Near-Wall Modelling: FLUENT Capabilities

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Modelling the Turbulent Transport

Modelling of the mean flow & near-wall behaviour are equally important in complex industrial flows

Mean Flow:

- Two-Equation Turbulence Models
- Reynolds-Stress Turbulence Closures
- Large-Eddy Simulation

Near-Wall Flow:

The thin viscous sublayer is very influential in determining wall friction and wall heat transfer in turbulent flows

- Low-Reynolds-Number Approach
- Wall Functions
Near-Wall Flow Phenomena

The treatment of wall boundaries requires particular attention in turbulence modelling due to the presence of the **viscosity-dominated region** which is responsible for extremely **sharp gradients** of mean and turbulent flow variables.

<table>
<thead>
<tr>
<th>DNS data (Kim et al)</th>
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Importance of the Near-Wall Turbulence

- Walls are main source of the shear & vorticity and are responsible for the turbulence generation and dissipation mechanisms.

- Accurate near-wall modelling is important for most engineering applications:
  - Successful prediction of frictional drag for external flows, or pressure drop for internal flows, depends on fidelity of local wall shear predictions.
  - Pressure drag for bluff bodies is dependent upon extent of separation.
  - Thermal performance of heat exchangers, etc., is determined by wall heat transfer whose prediction depends upon near-wall effects.
Importance of the Near-Wall Turbulence (2)

- Flows under pressure gradients, flow separation
Importance of the Near-Wall Turbulence (3)

- **Buoyancy-assisted and buoyancy-opposed flows**

**Examples of the velocity profiles distortions**
Structure of Turbulent Boundary Layer

Freestream

Edge of boundary layer

outer layer

fully-turbulent region or log-layer

inner layer

sublayer + buffer layer

Wall
Near-Wall Modelling Issues

- **High-Reynolds Number (HRN) $k$-$\varepsilon$ and RSM models have been derived for free shear flows and are valid in the mean part of the turbulent flow away from walls**
  - Some of the modelled terms in these equations are based on isotropic behavior
    - Isotropic diffusion ($\mu_t/\sigma$)
    - Isotropic dissipation
    - Pressure-strain redistribution
    - Some model parameters based on experiments of isotropic turbulence
  - Near-wall flows are anisotropic due to the presence of walls

- **Special near-wall treatments are necessary since equations cannot be resolved down to the walls in their original form due to the strong gradients in mean and turbulent quantities**
Flow Behaviour in the Near-Wall Region
Equilibrium Flows (1)

Concept of local equilibrium:

\[ \text{Turbulence Production} = \text{Turbulence Dissipation} \]

Examples:

Flow over a Flat Plate & Fully Developed Pipe Flow
Flow Behaviour in the Near-Wall Region
Equilibrium Flows (2)

• Velocity profile exhibits layer structure identified from dimensional analysis
  – **Inner layer:** viscous forces rule, \( U = f(\rho, \tau_w, \mu, y) \)
  – **Outer layer:** dependent upon mean flow
  – **Overlap region:** Log-law applies

• \( k \) production and dissipation are nearly equal in overlap layer
  – ‘turbulent equilibrium’

• dissipation >> production in sublayer region
Near-Wall Distribution of Mean Velocity

- In many circumstances, even far enough from the wall for viscous stresses to be negligible, one still finds a region where the local mean velocity $U$ depends only on

  $$U = f(\rho, \tau_w, \mu, y)$$

- Note, velocity is not dependent on the channel width $D$ or boundary layer thickness $\delta$; nor on “upstream history” (i.e. convection effects)

- Since $U = f(\rho, \tau_w, \mu, y)$ the Buckingham theorem tells us there are:

  5 (variables) – 3 (basic dimensions: M, L, T) = 2 dimensionless groups

  usually presented as:

  $$\frac{U}{\sqrt{\tau_w/\rho}} = f\left(\frac{y\sqrt{\tau_w/\rho}}{\nu}\right)$$

  ‘Law of the wall’

- The quantity $\sqrt{\tau_w/\rho}$ evidently has the dimension of velocity and it is usually written as $U_\tau$, the friction velocity
Near-Wall Distribution of Mean Velocity (2)

