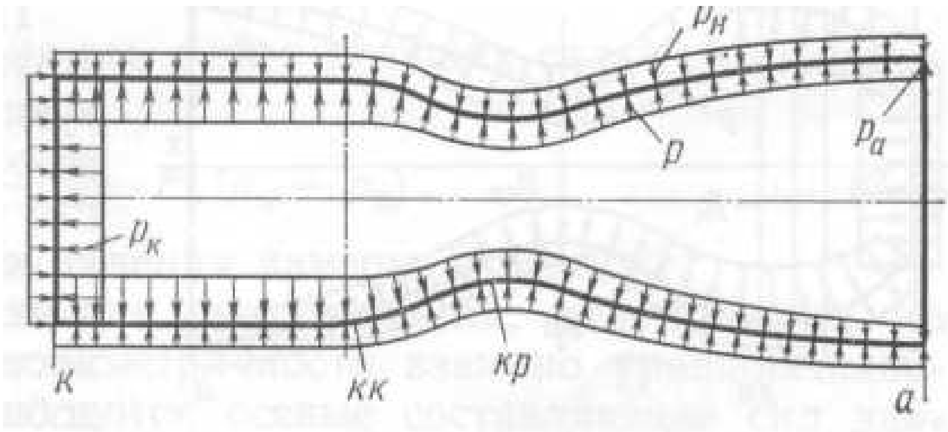
THE THRAST THEORY OF LPRE

1. THE COMMON DATA

Main role of a liquid rocket engine — creation of thrust P during certain period t. For each rocket there is a program of a thrust variation directionally a rocket flight, permitting at a minimum mass of a rocket to achieve the given terminal speed which, for example for ballistic missiles, determines range.

Without exact knowledge of value of thrust and its relation to those or other parameters it is impossible to create a rocket with optimum performances. First it is definable the thrust created by the chamber of a liquid rocket engine. Thrust of the chamber is resultant of gas dynamic forces which are operational on interior surfaces of the chamber at the expiration from it of substance and forces of ambient pressure, operational on its exterior surfaces.

Fig. 1. The forces which are operational on walls of the cylindrical combustion



chamber

From fig. 1 it is visible, that the alternating pressure varying from combustion-chamber pressure p_k up to pressure on a shear of the nozzle p_a acts on an interior surface of the chamber. Ambient pressure p_h acts on an exterior surface of the chamber.

Thrust of the chamber can be defined as resultant of the forces of pressure which are operational on interior and exterior surfaces of the chamber, or with the help of momentum equation.

Both methods are used for calculation of thrust of the chamber and have the characteristic advantages and shortages. The first method allows to understand a physical essence of the nature of thrust, to determine the parts of thrust obtained from separate parts of the chamber, and a place of their application, to estimate a degree of efficiency of separate elements of the chamber.

The second method allows to determine fast thrust of the chamber, but does not prize the nature of the gas dynamic phenomena happening inside the chamber.

At derivation of an equation of thrust we understand motion of gases as steadied and one-dimensional. Combustion materials which at nozzle flow are subject to a recombination, are substituted by perfect gas for which constant mean values of a parameter of the efflux k and a gas constant are selected. It is made so that the geometry of the nozzle and flow rate of gas were equal with geometry of the nozzle and exhaust velocity of real combustion materials. Friction and a heat exchange between gas and walls of the chamber neglect.

2. DEFINITION OF THRUST AS RESULTANT FORCES OF PRESSURE

For a conclusion of the formula of thrust we shall take advantage of the chamber of the arbitrary form (fig. 2).

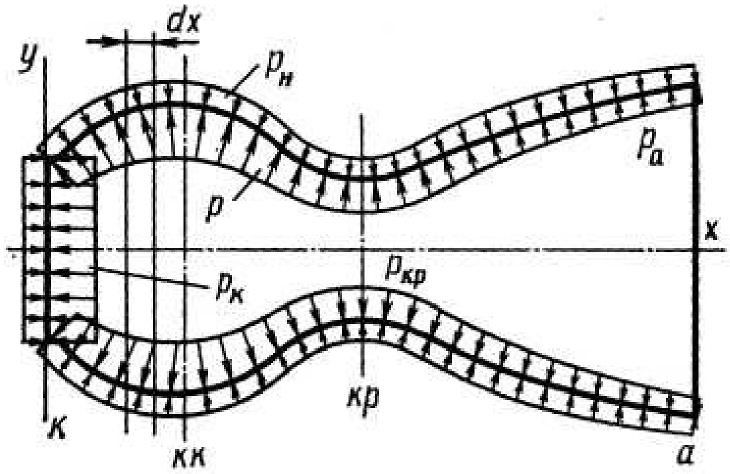


Fig. 2. The forces which are operational on walls of the chamber with the combustion chamber of the arbitrary form.

Pressure on a scarf of the nozzle as it also happens generally, differs from ambient pressure. By definition, thrust of the chamber is

$$P=\int_S p\cos(\widehat{nx})dS, \qquad (1)$$

where are n — a normal to a surface; x — an axis of the chamber; S — full (interior and exterior) surface of the chamber.

