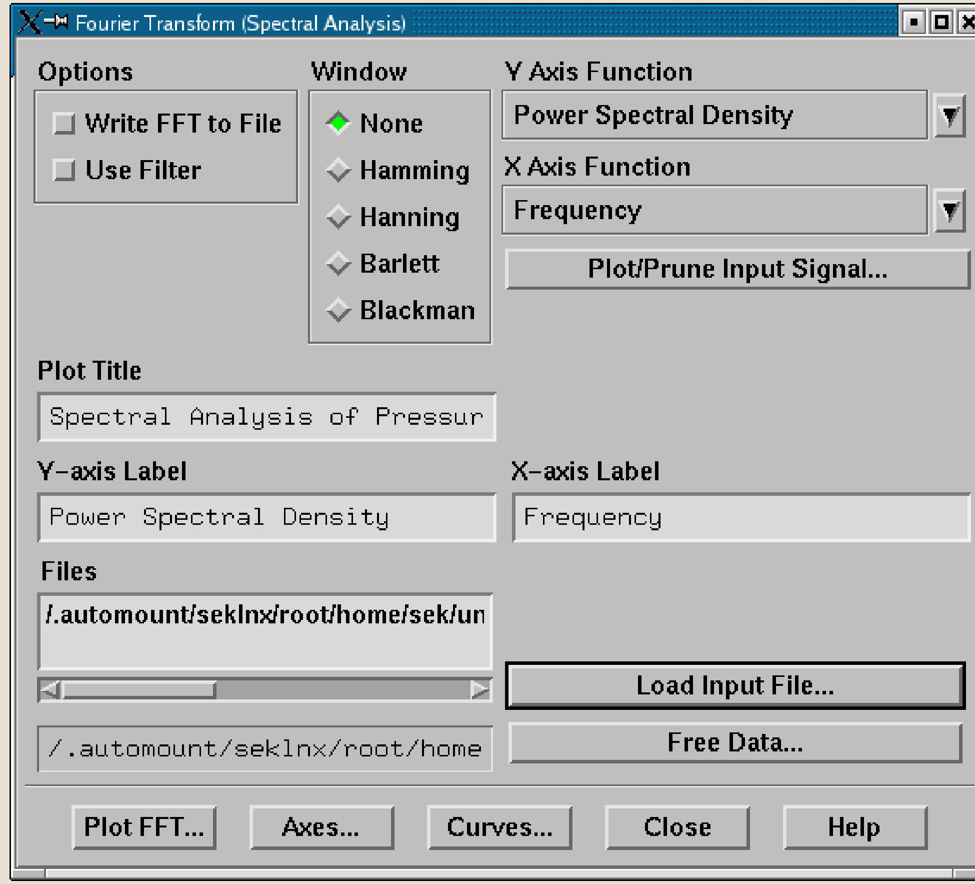


	C_D
Predicted (DES)	0.253
Measured	0.250



LES Capability in FLUENT Postprocessing

- Time-Averaging
  - Mean values of the solution variables.
  - RMS values of the solution variables
- Built-in FFT
  - Data conditioning
    - Pruning
    - Windowing
  - Power spectral density of arbitrary variables



Best Practice LES in FLUENT
General Tips

- Mesh
  - Use hex meshes for the best accuracy
  - Take advantage of hybrid meshing capability and local mesh refinement
- Spatial discretization
  - Stick with CD or BCD for momentum
  - Use the node-based gradient option
  - Use the upwind schemes (QUICK, MUSCL, SOU) for scalars
- Time discretization
  - Use the second-order scheme
  - Use the NITA/fractional-step method for incompressible or weakly compressible flows
  - With the fully-iterative scheme, use SIMPLEC (jack up URF's – 0.9 for pressure and momentum equations)
- SGS modelling
  - The dynamic Smagorinsky's model is a good starting point.
  - Consider DES for high-Re wall-bounded flows

Best-Practice LES with FLUENT Recommended Procedure (1)

- Compute the mean flow with steady RANS
- Superimpose the synthesized turbulence on the mean flow
  TUI: `/solve/initialize/init-instantaneous-vel`
- Switch to LES, select the SGS turbulence model of your choice
- Select the solver algorithm (e.g., ITA/NITA, FSM/PISO/SIMPLEC) and the discretization schemes
- Set the time-step (Δt) and adjust the solver parameters if needed (e.g., URF's, convergence criteria)
- Set the monitors with relevant global (e.g., forces/moments) and local quantities (e.g., velocity, pressure) of your choice
- Set the autosave of the data files (e.g., every a few hundreds time-steps)

Best-Practice LES with FLUENT Recommended Procedure (2)

- Save the viewgraphs of your choice for animation (contours of pressure, iso-surfaces of vorticity, second-invariant, etc.)
- Start the transient run and continue until a statistically steady state is reached
- Start sampling the data (to compute mean and r.m.s. values)

  GUI: Solve/Iterate… (click on the "Data Sampling for Time Statistics" button)
  GUI: Solve/Initialize/Reset Statistics to reset the data sampling
- Continue sampling for a sufficiently long period of time
- Post-process the results (mean, r.m.s., power spectra, etc.)

Closing Remarks

- RANS-based turbulence models are still the workhorse for industrial CFD, and will be so for some time
- In the coming years, high-fidelity CFD will rely upon less “modelling” and more “simulation”
- LES is rapidly becoming feasible for industrial flows
- FLUENT has been significantly upgraded recently in numerics and subgrid-scale turbulence modelling
- Our goal is to ramp up the overall efficacy of LES to the level where LES becomes a useful tool for industrial applications.