- Now define $U^+ = U / U_\tau$, $y^+ = y U_\tau / \nu$ and obtain a universal relationship for the near-wall flows

$$U^+ = f(y^+)$$

- While written especially for the turbulent region, it applies also to the viscous sub-layer if

$$f(y^+) = \rho y U_\tau / \mu = y^+$$

$$\frac{U}{U_\tau} = \frac{y \rho U_\tau}{\nu} \quad \text{or} \quad \frac{U}{y} = \rho U_\tau^2 = \tau_w$$

- In the fully turbulent region, while $U$ certainly depends on $\mu$, the difference in velocity between adjacent layers should not. Thus

$$\frac{\partial U}{\partial y} = U_\tau \frac{df}{dy^+} \frac{dy^+}{dy} = U_\tau \frac{U \rho}{df}$$

should be independent of $\mu$
Near-Wall Distribution of Mean Velocity (3)

- **This can only happen if:**
  \[
  \frac{df}{dy^+} = \text{Const} \cdot \left(\frac{y\rho U_\tau}{\nu}\right)^{-1}
  \]

- **After integration wrt** \(y^+\)
  \[
  f = U^+ = \frac{1}{K} \ln \left(y^+\right) + C
  \]

- **The edge of the viscous sublayer is where the laminar and the turbulent regions meet**

- **With** \(K = 0.42\) and \(C = 5.45\)
  \[
  y^+_v = \frac{y_v U_\tau}{\nu} \approx 11
  \]

- **\(y^+ < 5\) – laminar region**

- **\(y^+ > 35\) – fully turbulent logarithmic region**

\[
\frac{y_v U_\tau}{\nu} = \frac{1}{K} \ln \left(\frac{y_v U_\tau}{\nu}\right) + C
\]
Near-Wall Distribution of Temperature

- Reynolds' analogy between momentum and energy transport gives a similar logarithmic law for the mean temperature

- The usual thermal logarithmic law is

  \[ T^+ = \frac{1}{\chi} \ln\left( y^+ \right) + C_\theta \]

  \[ C_\theta = f(Pr) = f(\sigma) \]

- It is possible to replace the thermal log-law with the help of the hydrodynamic one

  \[ T^+ = \sigma_T \left( U^+ + \Pi \right) \]

  \[ \Pi \quad - \text{Pee function by Jayatilleke} \]
Flow Behaviour in the Near-Wall Region
Non-Equilibrium Flows (1)

- Equilibrium flows are very rare
- Great majority of flows are non-equilibrium

Examples of Non-Equilibrium Flows

- Flows developing under pressure gradients
- Buoyant flows

\[
\begin{align*}
\frac{dP}{dx} &= 0 \\
\frac{dP}{dx} &< 0 \\
\frac{dP}{dx} &> 0
\end{align*}
\]
Near-Wall Modelling Options

- Two distinct modelling strategies exist for the near-wall region
  - Low-Reynolds-number approach (fine mesh)
  - Wall-function approach (coarse mesh)

- The choice of modelling strategy depends on the application, the engineering question, available resources…
Low-Reynolds-Number Modelling

- Fine near-wall grid
- Includes near-wall damping terms

+ Accurate (using appropriate turbulence model)

- Slow convergence due to elongated cells
- High storage requirements

Too expensive for most industrial applications
Conventional Wall Functions

- Coarse near-wall grid: first node in fully-turbulent region ($y^* \geq 30$)
- Based on prescribed $U$ and $T$ profiles in near-wall cell

  + Fast solution (at least 10x faster than LRN)
  + Low storage requirements
  - Poor results in non-equilibrium flows
  - Near-wall cell size dependency
  - Does not take into account such effects as: pressure gradients, convective transport and gravitational forces

**Insufficient breadth of applicability**
Near-Wall Modeling Options in FLUENT

Wall-Function Approach

- Wall functions provide boundary conditions for momentum, energy, species and turbulent quantities

- The Standard and Non-equilibrium Wall Functions (StWF and NEWF) use the law of the wall

- Original flow equations are modified in accord with the chosen law-of-the-wall

![Wall Shear Stress](image)

\[ \tau_w = \frac{\partial U}{\partial y} \]

![Fick’s Law](image)

\[ q_w = -k \frac{\partial T}{\partial y} \]
Near-Wall Modeling Options in FLUENT
Wall-Function Approach (2)

**In the Near-Wall Row of Cells:**

- **Streamwise momentum equation:**

\[
\frac{D}{Dt} (\rho U) = \frac{\partial}{\partial y} \left( \left( + \frac{t}{\sigma} \right) \frac{\partial U}{\partial y} \right) + S_U
\]

\[ S_U \sim \tau_w = f(U_p, k_p, y_p) \]