To decide this integral, we shall divide thrust on four component, and the component of thrust conterminous on a direction with velocity vector of the efflux, we shall receive positive, and on the contrary:

$$-P=-P_1-P_2+P_3-P_4, \qquad (2)$$

where P_1, P_2, P_3, P_4 — there are the resultant force of pressure affixed on the head of the combustion chamber, to extending and narrowed down parts of the combustion chamber and to an extending part of the nozzle.

At definition resultant forces of pressure which act on the head of the combustion chamber, we count, that all fuel capacity is in the head of the combustion chamber. Here, as well as in a tank, the velocity of motion of a fluid is negligible small. The similar assumption will not affect end results of a conclusion, and calculations become simpler.

For definition of force P_1 we use the theorem of momentums for the volume of a fluid is in a cavity of the head. For this purpose we shall limit the head to a monitoring surface and we shall apply the superposed forces which are operational on the chosen head loop of a fluid (fig. 3 a. and b.).

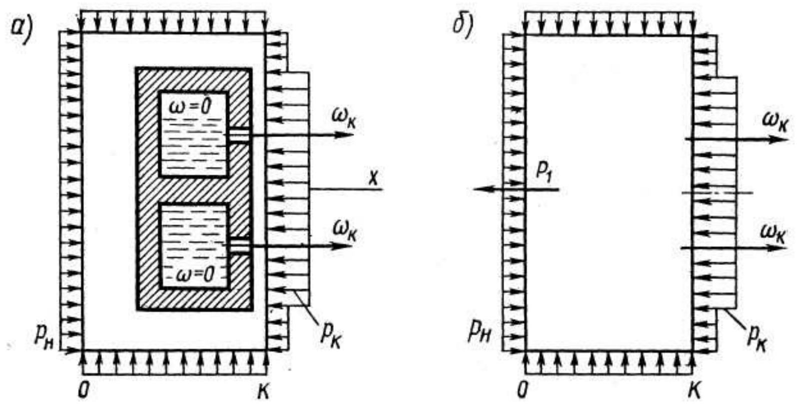


Fig. 3. The computational scheme for definition of force P_1

Forces of pressure of gases act on the chosen head loop in initial cross-section of the chamber p_k, force of ambient pressure p_h and required force P_1 with which head acts on the chosen head loop.

According to momentum equation in the form of the Euler the projection to an axis X of all superposed forces affixed on the considered head loop of a fluid, is equal to a projection of a modification of second momentum:

$$P_x = \dot{m}\left(w_{x_2} - w_{x_1}\right), \qquad (3)$$

where w_{x_1} and w_{x_2} — axial component velocities of motion of a fluid on an input and an exit in the chosen head loop.

In this case $w_{x_1} = 0$, $w_{x_2} = w_k$, then

$$P_x = \dot{m} w_k . \qquad (4)$$

At the same time a projection to an axis X of all superposed forces which are operational: on a chosen head loop,

$$P_x = -P_1 - F_k(p_k - p_h), \qquad (5)$$

where F_k — square of initial cross-section of the combustion chamber.

As the chamber is axially symmetric, all radial pressure component forces which act on walls of the chamber, are mutually equilibrated. Unbalanced there are axial pressure component forces which in the total give required force $-P_1$. The sign "-" means, that force is directed aside, inverse to a positive direction of an axis X. Further all forces conterminous on a direction with an axis X, we shall consider positive, and on the contrary.

Substituting in an equation (5) from (4) value P_x, we shall receive

$$-P_1 = \dot{m}w_k + F_k(p_k - p_h). \qquad (6)$$

For definition of force P_2 (fig. 4) add the axial component of pressure forces, affixed on extending part of combustion chamber between sections *k* and *кк* (see fig. 2),

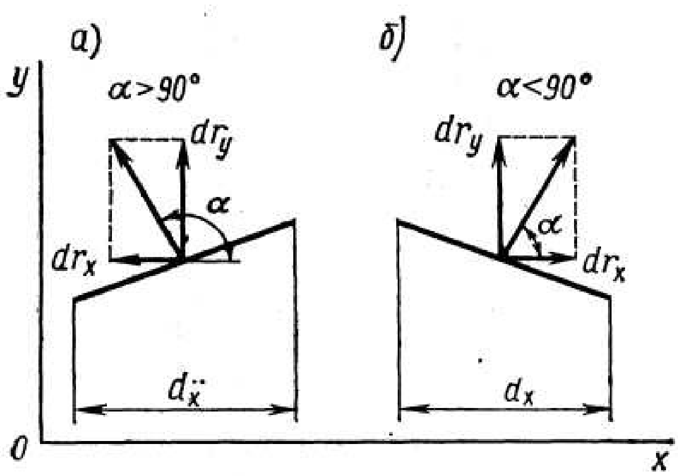


Fig. 4. The computational scheme for definition of forces P_3 and P_4

Axial projection of forces of the pressure which are operational on an elemental annulus of the combustion chamber with a breadth of dx is:

$$dP_2 = (p - p_h) dS \cos\alpha ,$$

where dS — a surface of an elementary cone a breadth dx; α — a an angle between positive directions of an axis x and a normal to a surface.

Taking into account, that $dS\cos\alpha$ there is a projection of a site dS to a plane, perpendicular axes, is definable force

$$- dP_2 = (p - p_h) dF ,$$

where $dF = dS\cos\alpha$.

Integrating $-dP_2$ on all surface of a considered segment of the combustion chamber, we shall receive

$$-P_2 = \int_{F_k}^{F_{kk}} (p - p_h) dF = \int_{F_k}^{F_{kk}} p dF - p_h (F_{kk} - F_k) .$$

Integrating piecemeal pdF, we shall discover

$$\int_{F_k}^{F_{kk}} p dF = p_{kk} F_{kk} - p_k F_k - \int_{p_k}^{p_{kk}} F dp .$$

Using equation of continuities

$$F = \frac{\dot{m}}{\rho w}$$

and Bernoulli's theorem

$$wdw = -\frac{dp}{\rho},$$

and also taking into account, that at steadied engine regime $\dot{m}$ = const, we shall receive

$$\int_{F_k}^{F_{kk}} pdF = p_{kk}F_{kk} - p_kF_k + \int_{w_k}^{w_{kk}} \dot{m}dw = p_{kk}F_{kk} - p_kF_k + \dot{m}(w_{kk} - w_k).$$

Substituting value of an integral $\int_{F_k}^{F_{kk}} pdF$ in an equation for force P_2 we shall discover

$$-P_2 = p_{kk}F_{kk} - p_kF_k + \dot{m}(w_{kk} - w_k) - p_h(F_{kk} - F_k) \qquad (7)$$

Forces P_3 (fig. 4, б) and P_4 (fig. 4, a) receive analogously:

$$P_3 = \int_{F_{kk}}^{F_{kp}} (p - p_h)dF = p_{kp}F_{kp} - p_{kk}F_{kk} + \dot{m}(w_{kp} - w_{kk}) \qquad (8)$$

$$-P_4 = \int_{F_{kp}}^{F_a} (p - p_h)dF = p_aF_a - p_{kp}F_{kp} + \dot{m}(w_a - w_{kp}) - p_h(F_a - F_{kp}) \qquad (9)$$

Substituting in (2) forces P_1 P_2 P_3, P_4 with the it is sign character, we shall receive the formula for calculation of thrust of the chamber

$$-P = \dot{m}w_a + F_a(p_a - p_h).$$

Usually in practice put have with an absolute value of thrust and a sign a minus lower.

$$P = \dot{m}w_a + F_a(p_a - p_h) \qquad (10)$$

The formula (10) can be received from an equation of momentums. On fig. 5 the control surface where the chamber is inside is shown.

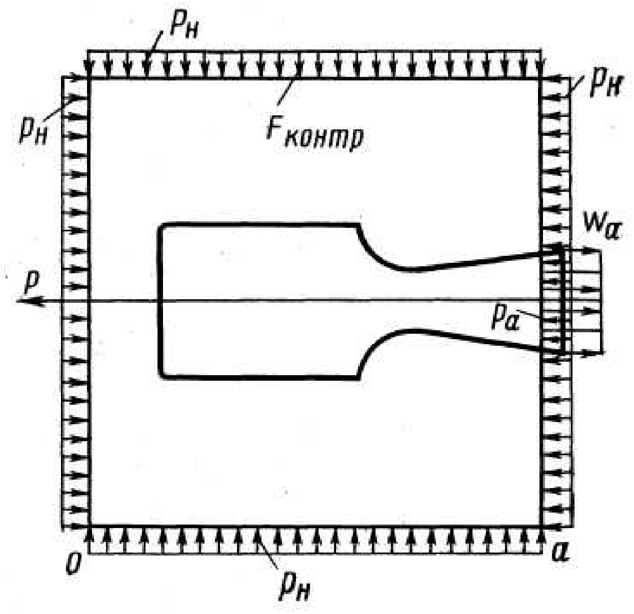


Fig. 5

One plane passes on a shear of the nozzle where all parameters are known.

Projection of all superposed forces affixed on a head loop, to an axis of the nozzle:

$$P_x = -P - F_a(p_a - p_h). \qquad (11)$$

Then from an equation of momentums, lowering a sign a minus, we shall receive the required formula of thrust.

$$P = \dot{m}w_a + F_a(p_a - p_h)$$

3. THE ANALYSIS OF THRUST FORMULA

The formula of the chamber thrust (10) is received by summation of the pressure forces operating on interior and exterior surfaces of the chamber:

$$P = \int_S p dS \cos(nx) = \dot{m}w_a + p_a F_a - p_H F_a$$

The integral can be separated into two integrals:

$$P = \int_{S_{внут}} pdS\cos(nx) - \int_{S_{вн}} pdS\cos(nx), \qquad (12)$$

from which the first characterises the thrust created by pressure forces, applied to an interior contour of the chamber, and the second - the thrust created by ambient pressure forces, applied to an exterior contour, i.e.