- **Energy equation:**

\[
\frac{D}{Dt} (\rho T) = \frac{\partial}{\partial y} \left( \left( \frac{1}{\sigma_\varepsilon} + \frac{t}{\sigma_k} \right) \frac{\partial T}{\partial y} \right) + S_T
\]

\[ S_T \sim q_w = f' (\tau_w, T_w, k_p, U_p) \]

- **Turbulent kinetic energy equation:**

\[
\frac{D}{Dt} (\rho k) = \frac{\partial}{\partial y} \left( \left( + \frac{t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right) + G_k - \rho \varepsilon
\]

\[ G_k \Rightarrow \overline{G_k} = f'' (\tau_w, k_p, y_n, y_v) \quad \varepsilon \Rightarrow \overline{\varepsilon} \]

- **Turbulent energy dissipation rate equation:**

\[
\frac{D}{Dt} (\rho \varepsilon) = \frac{\partial}{\partial y} \left( \left( \frac{t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y} \right) + \varepsilon \frac{\varepsilon}{k} G - \frac{\varepsilon^2}{k} \left( \varepsilon \frac{\varepsilon}{k} G \right)
\]

\[ \varepsilon_p = \frac{k_p^{3/2}}{l} = \frac{k_p^{3/2}}{C_1 y_p} \]

\[ C = 0.09, \quad C_{\varepsilon_1} = 1.44, \quad C_{\varepsilon_2} = 1.92, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3 \]
Standard Wall Functions

- **FLUENT uses Launder-Spalding Wall Functions**
  
  \[
  U = U(\rho, \tau, \mu, y, k)
  \]

  - Introduces additional velocity scale for ‘general’ application
    
    \[
    \begin{align*}
    U^i &= y^i \\
    U^i &= \frac{1}{\kappa K} \ln (E y^i)
    \end{align*}
    \]

    for

    \[
    \begin{align*}
    y^i &< y_v^i \quad \text{where} \quad U^i = U_P C^{1/4} k^{1/2} \\
    y^i &> y_v^i
    \end{align*}
    \]

  - Generally, \(k\) is obtained from solution of \(k\) transport equation
    
    - Cell center is immersed in log layer
    - Local equilibrium (production = dissipation) prevails
    
    \[
    \forall \quad \nabla k \cdot n = 0 \quad \text{at surface}
    \]

  - \(\varepsilon\) calculated at wall-adjacent cells using local equilibrium assumption
    
    \[
    \varepsilon = C^\mu 3/4 k^{3/2} / \kappa y
    \]

  - Wall functions less reliable when cell intrudes viscous sublayer
    
    - Forcing (Production = Dissipation) over (Production << Dissipation)
Wall functions become less reliable when flow departs from the conditions assumed in their derivation:

- **Local equilibrium assumption fails**
  - Severe $\nabla p$
  - Transpiration through wall
  - strong body forces
  - highly 3D flow
  - rapidly changing fluid properties near wall

- **Low-Re flows are pervasive throughout model**
- **Small gaps are present**
• **Log-law is sensitized to pressure gradient for better prediction of adverse pressure gradient flows and separation**

• **Relaxed local equilibrium assumptions for TKE in wall-neighboring cells**

\[
\frac{\tilde{U} C^{1/4} k^{1/2}}{\tau_w/\rho} = \frac{1}{K} \ln \left( E \frac{C^{1/4} k^{1/2} y}{\rho C^{1/4} k^{1/2}} \right)
\]

where

\[
\tilde{U} = U - \frac{1}{2} \frac{dp}{dx} \left[ \frac{y_v}{\rho k^{1/2}} \ln \left( \frac{y}{y_v} \right) + \frac{y - y_v}{\rho k^{1/2}} + \frac{y_v^2}{\rho k^{1/2}} \right]
\]

\( R_{ij}, k, \varepsilon \) are estimated in each region and used to determine average \( \varepsilon \) and production of \( k \)
Thermal Wall Functions

Temperature log-law

\[ T^+ = \sigma_T \left( U^+ + \Pi \right) \]

- Similar to the hydrodynamic wall functions - thermal wall functions in Fluent are based on \( y^* \), \( U^* \), \( T^* \) rather than \( y^+ \), \( U^+ \), \( T^+ \)

- within thermal viscous sublayer

\[ T^i = \Pr y^i + \frac{1}{2} \rho \frac{C^{1/4}}{k_p^{1/2}} \frac{U^2}{q_w} \]