$$P_{внут} = \int_{S_{внут}} pdS\cos(nx) = \dot{m}w_a + p_a F_a \qquad (13)$$

$$P_{вн} = \int_{S_{вн}} pdS\cos(nx) = p_H F_a \qquad (14)$$

It is necessary to be disassembled accurately in the thrust nature. The thrust created from interior contour $P_{внут}$, depends only on parameters of working process in the chamber. In case of independence of working process in the chamber from ambient pressure (a nozzle regime without shock waves in the nozzle, called by ambient pressure influence) this component characterizes a vacuum thrust, i.e. At $p_H = 0$, we have $P_{внут} = P_п$

The thrust created from exterior contour $P_{вн}$, characterizes influence only exterior pressure. From (10) follows that thrust increasing is possible at the expense of a propellant consumption and exhaust velocity of yields of combustion w_a. The greatest value of thrust at the parameters set of the chamber is reached in vacuum at $p_H = 0$. When the negative component of thrust $P_{вн} = p_H F_a$, is equaled to zero, thrust is

$$P_п = P_{внут} = \dot{m}w_a + p_a F_a \qquad (15)$$

Hence, thrust at any ambient pressure is connected (at without shock - nozzle regime) with thrust $P_{внут}$ by the ration

$$P = P_{внут} - p_H F_a = P_п - p_H F_a \qquad (16)$$

It is necessary to stop on one concept - a design regime of activity of the chamber nozzle at which pressure upon a nozzle shear is equal to ambient pressure. Then thrust is

$$P_p = \dot{m} w_a \qquad (17)$$

Large influence on thrust value has a chamber nozzle regime. Naturally, it is necessary to know, at what working conditions of the nozzle the chamber will advance the greatest thrust. At the set pressure in combustion chamber it is possible to appoint various expansion ratios of gases in the nozzle at which pressure upon a nozzle shear can be more, less or equally to ambient pressure. It is necessary to choose such expansion ratio at the set combustion-chamber pressure at which the chamber will advance the greatest thrust (to determine pressure upon a nozzle shear). From (10) it is not visible, at what expansion ratio (at p_H = const, $p_к$ = const, $\dot{m}$ = const) maximum value of thrust of the chamber is reached. Really, with expansion ratio magnification at $p_к$ = const exhaust velocity is

$$w_a = \sqrt{2\frac{k}{k-1}R_K T_K\left[1-\left(\frac{p_a}{p_K}\right)^{\frac{k-1}{k}}\right]}$$

it grows, but thus second member $F_a(p_a - p_H)$ in (10) decreases, becomes equal to zero at $p_a = p_H$ and at the further raise of expansion ratio, when $p_a \lhd p_H$ it becomes negative. With reduction of expansion ratio exhaust velocity w_a and member $\dot{m} w_a$ in the equation of thrust (10) decreases, and second member $F_a(p_a - p_H)$ increases.

It is necessary to know, at what expansion ratio of gas in the nozzle or pressure on a shear of the nozzle value of thrust will be maximum.

The thrust equation can be transformed to a such form

$$P = \dot{m} w_{пр} \qquad (18)$$

Where some reduced exhaust velocity is

$$w_{пр} = w_a + [(p_a - p_H)F_a]/\dot{m} \qquad (19)$$

From the formula (18) follows, that the thrust variation is determined only by a modification of the reduced exhaust velocity, because $\dot{m}$=const. To establish expansion ratio influence (dimensionless square of nozzle $F_a/F_{кр}$ or p_a at p_K = const), we will take a derivative (19) on variable p_a and we will equate the first derivative to zero. Then

$$\frac{dw_{пр}}{dp_a}=\frac{dw_a}{dp_a}+\frac{1}{\rho w_a}-\frac{p_a-p_H}{(\rho_a w_a)^2}\frac{d(\rho_a w_a)}{dp_a}=0.$$

Using the Bernoulli's equation for a compressible gas in the differential form

$$\frac{dw_a}{dp_a}+\frac{1}{\rho_a w_a}=0,$$

we have

$$\frac{dw_{пр}}{dp_a}=-\frac{p_a-p_H}{(\rho_a w_a)^2}\frac{d(\rho_a w_a)}{dp_a}=0. \qquad (20)$$

From (20) it is visible, that function $\frac{dw_{пр}}{dp_a}$ reaches extreme value at $p_H=p_a$.

For definition of extreme character we will take a flexion

$$\frac{d}{dp_a}\left(\frac{dw_{пр}}{dp_a}\right):$$

$$\frac{d^2w_{пр}}{dp_a^2}=-\frac{1}{\rho_a w_a}\frac{d(\rho_a w_a)}{dp_a}. \qquad (21)$$

From here $\frac{d^2w_{пр}}{dp_a^2}<0$ because multiplier $\frac{1}{\rho_a w_a}$ and $\frac{d(\rho_a w_a)}{dp_a}$ are more than zero whereas in a supercritical part of the nozzle the mass velocity and pressure in jet vary in one direction. Therefore the reduced velocity and consequently, and thrust reach maximum value at $p_a=p_H$. A similar nozzle regime name *design regime.* Nozzle regimes when $p_a>p_H$ or $p_a<p_H$ name *off-design regime.* The indicated conclusion can

be understood well if to consider thrust of the chamber as outcome of an operation of forces of pressure upon exterior and interior surfaces of the chamber.

Let there are three chambers which differ from each other only pressure upon a nozzle shear. On fig. 6 diagrammes of pressure profile of yields of combustion and ambient pressure for three nozzles operating on a supercritical part are presented (p_H = const for all three chambers).

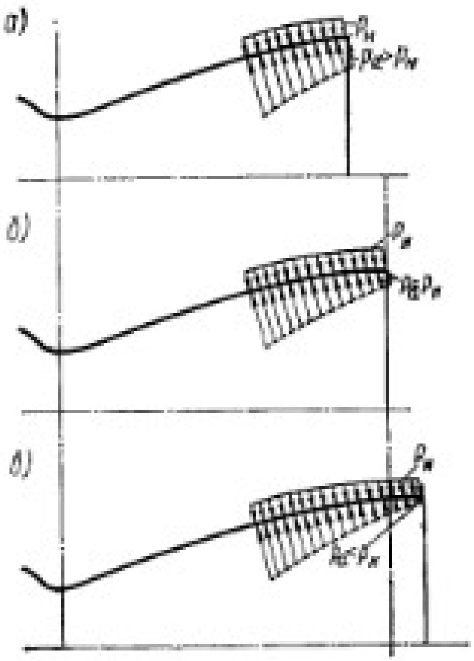


Fig. 6. Scheme of nozzle activities on conditions $p_a \triangleright p_H$, $p_a = p_H$, $p_a \triangleleft p_H$

On fig. 6a, the diagram of the nozzle with a gas underexpansion is presented, i.e. $p_a \triangleright p_H$. At lengthening of the nozzle to sizes, when in output nozzle section $p_a = p_H$ (Fig. 6. 6), thrust of the chamber is increased at value ΔP because pressure force of combustion nozzles yields operating on the oblong part, everywhere there are more than forces of ambient pressure.

In condition in the nozzle (fig. 6.) pressure of combustion products, since section where $p_a = p_H$ on walls everywhere there is less than ambient pressure. Therefore the projection to an axis x the forces of exterior pressure operating on a wall of the nozzle, will be more projections of the pressure forces operating on the same site of the nozzle from yields of combustion. As a result this part of the nozzle gives negative thrust. Thus, both on an underexpansion condition, and on a condition of reexpansion thrust of the chamber is less, than on a design regime.

For obtaining of the greatest value of thrust for the chamber with the set intrachamber parameters it is necessary to project the nozzle with a design regime of its activity. However most of chambers work on an off-design regime condition.

If it needs to design the nozzle with pressure upon a shear, rated for earth conditions the chamber would advance rated thrust only at start, i.e. at a surface of the Earth. Further, with rise on altitude, the chamber nozzle everywhere would work on a condition of an underexpansion and thrust of the chamber with the similar nozzle would be less in comparison with the nozzle at which it would be possible to execute a design regime at each altitude.

If we design the nozzle of the same drive with smaller pressure upon a shear, it would work in rated conditions only at one altitude where ambient pressure would be equaled to pressure upon a nozzle shear. Such nozzle to a design altitude would work in a reexpansion condition, and after a design altitude - in an underexpansion condition. The chamber with the similar nozzle will advance at all altitudes, except rated, thrust, smaller, than the chamber at which would be possible to execute at any altitude a design regime of activity of the nozzle. In this connection, it is important quantitatively to estimate losses in thrust on various off-design operational modes and to plan ways of their diminution.

Let's consider, as thrust with a modification of dimensionless square output nozzle sections (pressure upon a nozzle shear) varies at constant pressure to combustion chamber and environment.

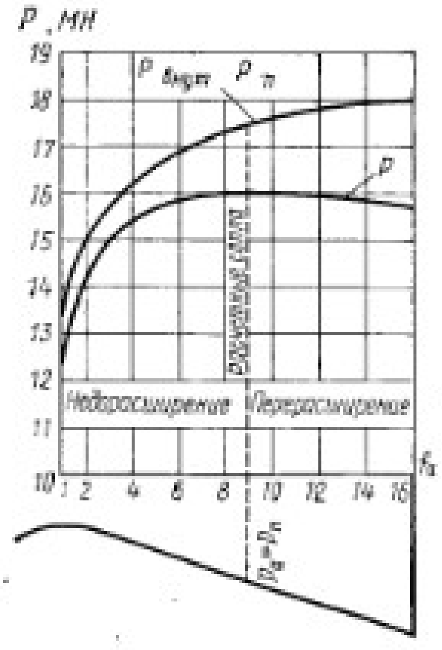


Fig. 7

On fig. 7 outcomes of calculations under formulas (10) and (13) are presented. Maximum value of thrust corresponds to a nozzle design regime when the nozzle geometry supplies expansion of gases to ambient pressure, i.e. $p_a = p_H$ ($\bar{f}_a = 9.0$; $p_K = 6$ MPa; k=1,15).