- in the fully turbulent region

\[ T^i = \Pr \left[ \frac{1}{K} \ln \left(Ey^i\right) + \Pi \right] + \frac{1}{2} \rho \frac{C^{1/4}}{k_p^{1/2}} \frac{U^2}{q} \left[ \Pr U_p^2 + \left( \Pr - \Pr_t \right) U_c^2 \right] \]

- terms account for heating due to viscous dissipation (supersonic flows)
Hydrodynamic log-law is not valid when the flow departs from the state of local equilibrium

Thermal log-law is expressed in terms of hydrodynamic one and inevitably suffers from the same limitations

\[ T^+ = \Pr_T \left( U^+ + \Pi \right) \]

Plus there is a dependence on molecular and turbulent Prandtl numbers

\[ \Pi = 9.24 \left( \frac{\Pr}{\Pr_T} \right)^{3/4} - 1 \left[ 1 + 0.28 \exp \left( -0.007 \frac{\Pr}{\Pr_T} \right) \right] \]

Example: Prandtl number for water ranges from 10 to 1.5 when the temperature changes from 0°C to 100°C
Thermal Wall Functions - Limitations (2)

- **Buoyancy-assisted flow in a pipe**
  - distortion of the velocity profile

  ![Diagram of buoyancy-assisted flow in a pipe]

  - distortion of the temperature profile

  ![Diagram showing temperature profile distortion]

\[ \frac{q_{wall}}{q_{wall}^{(a)}} \]
Buoyancy-Aided Upflow of Air in Heated Pipes

*(Experimental data of J. Li, 1994)*

Inlet: \( Re = 15023, \) \( Gr = 2.163 \times 10^8, \) \( Bo = 0.1124 \)

\[
\begin{align*}
\Delta & \text{ Exp.data of Li (1994)} \\
\hspace{1cm} & y_\alpha = 50 \\
\hspace{1cm} & y_\alpha = 75 \\
\hspace{1cm} & y_\alpha = 100 \\
\hspace{1cm} & y_\alpha = 150 \\
\text{Nu} & = 0.023 \text{ Pr}^{0.333} \text{ Re}^{0.8} \\
\text{LRN Calculation} &
\end{align*}
\]

LRN terms included

\( Re = 15000 \)

Inlet: \( Re = 49635, \) \( Gr = 1.745 \times 10^9, \) \( Bo = 0.0156 \)
Buoyant effects cause significant changes in Heat Transfer

**Dominant Factor:**

*Ability of buoyant forces to modify turbulent shear stress*

*Consequential changes in the turbulence production*

*Altered turbulent diffusivity*

*Impaired or enhanced heat transfer*

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**Graphs:**

- **Graph 1:**
  - **Y-axis:** $U/U_{bulk}$
  - **X-axis:** $(r-y)/r$
  - Lines:
    - Blue: neutral
    - Green dashed: moderate heating
    - Red dashed: severe heating

- **Graph 2:**
  - **Y-axis:** $\rho u'v'/(\rho U_{bulk}^2)$
  - **X-axis:** $(r-y)/r$
  - Lines:
    - Blue: neutral
    - Green dashed: moderate heating
    - Red dashed: severe heating
Near-Wall Modelling Options in FLUENT

Enhanced Wall Treatment (EWT)

- **Enhanced Wall Treatment** is a ‘Quasi-LRN’ approach which, ideally, requires fine grid
  - Combines the use of a two-layer zonal model and blended law-of-the-wall. Thus coarser mesh is less of a problem than in LRN models
  - Suitable for low-Re flows or flows with complex near-wall phenomena as it employs fine mesh and directly resolves turbulence variables all the way to the wall
  - Turbulence models are modified for the inner layer
  - Generally requires a fine near-wall mesh capable of resolving the viscous sub-layer (more than 10 cells within the inner layer)
  - The EWT is an option for the $k$-$\varepsilon$ and RSM turbulence models
Enhanced Wall Treatment (EWT) Two-Layer Model

- A blended two-layer model is used to determine near-wall $\varepsilon$ field:
  - Domain is divided into viscosity-affected (near-wall) region and turbulent core region
  - High Re turbulence model used in outer layer
  - ‘Simple’ turbulence model used in inner layer
- Solutions for $\varepsilon$ and $\mu_t$ in each region are blended