At reduction of the nozzle dimensionless square twice (a pressure buildup on a nozzle shear) losses in thrust in relation to a design regime are 4%, and at magnification twice thrust decreases for 0,5 %. Hence, by the nozzle activity with a loss underexpansion in thrust much more, than by activity of the nozzle with reexpansion, that it is easy to explain if to consider chamber's thrust as outcome of an operation of pressure forces upon the chamber exterior and interior surfaces.

The important conclusion can be made if to analyze pressure influence in combustion chamber on character of a thrust variation at a nozzle regime deviation from the design regime.

Than pressure in combustion chamber is more, thrust especially smoothly varies at a nozzle regime deviation from rated, and on the contrary. Hence, losses in thrust thus increase more intensively at chambers with smaller pressure the last speaks that with recompression in combustion chamber the share of thrust received from an exterior surface of the chamber, in comparison with the thrust received from an interior surface of the chamber, decreases. In a limit when combustion-chamber pressure going to infinity, value of its thrust going to value of a vacuum thrust, i.e. influence of static member $p_H F_a$ will be very small.

The large scoring in thrust would be given by the chamber if the dimensionless square of the nozzle at a pressure variation in the combustion chamber correspondes to a nozzle design regime. To make a design regime of the nozzle activity is very difficult. If for rockets engines with flight trajectories passes in a medium with pressure more than zero, basically such nozzles can be created, for the engines working in vacuum, to make an adjustable optimum nozzle it is impossible, as in it infinitely large output area sections that it is impossible to be reached.

4. SPECIFIC IMPULSE

The ration of the chamber thrust to mass flow of fuel is called *specific impulse of LPT thrust,* i.e.

$$I_s = \frac{P}{\dot{m}}. \qquad (22)$$

One of the important ways of LPT perfection and rocket as a whole is the raise of LPT specific impulse. We will consider the factors influencing specific impulse, and ways of its magnification. Having divided the left and right parts (10) on fuel mass flow $\dot{m}$ we will receive specific impulse

$$I_s = \frac{P_{внут}}{\dot{m}} - \frac{P_{вн}}{\dot{m}} = \left(w_a + \frac{p_a F_a}{\dot{m}} \right) - \frac{p_H F_a}{\dot{m}}. \qquad (23)$$

From (23) we see, that the specific impulse value depends on parameters of intrachamber process and ambient pressure. Maximum value of specific impulse at the set of the chamber parameters, as well as thrust, reaches at $p_H = 0$. Then the specific impulse is completely determined by intrachamber parameters:

$$I_{s\text{внут}} = \frac{P_{\text{внут}}}{\dot{m}} = w_a + \frac{p_a F_a}{\dot{m}}. \qquad (24)$$

In case of lack of a shock wave in the nozzle because of influence of an exterior back pressure the specific impulse received from an interior contour, is equal to specific impulse in vacuum, i.e.

$$I_{s\text{внут}} = I_{s\text{п}}$$

At a specific impulse raise equation the first member of (23), or the specific impulse from an interior contour of the chamber has great value.

Determinative of a specific impulse raise, received from an interior contour, the exhaust velocity raise is

$$w_a = \sqrt{2\frac{k}{k-1} R_K T_K \left[1 - \left(\frac{p_a}{p_K}\right)^{\frac{k-1}{k}}\right]},$$

which depends on a propulsive mass sort ($R_K T_K$ and k) and expansion ratios of gases in nozzle $\frac{p_a}{p_K}$. The more product $R_K T_K$ and expansion ratio of gases in the nozzle, the more is exhaust velocity.

Hence, for exhaust velocity raise, i.e. magnifications of specific impulse $I_{s\text{внут}}$, it is necessary to apply under all other equal conditions propellant with high value $\boxed{R_K T_K}$ and to increase expansion ratio of gases in the nozzle. For the set propellant it is possible only raise of expansion ratio of gases $\frac{p_a}{p_K}$ to pick up speed the expirations of gases from the nozzle. Last way of raise of specific impulse is widespread in practice.

To increase expansion ratio of gas in the nozzle it is possible by pressure decline on a nozzle shear, leaving constants combustion-chamber pressure, or by recompression in the combustion chamber, leaving constants pressure on a nozzle shear, or, at last, using both ways, considering assigning of the LPRE. On fig. 8 theoretical values of specific impulse on a design regime depending on expansion ratio of gases in nozzle $\frac{p_a}{p_K}$ for propellants oxygen-kerosene and a nitrogen acid-kerosene are presented .