The two regions are demarcated on a cell-by-cell basis in a dynamic, solution-adaptive way:
  - Turbulent core region $y^* > 200$
  - Viscosity affected region $y^* < 200$

Where $y^* = \rho k^{1/2} y / \mu$ and $y$ is the shortest distance to a nearest wall
Models used in two-layer zones

- **In the turbulent region, the selected high-Re turbulence model is used**
- **In the viscosity-affected region, a one-equation model is used**
  - \( k \) – equation is same as high-Re model
  - Length scale used in evaluation of \( \mu_t \) is not from \( \varepsilon \)
    - \( \mu_t = \rho C_{\mu} k^{1/2} l_{\mu} \)
    - \( l_{\mu} = c_l y (1 - \exp(-y^* / A_{\mu})) \)
    - \( c_l = \kappa C_{\mu}^{3/4} \)
  - Dissipation rate, \( \varepsilon \), is calculated algebraically and not from the transport equation
    - \( \varepsilon = k^{3/2} / l_\varepsilon \)
    - \( l_\varepsilon = c_l y (1 - \exp(-y^* / A_{\varepsilon})) \)
  - The two \( \varepsilon \) fields can be quite different along the interface in highly non-equilibrium turbulence
Two-Layer Zonal Model

- The $k$ – equation is solved throughout the whole domain, but $\varepsilon$ – field is determined by two different formulations

- The two $\varepsilon$ – fields can be quite different along the interface in highly non-equilibrium conditions

\[
\frac{D}{Dt}(\rho \varepsilon) = \mathbf{\dot{\varepsilon}} \cdot \mathbf{\dot{\varepsilon}}
\]

\[
\varepsilon = \frac{k^{3/2}}{\varepsilon}
\]
Blended $\varepsilon$ – equations

- The transition (of $\varepsilon$ – field) from one zone to another can be made smoother by blending the two sets of $\varepsilon$ – equations (Jongen, 1998)

$$
\lambda_\varepsilon \left[ a_P \varepsilon_P + \sum_{nb} a_{nb} \varepsilon_{nb} = (S_\varepsilon)_P \right] + \left( 1 - \lambda_\varepsilon \right) \left[ \varepsilon_P = \frac{k_P^{3/2}}{\varepsilon} \right]
$$

with

$$
\lambda_\varepsilon = \frac{1}{2} \left[ 1 + \tanh \left( \frac{Re_y - Re^{*}_y}{A} \right) \right]
$$

### Diagram:

- **Outer layer**
  - $\rho \frac{D\varepsilon}{Dt} = \overline{\tau} \cdot \overline{c} \rightarrow a_P \varepsilon_P + \sum_{nb} a_{nb} \varepsilon_{nb} = (S_\varepsilon)_P$

- **Inner layer**
  - $\varepsilon_P = \frac{k_P^{3/2}}{\varepsilon}$

- **Wall**
**Blended Turbulent Viscosity**

- **Turbulent viscosity, \( \mu_t \), is also blended using the individual formulations**

\[
\lambda \varepsilon (t)_{\text{outer}} + (1 - \lambda \varepsilon) (t)_{\text{inner}}
\]

\[
(t)_{\text{outer}} = \rho C \frac{k^2}{\varepsilon}, \quad (t)_{\text{inner}} = \rho C \sqrt{k}
\]

\[
Re_y A = c y \left[ 1 - \exp \left( - \frac{Re_y}{A} \right) \right]
\]

![Diagram of Outer and Inner layers with equations for turbulent viscosity](image)

- Outer layer
  - \( (t)_{\text{outer}} = \rho C \frac{k^2}{\varepsilon} \)

- Inner layer
  - \( (t)_{\text{inner}} = \rho C \sqrt{k} \)
Enhanced **Wall Treatment (EWT)**

**Enhanced Wall Functions**

- **Momentum boundary condition based on blended law-of-the-wall (Kader):**
  \[ u^+ = e^Γ u_{lam}^+ + e^{1/Γ} u_{turb}^+ \]

- **Similar blended ‘wall laws’ apply for energy, species, and \( ω \)**

- **Kader’s form for blending allows for incorporation of additional physics**
  - Pressure gradient effects
  - Thermal (including compressibility) effects
Blended Hydrodynamic Law-of-the-Wall

- **Mean velocity**

\[ u^+ = e^\Gamma u_{lam}^+ + e^{1/\Gamma} u_{turb}^+ \]