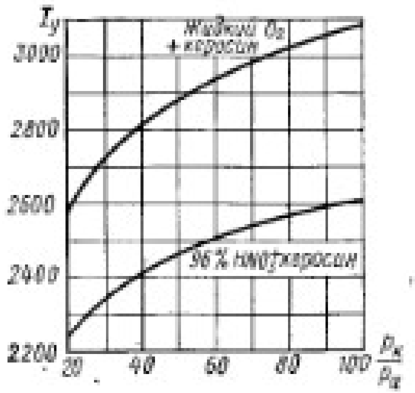


Figure 8

5. THE BASIC PARTS OF THRUSTS OF THE CHAMBER AND THE PLACE OF THEIR APPLICATION

Earlier rated relations for definition of the thrust created by all chamber have been given. Designer has to know what share of thrust is removed from this or that part of the chamber and what further capabilities of the thrust augmentation, received with 1 kg of fuel are available, i.e. specific impulse magnifications. It is necessary for an operational analysis of separate parts of the chamber, strength calculations, estimations of ways increasing of performances of chambers definitions of the optimal attachment point of the chamber. It is more convenient to present thrust how it is shown on fig. 9 for the analysis of components of thrust and a place of their application.

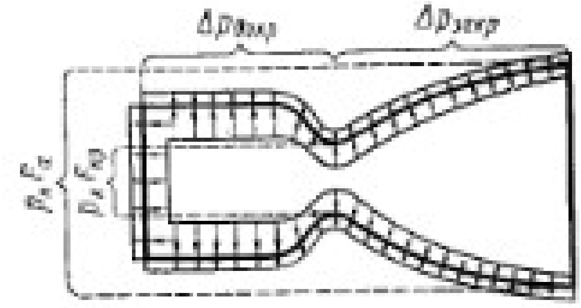


Fig.9. Site and components of thrust of the chamber

Let's carry out this analysis for ideal process in the chamber, considering, that the combustion chamber isobaric, i.e. combustion-chamber pressure is equal to stagnation pressure. As well as earlier, thrust of the chamber is determined by the equation (12). It is possible to present the first integral of the equation in a kind

$$P_{внут} = p_K F_{кр} + \Delta P_{докр} + \Delta P_{закр}, \qquad (25)$$

Where $p_K F_{кр}$ — the unbalanced force applied on a head of the combustion chamber; $\Delta P_{докр}$ - the force which has originated for the account mean pressure, operating on a ring element of head $(F_K - F_{кр})$, in comparison with the mean pressure operating on a subcritical part of the nozzle of the same square; $\Delta P_{закр}$ — the force operating on a supercritical part of the nozzle.

The second integral in (12) determines the force which has originated from ambient pressure on an exterior contour of the chamber, equal $p_H F_a$. Then thrust is

$$P = p_K F_{кр} + \Delta P_{докр} + \Delta P_{закр} - p_H F_a$$

For quantitative analysis of components of the thrust from an interior contour, it is expedient to convert the equation (15) as follows:

$$P_{внут} = \dot{m} w_a + p_a F_a = p_a F_a \left(1 + \frac{w_a \dot{m}}{p_a F_a} \right)$$

Further, using equalities

$$\dot{m} = \rho_a w_a F_a;$$

$$\frac{p_a}{\rho_a} = RT_a;$$

$$M_a^2 = \frac{w_a^2}{kRT_a}$$

where k— a special heat ratio, we will receive

$$P_{внут} = p_a F_a \left(1 + kM_a^2\right). \qquad (26)$$

Let's divide and multiply a right member of expression (26) on $p_К F_{кр}$, then

$$P_{внут} = \frac{p_a}{p_К} \frac{F_a}{F_{кр}} p_К F_{кр} \left(1 + kM_a^2\right) \qquad (27)$$

Let's inject concept about a *chamber thrust coefficient*

$$К_{тп} = К_{твнут} = \frac{P_{внут}}{p_К F_{кр}}, \qquad (28)$$

which determines, in how many time the thrust token from an interior surface of the chamber, is more than the thrust applied on a head of the chamber on the square, equal to a throat diameter.

Having divided the left and right sides of an equation (27) on $p_К F_{кр}$, we will receive

$$К_{твнут} = \frac{p_a}{p_К} \frac{F_a}{F_{кр}} \left(1 + kM_a^2\right) = \overline{F}_a \frac{1}{\varepsilon} \left(1 - M_a^2\right), \qquad (29)$$

where $\overline{F}_a$ - the dimensionless square of the nozzle; ε — expansion ratio of gases in the nozzle.

The formula (29) can be converted to convenient for the analysis and calculations view if all values entering in it to express or through number M_a, or through factor of velocity λ_a or through ration $\frac{p_a}{p_К}$. After simple transformations it is received:

$$К_{твнут} = \left(\frac{2}{k+1}\right)^{\frac{k-1}{2(k+1)}} \frac{1 + kM_a^2}{M_a^2 \sqrt{1 + \frac{k-1}{2} M_a^2}}; \qquad (30)$$

or

$$K_{\text{твнут}} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \frac{\lambda_a^2 + 1}{\lambda_a}; \qquad (31)$$

$$K_{\text{твнут}} = 2\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \frac{k}{\sqrt{k^2-1}} \sqrt{1-\left(\frac{p_a}{p_K}\right)^{\frac{k-1}{k}}} \left[1 + \frac{k-1}{2k} \frac{\left(\frac{p_a}{p_K}\right)^{\frac{k-1}{k}}}{1-\left(\frac{p_a}{p_K}\right)^{\frac{k-1}{k}}}\right]. \qquad (32)$$

Let's conduct quantitative analysis of components of thrust. From (30) — (32) it is visible, that the thrust coefficient depends on extent and an index of process of expansion. The thrust coefficient increases with magnification of expansion ratio of gas in the nozzle and reduction of an index of process of expansion.