- **Blended ‘wall laws’ for temperature and species as well**

\[ \Gamma = -\frac{a(y^+)^4}{1 + b y^+} \quad , \quad a = 0.01 \ c \]

\[ b = \frac{5}{c} \quad , \quad c = \exp\left(\frac{E''}{E} - 1\right) \]

where

\[ u_{lam}^+ = y^+ \]

\[ u_{turb}^+ = \frac{1}{K} \ln\left(\frac{E}{y^+}\right) \]
**Mean temperature**

\[
\Gamma = - \frac{a (\sigma y^+)^4}{1 + b \sigma^3 y^+}, \quad a = 0.01 \quad c, \quad b = \frac{5}{c}, \quad c = \exp \left( \frac{E''}{E} - 1 \right)
\]

where

\[
t^+ = \frac{|T - T_w| \rho c_p u^i}{q_w},
\]

\[
t_{lam}^+ = \sigma y^+
\]

\[
t_{turb}^+ = \sigma_t (u_{turb}^+ + P)
\]

\[
P = 9.24 \left[ \left( \frac{\sigma}{\sigma_t} \right)^{3/4} \right] \left[ 1 + 0.28 \exp \left( -0.007 \frac{\sigma}{\sigma_t} \right) \right]
\]
‘Wall-Law’
Sub-models and Options

- "Pressure Gradient Effects" option
  - Always available - deactivated by default

- "Thermal Effects" option
  - Available only when energy equation is turned on - deactivated by default
  - Accounts for
    - Non-adiabatic wall heat transfer effects
    - Compressibility effects - takes effect when ideal-gas option is chosen
• **The base laws-of-the-wall** (mean velocity and temperature) are modified using (White and Christoph, 1972):

\[
\tau \approx \tau_w + \frac{dp}{dx} y, \quad \alpha = \frac{\nu_w}{\tau_w u_\tau} \frac{dp}{dx}
\]

— “Pressure Gradient Effects” option

\[
\frac{du^+}{dy^+} = \frac{1}{\kappa y^+}
\]

— “Thermal Effects” option

\[
\frac{\rho_w}{\rho} \approx 1 - \beta u^+ - \gamma u\]

\[
\beta = \frac{\sigma_i q'' w u_\tau}{c_p \tau_w T_w}, \quad \gamma = \frac{\sigma_i u_\tau^2}{2C_p T_w}
\]

\[
\text{heat transfer parameter,} \quad \text{compressibility parameter}
\]
Enhanced Wall Treatment
Wall Boundary Conditions

- Zero normal-gradient is applied to TKE at walls
  \[
  \frac{\partial k}{\partial n} = 0
  \]

- Other turbulence quantities at wall-adjacent cells are computed, whenever possible, using the blended two-layer formulation
  - Production of TKE at wall-adjacent cells is computed using the velocity gradient given by the blended law-of-the-wall for mean velocity

- Dissipation rate, \( \varepsilon \), at wall cells is computed using the inner layer formula

- EWT is the only near-wall option available for the \( k-\omega \) models
Enhanced Wall Treatment
Some Results

- **Fully-Developed Channel Flow** ($Re_t = 590$)
  
  For fixed pressure drop cross periodic boundaries, different near-wall mesh resolutions yielded different volume flux as follows:

<table>
<thead>
<tr>
<th></th>
<th>$y^+ = 1$</th>
<th>$y^+ = 4$</th>
<th>$y^+ = 8$</th>
<th>$y^+ = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>StWF</td>
<td>12.68</td>
<td>13.77</td>
<td>16.77</td>
<td>19.08</td>
</tr>
<tr>
<td>EWT</td>
<td>18.31</td>
<td>17.58</td>
<td>17.70</td>
<td>18.48</td>
</tr>
</tbody>
</table>

- The enhanced near-wall treatment gives a much smaller variation for different near-wall mesh resolutions compared to the variations found using standard wall functions.
For standard or non-equilibrium wall functions, each wall-adjacent cell’s centroid should be located within:

\[ y_p^+ \approx 30 - 300 \]

For the enhanced wall treatment (EWT), each wall-adjacent cell’s centroid should be located:

- Within the viscous sublayer, \( y_p^+ \approx 1 \), for the two-layer zonal model:
- Preferably within \( y_p^+ \approx 30 - 300 \) for the blended wall function

How to estimate the size of wall-adjacent cells before creating the grid:

- \( y_p^+ \equiv y_p u_\tau / \nu \Rightarrow y_p \equiv y_p^+ \nu / u_\tau \), \( u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}} = U_e \sqrt{c_f / 2} \)
- The skin friction coefficient can be estimated from empirical correlations:
  - Flat Plate: \( \frac{C_f}{2} \approx 0.037 \text{ Re}_L^{-1/5} \)
  - Duct: \( \frac{C_f}{2} \approx 0.039 \text{ Re}_D^{-1/4} \)
Near-Wall Modelling: Recommended Strategy

- **Use StWF or NEWF in high-Re applications (Re > \(10^6\)) where you cannot afford to resolve the viscous sub-layer**
  - Use NEWF for mildly separating, reattaching, or impinging flows

- **You may consider using EWT if:**
  - Near-wall characteristics are important.
  - The physics and near-wall mesh of the case is such that \(y^+\) is likely to vary significantly over a wide portion of the wall region.

- **Try to make the mesh either coarse or fine enough to avoid placing the wall-adjacent cells in the buffer layer** \((y^+ = 5 \sim 30)\)
Near-Wall Modelling: Low-Reynolds-Number Modelling

Damping Function LRN Models

- **Original $\varepsilon$ – equation for HRN flows**

\[
\frac{D}{Dt}(\rho \varepsilon) = \frac{\partial}{\partial x_j} \left[ \left( 1 + \frac{t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon 1} \frac{\varepsilon}{k} G_k - \rho C_{\varepsilon 2} \frac{\varepsilon^2}{k}
\]

- **Std. $k$-$\varepsilon$ model modified by damping functions:** $f$, $f_1$, $f_2$

\[
\varepsilon \ - \ transport \ equation
\]

\[
\frac{D}{Dt}(\rho \varepsilon) = \frac{\partial}{\partial x_j} \left[ \left( 1 + \frac{t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + f_1 \cdot C_{\varepsilon 1} \frac{\varepsilon}{k} G_k - f_2 \cdot \rho C_{\varepsilon 2} \frac{\varepsilon^2}{k}
\]

- **$t$ = turbulent viscosity**

\[
t = \rho \ f \ C \ \frac{k^2}{\varepsilon}
\]

- **$k$ and $\varepsilon$ equations solved on fine mesh (required) right to the wall**
Typical damping functions

- **Damping functions written in terms of Reynolds numbers:**

  \[
  \mathcal{R}_t = \frac{\rho k^2}{\mu \varepsilon} \quad ; \quad \mathcal{R}_y = \frac{\rho \sqrt{k} y}{\mu} \quad ; \quad \mathcal{R}_\varepsilon = \frac{\rho (\mu \varepsilon / \rho)^{1/4}}{\mu} y
  \]

- **e.g., Abid’s model:**

  \[
  f = \tanh \left( 0.008 \, Re_y \right) \left( 1 + 4 \, Re_{t}^{-3/4} \right)
  \]

  \[
  f_1 = 1
  \]

  \[
  f_2 = \left[ 1 - \frac{2}{9} \exp \left( -\frac{Re_t^2}{36} \right) \right] \left[ 1 - \exp \left( \frac{Re_y}{12} \right) \right]
  \]
Low-Re Model Fundamentals

- **Most models try to achieve asymptotic consistency**

\[
\begin{align*}
    u & \approx A(x, z, t) y + O(y^2) \\
    v & \approx B(x, z, t) y^2 + O(y^3) \\
    w & \approx C(x, z, t) y + O(y^2)
\end{align*}
\]

as \( y \to 0 \)

\[
\begin{align*}
    k & \approx \frac{1}{2} \left( A^2 + C^2 \right) y^2 + O(y^3) \\
    \varepsilon & \approx \nu \left( A^2 + C^2 \right) + O(y) \\
    \tau_{xy} & \approx AB y^3 + O(y^4)
\end{align*}
\]

- **The models usually end up looking like**

\[
\begin{align*}
    U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} &= \nu_t \left( \frac{\partial U}{\partial y} \right)^2 - \varepsilon + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] \\
    U \frac{\partial \tilde{\varepsilon}}{\partial x} + V \frac{\partial \tilde{\varepsilon}}{\partial y} &= C_{\varepsilon_1} f_1 \frac{\varepsilon}{k} \nu_t \left( \frac{\partial U}{\partial y} \right)^2 - C_{\varepsilon_2} f_2 \frac{\varepsilon^2}{k} + E + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \tilde{\varepsilon}}{\partial y} \right]
\end{align*}
\]

where \( \varepsilon = \varepsilon_0 + \tilde{\varepsilon} \) and \( \nu_t = C f k^2 / \tilde{\varepsilon} \)
LRN $k$-$\varepsilon$ Models