Having divided all terms (25) on $p_K F_{\text{кр}}$, we will discover

$$K_{\text{твнут}} = K_{\text{твнут1}} + K_{\text{твнут2}} + K_{\text{твнут3}}, \qquad (33)$$

where

$$K_{\text{твнут1}} = \frac{p_K F_{\text{кр}}}{p_K F_{\text{кр}}} = 1;$$

$$K_{\text{твнут2}} = \frac{\Delta P_{\text{докр}}}{p_K F_{\text{кр}}};$$

$$K_{\text{твнут3}} = \frac{\Delta P_{\text{закр}}}{p_K F_{\text{кр}}}.$$

It allows to us to size up a role of separate elements of the chamber in thrust creation.

We will observe the chamber without a supercritical part of the nozzle for definition of a share of the thrust token from a subcritical part of the nozzle. Having assumed $M_a = 1$ in (30) expression $\lambda_a = 1$ or in (31), we will receive a thrust coefficient of the chamber without for a critical part of the nozzle:

$$K_{\text{твнут}\lambda=1} = 2\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}. \qquad (34)$$

In this case $K_{\text{твнут}\lambda=1} = K_{\text{твнут1}} + K_{\text{твнут2}}$.

Thrust coefficient $K_{твнут\lambda=1}$ depends only on an index of process of expansion, as in the presence of a subcritical part of the nozzle the combustion-chamber pressure ration to pressure in critical section is a constant

Thrust coefficient $K_{твнут\lambda=1}$ has following values at various k:

k	1,10	1,15	1,20	1,25	1,30
$K_{твнут\lambda=1}$	1,206	1,224	1,230	1,250	1,260

Hence, the part of thrust of a subcritical part of the nozzle makes 20 – 26 % from thrust $p_K F_{кр}$ or $K_{твнут2} = 0{,}2 \div 0{,}26$. The thrust coefficient of a supercritical part of the nozzle

$$K_{твнут3} = K_{твнут} - K_{твнут\lambda=1}. \qquad (35)$$

The share of the thrust token from a supercritical part of the nozzle, depends, as it is visible from (30) — (32), from expansion ratio of gases in the nozzle and is increased with growth of the last. The share the thrust, removed from a supercritical part of the nozzle, makes 0,25 — 0.55 at changing $\frac{p_K}{p_a}$ from 10 to 100 and k = 1,15.

From the resulted data it is visible, that the share of thrust of the nozzle attains 80% from main part of thrust, i.e. a nozzle role in thrust and specific impulse creation is exclusively great. A part of thrust $p_K F_{кр}$ has received a title of main part of thrust of the chamber in those days when expansion ratio of gases in the nozzle was small and the nozzle role in creation of thrust of the chamber was considerably smaller, than part $p_K F_{кр}$.

It is necessary to underline, that a share of thrust which is created by nozzles, will continuously increase, especially for the LPRE of high-altitude stages of multistage missiles and satellites. Rocketry development is connected with continuous growth of specific impulse of a rocket engine. One of effective ways of growth of specific impulse is raise of extent expansion of gases in the nozzle. Now there are chambers of the LPRE, at which expansion ratio of gases 500 ÷ 4000. Therefore the share of the thrust token from the nozzle continuously grows and can exceed value of the main part of thrust. Naturally, more and more attention it will be given to perfection of the nozzle end of the chamber.

Therefore it is necessary to know, what share of thrust in a limit can be given by the chamber nozzle as a whole and by a supercritical part separately. For this purpose, having assumed in the formula (32) $\frac{p_a}{p_K} \to 0$, we will receive the maximum value of a thrust coefficient

$$K_{твнут\,max} = 2\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \frac{k}{\sqrt{k^2-1}} . \qquad (36)$$

The maximum value of the thrust coefficient, token from a supercritical part of the nozzle,

$$K_{твнут3} = K_{твнут\,max} - K_{твнут\lambda=1} . \qquad (37)$$

Outcomes of calculations at various k are resulted below:

k	1,10	1,15	1,20	1,25	1,30
$K_{твнут}$	2,88	2,45	2,23	2,08	1,98
$(K_{твнут} - K_{твнут1})_{max}$	1,88	1,45	1,23	1,08	0,98
$K_{твнут3\,max}$	1,67	1,23	1,00	0,83	0,72

As it is visible from the resulted outcomes, the part of the thrust received from the nozzle of the chamber $(K_{твнут} - K_{твнут1})_{max}$ - at $\frac{p_a}{p_K} \to 0$ and k = 1,15 (the closest for modern fuels an index of process of expansion), exceeds the basic component of thrust in one and a half time. It is necessary to pay attention, that the intensive increase of a share of the thrust removed from a supercritical part of the nozzle, lies in engineeringly realizable limits of expansion ratio (an order of several thousand units). Therefore the share of the thrust removed from the nozzle, will exceed the basic component of thrust