- Several full low-Re $k$-$\varepsilon$ models now available
  (not documented or officially supported)
  - Lam Bremhorst
  - Launder-Sharma
  - Abid
  - Chang et al.
  - Abe-Kondo-Nagano

- Enables modeling of low-Re effects including transitional flows

- Features not visible in GUI, but must be accessed via TUI
Two-Layer Near-Wall RSM Model

- Two-layer zonal modeling approach combining
  - Launder & Sharma’s low-Re model
  - Wolfstein’s one-equation model

\[
\frac{Dk}{Dt} = \frac{\partial}{\partial x_i} \left( \frac{t \frac{\partial k}{\partial x_i}}{\sigma_k \frac{\partial x_i}{\partial x_i}} \right) + \frac{\partial U_i}{\partial x_j} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - C_D \rho k^{3/2}
\]

- RSM can be used with fine near-wall meshes
- RSM can be used for low-Re turbulent flows
In second-moment-closures explicit transport equations for the Reynolds stresses are solved:

\[
\frac{D \overline{u_i u_j}}{Dt} = P_{ij} + G_{ij} + \varphi_{ij} + d_{ij} - \varepsilon_{ij}
\]

Two different models of this type are implemented in Fluent

- ‘Basic’ second-moment-closure model
- Quadratic pressure strain model

Pressure Strain term:

\[
\varphi_{ij} = \varphi_{ij}^1 + \varphi_{ij}^2 + \varphi_{ijw}
\]
Near-Wall Adjustments in RSM models (2)

• **Wall reflection term**

• If EWT option is chosen and grid is fine then LRN modifications are applied to all three pressure redistribution terms

\[ \varphi_{ij1}, \varphi_{ij2}, \varphi_{ijw} = f(Re_T, a_{ij}) \]

• **Quadratic pressure strain model does not require wall-reflection terms**

• **Quadratic pressure strain model cannot be employed when EWT is used**
• Durbin (1990) suggests that wall normal fluctuations, $v^2$, are responsible for near-wall transport.

• $v^2$ behaves quite differently than $u^2$ and $w^2$:
  – attenuation of $v^2$ is a kinematic effect
  – damping of $u^2$ is a dynamic effect

• Model $t \sim \overline{v^2} T$ instead of $t \sim kT$

• Requires two additional transport equations:
  – equation for wall-normal fluctuations, $v^2$
  – equation for an elliptic relaxation function, $f$
\[ V^2 F k-\varepsilon \text{ model equations} \]

\[ \text{\varepsilon-transport equation} \]
\[ \rho \left( \frac{D \varepsilon}{Dt} \right) = \frac{\partial}{\partial x_j} \left( \varepsilon t \sigma_{\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{1}{T} \left( C_{1\varepsilon} t S^2 - \rho C_{2\varepsilon} \varepsilon \right) \]

\[ \text{\(V^2\)-transport equation} \]
\[ \rho \left( \frac{D V^2}{Dt} \right) = \frac{\partial}{\partial x_j} \left( V^2 t \sigma_k \frac{\partial V^2}{\partial x_j} \right) + \rho k f - \rho \frac{V^2 \varepsilon}{k} \]

\[ \text{relaxation equation} \]
\[ f - L^2 \frac{\partial^2 f}{\partial x_j \partial x_j} = \frac{C_1}{T} \left( \frac{2}{3} - \frac{V^2}{k} \right) + C_2 \frac{t S^2}{\rho k} + \frac{|N-1| V^2}{kT} \]

\[ \text{scales} \]
\[ T = \max \left[ \frac{k}{\varepsilon}, 6 \sqrt{\frac{V}{\varepsilon}} \right] ; \quad L^2 = C_L^2 \max \left[ \frac{k^3}{\varepsilon^2}, C_\eta \sqrt{\frac{V^3}{\varepsilon}} \right] \]
• Very promising results for a wide range of flow and heat transfer test cases
  – at least as good as the best of the damping function approaches in most test cases

• Still an isotropic eddy-viscosity model
  – Can be extended for RSM

• Needs 2 additional equations, so requires more memory and CPU than damping functions
Thank You